Relations Of Art and Mathematics in The Renaissance: The Application of Mathematics in Works of Leonardo Da Vinci

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Abstract. There is a close relationship between art and science in the Renaissance. Especially the relationship between art and mathematics. The relationship between mathematics and art has been discussed many times in the past, but there is barely a summary of the application of various mathematical methods in works of art. This paper will focus on Leonardo da Vinci's Mona Lisa, Vitruvian Man, and Last Supper to discuss the usage of the golden spiral, golden ratio, linear perspective, aerial perspective, and geometric figures in works in the Renaissance. Through the connection between these works and mathematical methods, this paper finds that mathematics is widely embodied in Renaissance artworks, and Renaissance artists can apply mathematics to their works to present a more perfect composition and the ratio of figures. The presentation of the deep meaning of artworks can also be presented to the viewer through mathematical language. Moreover, this method of applying mathematics in works of art is also widely used by artists of modern art, among which abstractionism and cubism use mathematical methods the most. In view of this result, the author believes that mathematics and art have always influenced each other in the past history, whether in form or in thoughts.

Keywords: Mathematics; Leonardo da Vinci; Renaissance; art; science.

1. Introduction

During the Renaissance, there was a close relationship between mathematics and science [1]. Anatomy, engineering, mathematics, and other sciences are all related to artistic works and methods used in artistic creation [1]. This paper will focus on the mathematical application in art. This is because during the Renaissance, art had a great influence on the development of mathematics, and mathematics was also a breakthrough for artists in artistic expression methods [2]. In the past research and papers, the public has explored the subsequent influence of different scientific fields in art. Former researchers have dabbled in perspectives, the golden ratio, and other artistic expression methods based on mathematical common methods. In addition to the application of artistic expression methods in works of art, the appearance of Mathematical formulas and graphs behind these methods are also explained in detail. However, most of these exploratory papers only focus on the effect of one mathematical method on artistic expression. There are not many authors who would sum up three or more mathematical techniques with Renaissance works of art all in one paper. The author of this paper believes that summarizing the most common connections between mathematics and works of art in the Renaissance in one article will have significance in the research results summary. Meanwhile, academics who want to know the application of mathematics in works of art can quickly understand the relationship between them. Because it is too wide to explore the relationship between Renaissance mathematics and the different works of different artists, the author will only focus on the mathematical methods present in Leonardo da Vinci's works. This paper will start with the development of art and mathematics in the Renaissance and the life of Leonardo da Vinci. Then, it will discuss the influence of mathematics methods, which are golden spiral, golden ratio, linear perspective, aerial perspective, and Platonic solids, on Leonardo's techniques through four works: Mona Lisa, Vitruvian Man, Last Supper and the Skeletal Version of the Icosidodecabedron. At the end of this paper, the author will summarize the influence of mathematics on Renaissance art and later art. With this idea, this paper is able to present the application of mathematics in Leonardo da Vinci's works and the overall influence of mathematics on art in the Renaissance and later periods.

2. Background

The Renaissance took place from the 14th century to the 16th century, and it was a European ideological and cultural movement [1]. Works of art declined in the centuries after the Greek and Roman periods, before the Renaissance [1]. Therefore, when art rises again, people call it the "Renaissance". This artistic revolution began in Milan and Venice and then spread to European countries [1]. Although there was still no such concept of art during the Renaissance, artists were not just the craftsmen of court and chapel [2]. They became the symbol of aesthetics, recording environment, still life, and beauty. Renaissance became one of the most important ideological emancipation movements in European history, promoting European culture's development in the later period [3]. At the same time, the Renaissance prepared people for the bourgeois revolution [3].

Moreover, mathematics made great progress in the Renaissance under the impetus of schools of art and some of the artists. Influenced by the perspective invented by Filippo Brunelleschi, many artists began to apply perspective to their works. The golden ratio is also widely used in works, which makes the ratio of a picture more harmonious. In addition, there are some other facades, cubes, and equation solutions that are inextricably linked with painters and architects in the Renaissance [4]. Nowadays, when the public mentions the representative painters of the Renaissance, Leonardo da Vinci, Michelangelo, and Raphael will be the first to come to mind [3]. Among them, is Leonardo da Vinci, who has an in-depth study of mathematics and is even a natural scientist and engineer himself [5]. As a result, During the Renaissance, mathematics played an important role in the development of art. There was an interactive relationship between art and mathematics.

