

Ecosystem Assessment Model Based on Multi-Population Complex System Dynamics and Simulated Annealing Algorithm

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Abstract. This study explores the unique phenomenon of sex ratio variability in sea lampreys and its ecological significance. Through extensive research on the life cycles and habits of sea lampreys, we developed three distinct mathematical models to analyze the impact of this adaptability on population dynamics and ecosystem stability. The Lotka-Volterra Difference Model revealed that a variable sex ratio can enhance ecosystem resilience by reducing the amplitude of periodic fluctuations and delaying system collapse by 75%, although it accelerates collapse under extreme resource scarcity. The Food Web Model further confirmed the positive effects of sex ratio variability on ecosystem resilience and stability. Conversely, the Multi-population Complex System Dynamics Model, in conjunction with a sensitivity analysis, emphasized the delicate balance between early competition advantages and long-term competitiveness challenges. Overall, our findings elucidate the complex role of sex ratio variability in sea lampreys, offering insights into its ecological importance and potential management implications for maintaining biodiversity and ecosystem health.

Keywords: Lamprey; Lotka-Volterra Model; Difference Model; System Dynamics Model; Monte Carlo Method; Simulated Annealing Algorithm.

1. Introduction

This study delves into the ecological and evolutionary phenomenon of sex ratio variability in sea lampreys, a feature that distinguishes them from many other species. By developing and applying four sophisticated mathematical models, we aim to unravel the complex implications of this trait for lamprey population dynamics and broader ecosystem stability.

Our investigation is motivated by the need to better understand and manage the ecological impacts of sea lampreys, especially given their significant role as invasive predators in the Great Lakes, where they have caused substantial ecological disruption and economic damage. Despite efforts to control their population through various means, the challenge persists [1], underscoring the importance of innovative approaches to managing their impact. This research contributes to the field by offering new insights into the adaptive mechanisms of sea lampreys and their ecological consequences, potentially informing more effective strategies for biodiversity conservation and ecosystem management. Through a combination of theoretical analysis and empirical data, we explore the nuanced effects of sex ratio variability on ecological resilience, population stability, and competitive dynamics, highlighting its potential both as a mechanism for adaptation and a factor in population vulnerability.

2. Lotka-Volterra Difference Model

2.1. Predator (Sea Lamprey) - Prey (Plankton, etc.) Model

2.1.1 Model of unchanged sex ratio capacity

We hypothesize that in parallel time and space, there is a population of sea lampreys that is otherwise identical in terms of sex and inability to change in response to the environment, which we refer to here as the sea lampreys-1 population.

Establishment of the Lotka-Volterra model, which is the most basic of ecological studies, to discuss and examine the relationship between the predator sea lampreys-1 and the food source food of sea lampreys, such as the predator (zooplankton).

Since sea lampreys are migratory fish, the most significant feature of the reproduction of these organisms, which should not be ignored, is that the population spawns and reproduces at a fixed time of the year (and after spawning, both parents die) [3,4]. We should adopt a Discrete Model with a one-year time period N .

The Lotka-Volterra Difference Model is established as follows (1):

$$\begin{cases} N_{l1}(k) = (r_1 FM_1(k-1) + \alpha N_{f1}(k-1))N_{l1}(k-1) \\ N_{f1}(k) = (r_2 - \beta N_{l1}(k-1))N_{f1}(k-1) \\ FM_1(k) \equiv 0.44 \end{cases} \quad (1)$$

In general, a stable ecosystem has a natural growth rate $r_1 FM < 1$ for the number of predators lamprey-1 $N_{l1}(k)$, which has the effect of mortality otherwise the number of predators would have been growing and not decreasing. Similarly, the natural rate of increase of $N_{f1}(k)$ is $r_2 > 1$. The other coefficients α, β depict the interactions between populations.

Note that here we deliberately denote the natural growth rate of the predator lamprey-1 as $r_1 FM$ in order to reflect the effect of sex ratio on the population growth rate and to facilitate comparisons later in the paper.

