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From "Understanding" to "Learning Independently": Cultivating Autonomous Learning Skills in Theory of Computation Course

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Abstract: Theory of Computation is a foundational course that requires students to understand formal mathematical definitions and apply logical reasoning independently. However, understanding a teacher's explanation does not necessarily mean that students can interpret mathematical expressions or solve problems without assistance. This paper examines the cultivation of autonomous learning skills in a Theory of Computation course, focusing on the relationship between knowledge understanding and independent learning. The paper discusses the importance of distinguishing between understanding knowledge through instruction and developing the ability to understand knowledge independently. The paper then presents a layered approach to interpreting mathematical definitions, connecting formal expressions with written explanations and accessible oral interpretations. The approach aims to reduce initial barriers to understanding while helping students gradually return to formal definitions and interpret them independently. The paper argues that effective instruction should not only make knowledge understandable but also help students develop the capacity to understand and apply knowledge independently.

Keywords: Theory of Computation, Autonomous Learning, Mathematical Definitions, Layered Interpretation, Instructional Design.

1. Introduction

Theory of Computation is an important foundational course for students in computer science and artificial intelligence. Its content involves formal definitions, mathematical notation, logical reasoning, and theoretical methods. Unlike learning activities that rely primarily on memorization or the accumulation of factual knowledge, studying Theory of Computation requires students to understand concepts precisely, interpret formal definitions, and apply appropriate rules to derive conclusions. Students must not only know what a definition or theorem states, but also understand why it is formulated in a particular way, what it means, and how it can be used to solve problems.

In classroom practice, students frequently encounter a problem when studying mathematical subjects: understanding a teacher's explanation does not necessarily mean that they can independently complete the corresponding learning tasks. When a teacher explains an abstract mathematical definition in accessible language, students may feel that they have mastered the concept. When a teacher demonstrates a derivation step by step, students may find the reasoning straightforward. However, when they encounter a similar problem without the teacher's guidance, they may struggle to interpret the original definition or identify an appropriate reasoning strategy [1].

This phenomenon highlights an important distinction between understanding knowledge through instruction and developing the ability to understand and apply knowledge independently. Classroom teaching should therefore aim not only to make mathematical concepts understandable but also to help students develop the ability to analyze problems, interpret formal expressions, and carry out logical reasoning on their own [2].

Drawing on the instructional design of a Theory of Computation course, this paper examines how teaching can extend beyond knowledge transmission to the cultivation of autonomous learning skills. It first discusses the distinction between understanding knowledge through teacher explanations and developing the ability to understand knowledge independently. It then focuses on the layered interpretation of mathematical definitions, exploring how formal expressions, written explanations, and oral interpretations can be connected to support students' understanding. The central concern is how teachers can provide appropriate assistance while gradually enabling students to interpret mathematical definitions and apply their knowledge independently.

2. From Knowledge Transmission to Competency Development

2.1. Understanding Does Not Necessarily Mean Mastery

In mathematics education, teacher explanations play an important role in helping students understand abstract concepts and follow logical arguments. However, understanding a reasoning process that has already been completed by a teacher differs fundamentally from being able to construct that reasoning independently [3].

When explaining a mathematical definition, a teacher has usually completed the necessary analysis and organization in advance. When demonstrating a derivation, the teacher has already selected an appropriate starting point, identified the relevant rules, and organized the reasoning process. Students need only follow the established line of thought. Although this process can facilitate comprehension, it does not automatically develop their ability to analyze problems independently.

For example, after hearing an accessible explanation of an abstract mathematical definition, students may understand its intuitive meaning. Nevertheless, they may still have difficulty

interpreting the original formal definition without assistance, particularly when they need to explain the significance of each condition. Similarly, students may successfully follow a derivation in class but struggle to determine the starting point, select appropriate inference rules, or complete the reasoning independently.

Teachers therefore need to distinguish between two instructional objectives: helping students understand knowledge and developing their ability to understand knowledge independently. The former is an essential foundation for the latter, but it cannot replace it.