3. Artist Introduction

Leonardo da Vinci was born in Vinci town in 1452 [5]. He became a pupil in Verrocchio Studio and showed his artistic talent in 1470 [5]. After becoming a painter, he was funded by the House of Medici and was active in Milan, Rome, Florence, and other places in his later period [5]. In his career, he perfectly combined painting skills with scientific knowledge [5]. He added many skills about human anatomy, perspective, light and shade, composition, and so on based on his scientific knowledge. He made important discoveries in military, civil, water conservancy, and mechanical engineering [6]. Thus, his life has influenced the future development of art, geology, physics, and biology [5]. In his later years, he moved to France and died in France in 1519 [5]. In the aspect of painting theory and research, it can be seen from the Notebook of Leonardo and his painting theory that he had discussed the effect of applying perspective, light, and shadow to the human body's dynamic expression and natural scene in painting. During the Renaissance, he still advocated that painting was a noble art, even higher than other arts [6]. He believed that painting is a science and a reflection of reality [6]. He invented aerial perspective based on the study of linear perspective. At the same time, he believed that human anatomy could truly restore human form in paintings [5]. When he painted the Mona Lisa, he also used the sfumato for the first time, which created a new way of coloring oil painting. Although Leonardo left no more than 20 completed works, he still has more than 7,000 pages of manuscript notes and unfinished paintings [5]. Among them, his oil painting masterpieces are the Mona Lisa, the Virgin of the Rock, the Lady with an Ermine, and so on. His most representative fresco is the well-known Last Supper, which was painted in Santa Maira Delle Grazie, a church in Milan. Besides, Leonardo always thought that mathematics was a beautiful language. There is a quote from Leonardo:

“The merit of painting lies in the exactness of reproduction. Painting is a science, and all sciences are based on mathematics. No human inquiry can be a science unless it pursues its path through mathematical exposition and demonstration [7].”

He used to design pictures and compositions based on geometric figures. He worked with Luca Pacioli on arithmetic integration, and Leonardo painted many illustrations for Pacioli’s book [8]. He also used Plato's and Euclid's works to make himself more mature in geometric perspective skills [9]. He has had a great interest in the golden ratio all his life and used the golden spiral in the Mona Lisa.

In short, Leonardo used scientific methods to develop his art to the peak. He went down in history and enlightened the newcomers.

4. Mathematical Laws in Leonardo's Work

The golden spiral (1:1.618) is a spiral based on the Fibonacci series, also a type of golden ratio [10]. Generally speaking, the golden spiral is to locate the golden ratio between 3 points and form a spiral subsequently. Despite the fact that the golden spiral is a part of the golden ratio in arithmetic, it is widely used in works of art. Mona Lisa is Leonardo's most skillful painting in the utilization of the golden spiral. The spiral started from her nose, went through her chin, crossed the top of her head, and ended in her right wrist. What's more, the distance between her eyebrows, chin, and the neckline of her garment, the distance between the right side and left side of her face, and the proper edge of the panel, and the distance between the top of the panel, her hairline and chin can prove it [10]. There is no evidence to prove that Leonardo intentionally used the golden spiral to create this painting. However, it shows the high diploma of painting skills Leonardo has integrated the golden ratio into his artwork. In this painting, Leonardo used a pyramid shape to compose the scene. The overall posture of the figure forms an isosceles triangle from the top of her head to both sides of her body. Her hands had been folded in front of her. Even though her body leans slightly to the viewer's left, the face is facing the viewer. This pyramid composition and her sitting posture make viewers suppose that she appears very calm, consistent, and gentle. Moreover, the pyramid shape has constantly been taken into consideration by means of the general public to be very similar to the ratio of the golden section. Leonardo also used aerial perspective in the Mona Lisa. When observing the background in the Mona Lisa, it can be seen that with the change of background distance, the color becomes lighter, and the tone becomes bluer. When the background becomes blurred, it highlights the main figure. There are also different perspectives on the different horizontal lines on both sides of the Mona Lisa. This leads the audience to feel that its position has changed because of different perspectives when they appreciate this painting. This adds depth and mystery to the painting. Furthermore, shadows and light also make the effect of aerial perspective more obvious and the figure more vivid.

Regarding the golden ratio, it is worth mentioning that Leonardo studied the human body of men in Vitruvian Man. In this sketch, he found the connection between the human body and the golden ratio [11]. When a person takes his feet and fingers as the ends of points and extends them to the most open position, his hands, and feet swing up and down from his head and toes respectively [12]. The swing of his limbs can be connected into a circle shape. When his arms are placed flat, his feet are straight, and his head, feet, and fingers are regarded as the ends of the point, the limbs of the person will form a square shape [12]. This is the embodiment of the statement that the human body itself has the perfect proportion.

In The Last Supper, Leonardo used aerial perspective and linear perspective. This painting uses the traditional line-by-line composition convention. Linear perspective is a perspective method based on the knowledge of optical and geometric principles by using the triangular system between the eyes and the observed object. In general, the painting itself is flat. Nevertheless, by moving different figures on it, different orthogonal and vanishing points will appear [13]. Jesus is in the center of the audience's line of sight, and the vanishing point of one-point perspective gathers on Jesus' head [13]. The usage of linear perspective can also be seen on the floor and ceiling in the painting. Setting the vanishing point on top of Jesus can highlight the theme of Jesus and make the audience notice that he is the most important figure at first sight [14]. Additionally, the lines from Jesus' head to shoulder to his hands form a triangle or a pyramid shape. The top of this pyramid, which refers to the head of Jesus, is displayed in the middle window of the three windows behind Jesus. Leonardo used focus perspective to focus all the audience's eyes on Jesus, and the walls on both sides extended backward to further strengthen and highlight the most significant focus point of the whole painting [14]. The blue sky and white clouds that can be seen outside the window also use aerial perspective to enhance the depth of the picture [14].