2.1.2 Model of changed sex ratio capacity

Based on the biological characteristics of Sea Lampreys, the assumption is given that the sex ratio of Sea Lampreys-1 is fixed to the sex ratio of the Sea Lampreys population in a situation of food sufficiency (i.e., the most favorable environment for survival): males account for 56% of the population, and females, i.e., 44%. [2] As emphasized earlier, the reproduction of organisms is strongly correlated with females, so we set the dependent variable as the proportion of female individuals in the overall population size.

We build a new model:

$$\begin{cases} N_{l2}(k) = (r_1 FM_2(k-1) + \alpha N_{f2}(k-1))N_{l2}(k-1) \\ N_{f2}(k) = (r_2 - \beta N_{l2}(k-1))N_{f2}(k-1) \\ RA(k) = \frac{N_{f2}(k)}{N_{l2}(k)} \\ FM_2(k) = \begin{cases} 0.22, RA(k) \leq \mu_1 \\ 0.44 - \frac{0.22}{\mu_2 - \mu_1} [\mu_2 - RA(k)], \mu_1 \leq RA(k) \leq \mu_2 \\ 0.44, RA(k) \geq \mu_2 \end{cases} \end{cases} \quad (2)$$

We add assumptions: When $RA(k) \geq \mu_2$, it indicates that food is abundant, and the maximum proportion of females can reach 0.44; otherwise, when food is scarce, the proportion of females will decrease to 0.22.

After mathematical analysis of the model, we concluded that by fixing the values of μ_1, μ_2 , the actual meanings they originally represented could be accommodated by the other parameters. We

manually selected $\mu_1 = 0.4, \mu_2 = 2$, which had the best relative results and more stable results. These values will be held constant in subsequent numerical simulations and solution processes.

2.2. Monte Carlo algorithm to validate models

2.2.1 Model Results

The solution to the stabilization case of the unaltered sex ratio capacity model is:

$$N_{l1}(k) = N_{l1}(k-1) = \frac{r_2-1}{\beta}, N_{f1}(k) = N_{f1}(k-1) = \frac{1-0.44r_1}{\alpha}. \quad (3)$$

The solution (central tendency of the fluctuations) for the stable case of the capacitated model is:

$$N_{l2}(k) = N_{l2}(k-1) = \frac{r_2-1}{\beta}, N_{f2}(k) = N_{f2}(k-1) = \frac{1-r_1FM_2(k-1)}{\alpha} \quad (4)$$

$$FM_2(k) = FM_2(k-1) = 0.44 + \frac{0.22[\beta(1-0.44r_1) - \mu_2\alpha(r_2-1)]}{(\mu_2-\mu_1)\alpha(r_2-1) + 0.22\beta r_1}$$

Combining the conditions of the above parameters, our mathematical analysis for $FM_2(k)$ yields the following results:

$$0.44 < FM_2 < \frac{1}{r_1} \quad (5)$$

$$N_{f1}(k) > N_{f2}(k) > 0$$

2.2.2 Monte Carlo algorithm to validate the theoretical model

Ten thousand different sets of parameters were randomly substituted according to the Monte Carlo algorithm for a time span of 200 years (cycles). Three different outcomes inferred from this model were successfully obtained. In turn, whether two systems will collapse or not can constitute three scenarios, which we discuss below in a categorized manner:

Scenario 1: Neither of the systems composed of Lamprey-1 and Lamprey-2, respectively, are headed for collapse, but with a probability of only 5%. Below are images of several typical scenarios:

- ① Lamprey-2's population size tends to fluctuate steadily, while Lamprey-1's population size likely leads to eventual collapse.
- ② This situation represents a very adequate food supply for that environment, resulting in nearly identical performance of the two systems over a long period of time.