Teaching Theory of Computation should consequently address not only students' mastery of concepts, definitions, and reasoning procedures, but also their ability to read mathematical expressions independently, analyze problem conditions, select appropriate rules, and complete theoretical tasks without continuous guidance.

2.2. From Learning Outcomes to Learning Processes

Traditional knowledge-centered instruction tends to focus on the content being taught: which definitions have been explained, which difficult concepts have been clarified, which examples have been demonstrated, and whether students have remembered the relevant conclusions. These concerns are important, but the process through which students develop understanding deserves equal attention.

To understand a mathematical definition, students must do more than recognize its wording. They need to analyze its symbols, distinguish its conditions, understand the relationships among its components, and construct an integrated interpretation. Similarly, mastering a reasoning method requires more than remembering a sequence of rules. Students need opportunities to attempt a solution, encounter difficulties, reconsider their approach, and verify their conclusions.

If teachers eliminate every difficulty in advance and provide every reasoning step directly, students may experience a smooth learning process while losing valuable opportunities for independent thought and exploration.

Instructional design should therefore establish a productive balance between teacher explanation and student reasoning. Teachers should provide sufficient support to prevent students from remaining trapped in confusion. At the same time, they should preserve opportunities for students to analyze problems and attempt solutions independently, enabling them to learn not only the subject matter but also the methods required to acquire knowledge.

2.3. Autonomous Learning as an Important Instructional Objective

Autonomous learning does not mean that students must learn entirely without teachers, nor does it simply mean assigning more independent study. Rather, it involves developing the ability to identify questions, interpret concepts, organize knowledge, evaluate judgments, and solve problems actively.

In a mathematical foundation course such as Theory of Computation, autonomous learning can be reflected in several abilities: students should gradually become capable of reading mathematical definitions independently rather than relying on teachers to explain every expression; they should attempt to analyze problems before instruction instead of immediately waiting for teachers to provide solutions; they should use exercises to identify gaps in their understanding and address those gaps through review and further study; they should develop personalized notes that integrate classroom explanations, individual reflections, derivations, and unresolved questions; they should be able to examine explanations and established conclusions critically, raise questions when necessary, and attempt to justify their judgments.

These abilities cannot be developed simply by increasing the amount of content delivered in class. They require sustained attention throughout the teaching process. Accordingly, this paper treats autonomous learning as a central instructional objective and organizes its teaching strategies around that goal.

3. Lowering the Barrier to Understanding: Layered Interpretation of Mathematical Definitions

3.1. Helping Students Approach Abstract Mathematical Definitions

Mathematical definitions are characterized by precision, conciseness, and formal structure. Through mathematical notation and rigorous logical relationships, definitions specify concepts clearly and reduce the ambiguity associated with natural language. However, for students who are not yet familiar with the relevant notation and logical structures, formal expressions can also become obstacles to understanding [4].

When teaching Theory of Computation, instructors need to do more than explain the apparent meaning of a definition. They should help students understand why it is formulated in a particular way, how its conditions relate to one another, and what role it plays in solving problems.

To address these challenges, the instructional design adopts a layered approach to explanation, connecting formal mathematical definitions with written interpretations and more accessible oral explanations. Students are guided to examine the original mathematical definition and become familiar with its formal representation. They consider the accompanying written explanation, which helps them grasp the basic meaning. When certain aspects remain difficult to understand, the teacher provides a more direct explanation in everyday language, helping students establish connections between mathematical symbols and their intuitive meanings [5].

This approach is not simply a repetition of the textbook. It provides different levels of assistance in response to different levels of difficulty.

3.2. From Mathematical Language to Natural Language

In classroom instruction, the interpretation of mathematical definitions can be compared to the annotation and further explanation of classical texts. Classical texts often express ideas concisely and precisely, yet later readers may struggle to understand them. Annotations can help clarify the original text. If the annotations themselves remain difficult to understand, further explanations may be necessary.