In the "De Divina proportione" written by Pacioli, Leonardo drew sketches for Platonic solids in the book [9]. One of them is the Skeletal Version of the Icosidodecabedron [9]. Geometry is continuous for Leonardo, and he studied geometric entities. When he discovered the continuity of octahedron, dodecahedron, icosahedron, and Icosidodecabedron, it aroused Leonardo's interest [15]. Because when these polyhedrons are regularly cut off, a new regular polyhedron will be formed [15]. Leonardo's in-depth study of Platonic solids and sketching highlighted his passion for mathematics and geometry. His polyhedron with many details and vivid three-dimensional appearance also gave mathematicians a template to analyze, study, and draw. This polyhedral pattern was later widely used in decorative patterns of floors, ceilings, and houses.

5. The Relationship between Art and Mathematics

5.1. Correlation

During the Renaissance, more artists awakened and regarded artists as a noble profession [2]. They studied the perspective and golden ratio in mathematics in detail. This shows the influence of mathematics. Artists began to think about how to draw the three-dimensional world on two-dimensional panels through the combination of mathematical skills and artistic theory [14]. Artists in the Renaissance were influenced by Greek philosophy and thought that everything could be represented geometrically [3]. As a result, artists of the Renaissance, especially architects and engineers, studied mathematics. Some of these artists can even be called mathematicians. In addition to the common linear perspective, aerial perspective, and other perspectives, some artists also use more unusual perspectives through mathematical means. In The Ambassador of Hans Holbein, the Younger, there is an oblique figure directly below the picture. This figure is a skull, but the audience can't see it until they look at the picture from the right. This special technique is called “anamorphose” [12]. This technology was very popular during the Renaissance [15]. This visual illusion used a second perspective [16]. The two main figures in the painting are the perspective seen by the audience when they watch the painting. And this skull is the perspective of the audience looking at the painting from the right side [12]. This is an application of projective geometry. It leads to the feeling that when the audience looks at the painting from each side, the skull's eyes are fixed on them. It creates a creepy atmosphere for the audience. In the late Renaissance, more mathematical methods and artistic methods were merged by artists and applied to artworks. It is precisely because Renaissance artists had this thought of using mathematical language and ideas to change the expression form and deep meaning of art, there were more unconventional and artistic movements appeared in the later periods [3].

5.2. Profound Influence

In modern art, there are artistic movements such as cubism and surrealism. These all use mathematical methods and ideas. Cubism uses the concepts of geometric body and polyhedron. Surrealism uses the spirit of connecting illusion with reality [14]. Non-Euclidean geometry, four-dimensional space, and other mathematical terms also have been used in modern art [15]. For example, abstractionism is influenced by four-dimensional space, and the application of the golden ratio in Mondrian's Composition with Red, Yellow, and Blue. The appearance of four-dimensional space also brings infinite prospects to the future of the art field [16]. During the Renaissance, artists brought a three-dimensional world into two-dimensional space through the mathematical method of perspective [15]. Is it possible to break through the four-dimensional space into the three-dimensional space through updated mathematical methods in the future? The representative of surrealism, Salvador Dali was invited to paint the Crucifixion, in which the cross shows a four-dimensional facade that appears in three-dimensional space [17]. This makes the picture look unusually stereoscopic. To recapitulate, from form to thought, mathematics, and art have had a profound influence on each other all the time.

6. Conclusion

Through discussion and summary, this paper finds that Leonardo da Vinci's application of mathematics in his works of art is very common and extensive, and he invented perspective to present his works. This proves that Leonardo is very proficient in mathematics and artistic skills and can expand the technique presentation through mathematical methods. This is not only the ability that Leonardo had, but also the artists of the Renaissance were all qualified mathematicians. They can combine the knowledge they have learned in the field of mathematics with mature artistic painting techniques to present works that are more accurate, more aesthetic, and people still marvel at. Through the use of the golden ratio, perspective, and geometry in Leonardo da Vinci's works, people can also know that artists in the Renaissance used mathematical language and ideas to change the expression of art, and even applied it to the expression of deeper meanings of works of art. This shows the deeper connection between mathematics and art. At the same time, this influence has been extended to modern art. Many artistic movements in the later period adopted mathematical methods and ideas to present works of art with different media and meanings. These theories have been discussed, but they have not been combined and summarized their relationship and influence on later generations. Combining these findings, academics who are interested in the relationship between art and mathematics, especially those interested in the Renaissance, can learn more about the mutual relationship between them through this paper. Academics who want to learn the application of any mathematical method in Leonardo da Vinci's works, such as the golden ratio, perspective, and geometry, can find useful information in this paper and use it for new research and study. People who only want to know the influence of mathematics on modern art can also get some simple concepts from this paper. This paper only discusses the application of mathematics in Leonardo da Vinci's works, which is limited and may not be representative enough for some artists of the same period. When other researchers continue to further study the relationship between mathematics and art, the application of mathematics in other Renaissance painters or painters of modern art can be studied deeply. There are still many gaps in this field that can be filled.

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