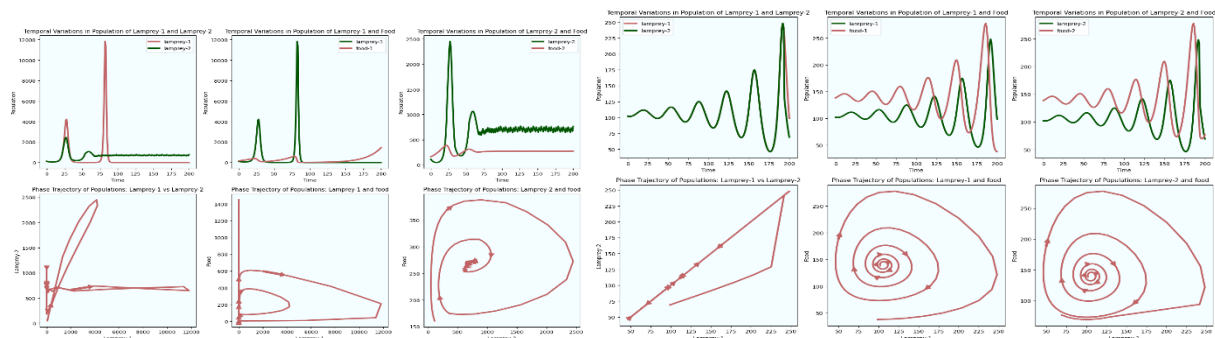


Fig. 1 Different and Identical Fluctuation Trajectories

Scenario 2: Lamprey-1 collapses before Lamprey-2 with a probability of 75%. This is the scenario with the highest probability of occurrence, and the one that best fits the inferences of our model.

- ① Lamprey-2 quickly comes to the case of stable fluctuations (very close to the ideal equilibrium point), while lamprey-1 is on the verge of collapse.
- ② Both are headed for collapse, but lamprey-2 holds out significantly longer than lamprey-1.

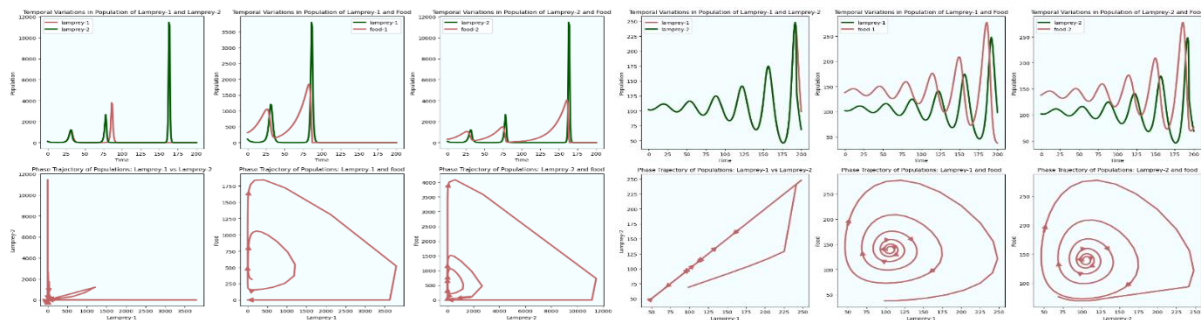


Fig. 2 Stable, Collapse and Both Collapse

Scenario 3: Lamprey-2 collapses before Lamprey-1 with a probability of 20%, there is only one classical scenario for this scenario:

- ① The fluctuations of lamprey-2 are significantly smaller than those of lamprey-1, but lamprey-2 goes to collapse faster.