A similar process applies to mathematical definitions. The formal definition provides the original expression, the written explanation offers an initial interpretation, and the teacher's

oral explanation provides further clarification based on students' actual learning difficulties.

This analogy helps students understand the relationship among different levels of explanation. The mathematical definition remains the formal expression to be mastered, while written and oral interpretations serve as tools for understanding it.

In practice, the teacher focuses particularly on the meanings of abstract symbols and attempts to translate expressions that students find difficult into more accessible language. This allows students to develop an intuitive understanding before examining the precise conditions and logical relationships embodied in the formal definition.

However, natural-language explanations cannot fully replace mathematical definitions. Although natural language is accessible, it may be ambiguous and may fail to preserve all the precise conditions required by a formal definition. If students remember only the teacher's informal explanation without mastering the original definition, they may still misunderstand the concept when solving problems independently.

The purpose of layered interpretation is therefore not to weaken mathematical rigor, but to build a bridge that enables students to understand rigorous mathematical expressions.

3.3. From Reliance on Explanation to Independent Interpretation

The most important aspect of layered explanation is not how effectively teachers translate mathematical definitions into everyday language. It is whether students can eventually return to the original definitions and understand them independently.

The instructional design explicitly emphasizes that students should not remain permanently dependent on the teacher's oral explanations. Natural-language interpretation is a means of facilitating understanding, while independent comprehension of formal mathematical definitions remains the ultimate objective.

This principle reflects an important function of instructional scaffolding: support should enable students to accomplish tasks that are initially difficult for them, but it should not become a permanent substitute for their own abilities.

At the beginning of learning, students may need detailed explanations of mathematical symbols and assistance in understanding definitions through natural language. As learning progresses, they should gradually learn to analyze the conditions in a definition, understand the logical relationships among their components, and extract the required information directly from formal expressions.

From this perspective, successful teaching is demonstrated not only when students understand an explanation in the classroom, but also when they can approach similar problems in the future with less assistance.

Layered interpretation and autonomous learning are therefore not separate instructional objectives. The former provides an entry point into mathematical knowledge, while the latter requires students to develop the ability to understand that knowledge independently.

4. Conclusion

Teaching Theory of Computation should aim not only to help students understand essential mathematical concepts, but also to develop their ability to interpret formal definitions and apply logical reasoning independently. Understanding a teacher's explanation is an important step in learning, but it does not necessarily indicate that students can complete similar tasks without assistance.

To address this challenge, this paper emphasizes the distinction between understanding knowledge through instruction and developing the ability to understand knowledge independently. It also discusses a layered approach to interpreting mathematical definitions, connecting formal expressions with written explanations and accessible oral interpretations. Such an approach can help students overcome initial difficulties while preserving the precision and rigor of mathematical definitions.

The ultimate purpose of instructional support is not to make students permanently dependent on teachers' explanations, but to help them gradually develop the ability to understand mathematical expressions and solve problems on their own. Teachers should therefore consider not only whether students understand the material presented in class, but also whether they can apply what they have learned independently.

The transition from understanding through explanation to independent understanding is an important objective of teaching mathematical foundation courses. It shifts the focus of instruction from the successful delivery of knowledge to the development of students' capacity for continued learning and independent problem-solving.

References

[1] Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory Into Practice*, 41, 64–70. https://doi.org/10.1080/00405840209543995

[2] Wu, H. L., & Leng, F. L. (2025). Instructional prelude of theory of computation: The history of algebra. *International Journal of Education and Humanities*, 21, 67–69.

[3] Edwards, B. S. (2004). Surprises from mathematics education research: Student (mis)use of mathematical definitions. *The American Mathematical Monthly*, 111, 411–424. https://doi.org/10.1080/00029890.2004.11920090

[4] Torkildsen, H. A. (2023). Toward a unified account of definitions in mathematics education research: A systematic literature review. *International Journal of Mathematical Education in Science and Technology*, 56, 1–28. https://doi.org/10.1080/0020739X.2023.2200248

[5] Jiang, W. S. (2023). *Mathematical elements*. Tsinghua University Press.