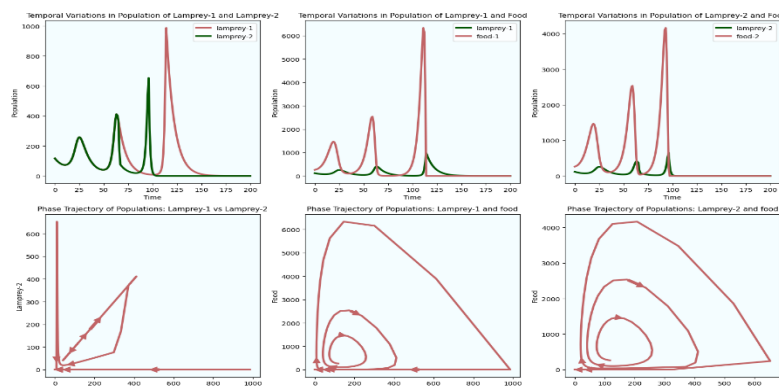


Fig. 3 System Two Collapses Earlier Than One

The above classical scenarios have two major points in common:

- (1) The fluctuations of lamprey-2 and food are significantly smaller and more stable. (2) The changes in population size of lamprey-1 and lamprey-2 in the early stage are highly coincident. The reason for this is that our starting point was uniformly set to a situation where FOOD was abundant and sea lamprey numbers were low.

By observing and comparing the developmental changes of lamprey-1 and 2, we conclude that lamprey-2 is able to survive better with low resource availability, validating the inference of our model. It is also able to stabilize the population size with greater probability.

By comparing the probability of which of the two collapses first, we find that the probability of the lamprey-2 system surviving longer than the lamprey-1 system is close to 50%. At the same time, having the ability to change sex helps lamprey populations to better maintain population size stability, and also indirectly helps to maintain food size stability. In other words, it contributes to the stability of the ecosystem.

2.3. Higher Predator (Humans, etc.) –Predator– Prey Model

Considering the composition of the food chain, the following is a model in which we introduce the natural enemies of sea lampreys, an organism that is higher in the food chain.

In this model, we will add a discussion of the impacts of sea lamprey populations being captured by higher level predators. That is, a discussion of the impacts caused by sea lamprey populations on the upper levels of the ecosystem.

The modeling is as follows:

$$\begin{cases} N_l(k) = [r_1 FM(k-1) + \alpha N_f(k-1) - \theta N_e(k-1)]N_l(k-1) \\ N_f(k) = [r_2 - \beta N_l(k-1)]N_f(k-1) \\ N_e(k) = [r_3 + \lambda N_l(k-1)]N_e(k-1) \\ S(k) = \theta N_e(k-1)N_l(k-1) \\ RA(k) = \frac{N_f(k)}{N_l(k)} \\ FM_2(k) = \begin{cases} 0.22, RA(k) \leq \mu_1 \\ 0.44 - \frac{0.22}{\mu_2 - \mu_1} [\mu_2 - RA(k)], \mu_1 \leq RA(k) \leq \mu_2 \\ 0.44, RA(k) \geq \mu_2 \end{cases} \end{cases} \quad (6)$$

The solution to the stability case, if it exists, is extremely demanding on the parameters.

On the basis of the model assumptions, we have likewise substituted a large number of random parameters to obtain the typical situation as shown below:

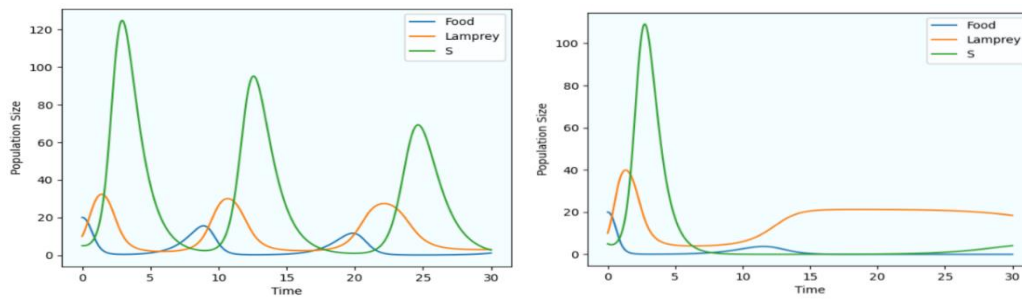


Fig. 4 Most cases and Theoretical scenarios

By looking at the Fig. 4 image it can be concluded that changes in the population of the sea lamprey also significantly affect the organisms at the top of the food chain. Since lamprey can change its sex, its own population size is more stable, which provides a more stable benefit $S(k)$ to his natural predators, or species that can prey on it, including humans.

3. Structural Equation Modeling

In this model, we further optimize the previous model. The previous model mainly discusses the impacts on the ecosystem when the survival rates of male and female sea lampreys are the same. In this part, we will consider the case of different survival rates of males and females in a more practical way.

3.1. Logistic Function Difference Model

In this model, we focus on the differential effects of the male and female lamprey themselves on the population as well as on the rest of the food web. It is hypothesized that the competitive advantage of males leads to a higher survival rate, that females have a higher predation coefficient on food because they require more resources for reproduction, and so on.

In addition, we believe that in a real ecosystem, the population size of advanced predators (natural enemies of lamprey) tends to be stable by itself, and with a variety of food sources that are minimally affected by a single species. Therefore, we use the difference form of the logistic function to describe its quantity:

$$\left\{ \begin{array}{l} N_l(k) = \{r_1 FM(k-1) + [\alpha_1 FM(k-1) + \alpha_2] N_f(k-1) \\ \quad - [\theta_1 FM(k-1) + \theta_2] N_e(k-1)\} N_l(k-1) \\ N_f(k) = \{r_2 - [\beta_1 FM(k-1) + \beta_2] N_l(k-1)\} N_f(k-1) \\ N_e(k) = \{r_3 + [\lambda_1 FM(k-1) + \lambda_2] N_l(k-1)\} [1 - \frac{N_e(k-1)}{N_{e-max_e}}] \\ RA(k) = \frac{N_f(k)}{N_l(k)} \\ FM(k) = \begin{cases} 0.22, RA(k) \leq \mu_1 \\ 0.44 - \frac{0.22}{\mu_2 - \mu_1} [\mu_2 - RA(k)], \mu_1 \leq RA(k) \leq \mu_2 \\ 0.44, RA(k) \geq \mu_2 \end{cases} \end{array} \right. \quad (7)$$

3.2. Model Analysis and Inference

Based on our modeling assumptions, we have the following speculations:

$$RA \downarrow \xrightarrow{\text{make}} FM \downarrow \xrightarrow{\text{make}} r_1 FM \downarrow \xrightarrow{\text{make}} N_l \downarrow \xrightarrow{\text{make}} RA \uparrow$$

$$RA \uparrow \xrightarrow{\text{make}} FM \uparrow \xrightarrow{\text{make}} r_1 FM \uparrow \xrightarrow{\text{make}} N_l \uparrow \xrightarrow{\text{make}} RA \downarrow$$

This will lead to a stabilizing cycle for the entire ecosystem, so the inference of the model is that lamprey populations with the ability to alter sex ratios help to simultaneously help the ecosystem improve both resilience stability and resistance stability.

3.3. Numerical simulation by Monte Carlo algorithm

We still got the following typical cases by randomly selecting a large number of parameter samples:

- ① The most ideal situation: the fluctuation of population size of food, lamprey and enemy all show stable periodicity, and the change of sex ratio also tends to stabilize at the maximum value. The ability of sex change of lamprey can help the resource availability of the environment where it lives is suitable and stable.
- ② The proportion of lamprey females has roughly stabilized at the midpoint of its value range, and the population size fluctuations of food, lamprey, and enemy have all stabilized.
- ③ This situation forms a different cyclical outcome from the first - where the sex ratio of lamprey has remained cyclical. But it still constitutes a stabilizing outcome.

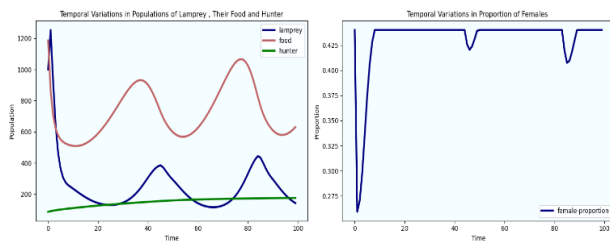


Fig. 5 Scenario 1 - Periodic Variations

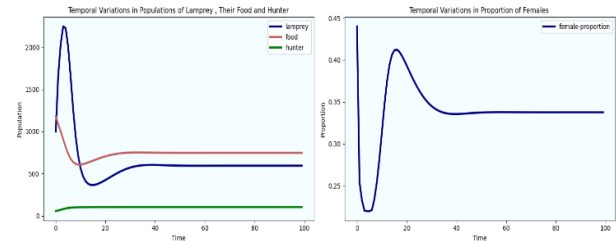


Fig. 6 Scenario 2 - Tending Towards Stable

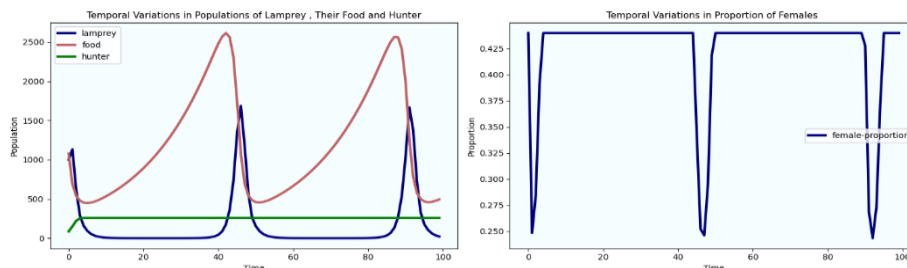


Fig. 7 Scenario 3 - More Intense Fluctuations

In addition, the images of the collapse results are not different from the previous model, and the significant decrease in the probability of collapse is sufficient to demonstrate that lamprey sex change ability has a positive effect on ecosystem stability, so the results will not be shown again.

3.4. Impact on Ecosystem Stability

The ability of sea lamprey populations to change sex ratios dependent on resource availability in the environment creates a complete feedback loop. Through this loop, the stability of the ecosystem can be enhanced. The details are as follows:

3.4.1 Construction of the feedback loop

If food resources decrease, the proportion of males will increase, reducing the pressure on food resources, thus helping the ecosystem to return to a balanced state.

3.4.2 Ecosystem stabilization

Through this feedback loop, sea lamprey populations may help the ecosystem return to equilibrium more quickly and increase the probability of ecosystem stabilization. This is because sex-ratio adjustments in sea lamprey populations may help stabilize ecosystems by relieving pressure on environmental resources.

3.4.3 Speed of ecosystem recovery

Sex ratio adjustments in sea lamprey populations may also accelerate ecosystem recovery to a state of equilibrium. This is because sex-ratio adjustments can quickly reduce the pressure on environmental resources, thus speeding up ecosystem recovery.

3.4.4 Impacts on other species

Sex ratio adjustments in sea lamprey populations may also affect other species. If sex ratio adjustments in sea lamprey populations lead to an increase in food resources, it may affect the survival and reproduction of their predators and prey, thereby maintaining ecosystem stability.

4. Dynamics Models of Multi-Species Complex Systems

4.1. Establishment of Complex System Models

4.1.1 Modeling

Consider adding a parasite with the same ecological niche as Sea Lamprey, which will compete with Sea Lamprey for food and has a subscript of c (competitor) in the model[5]. It may be assumed that its female ratio is the upper limit of 0.44 for Sea Lamprey.

$$\begin{cases} N_l(k) = [r_1 FM(k-1) + \alpha_l N_f(k-1) - \theta_l N_e(k-1) + \eta_l N_e(k-1)] N_l(k-1) \\ N_f(k) = [r_2 - \beta_l N_l(k-1) - \beta_c N_c(k-1)] N_f(k-1) \\ N_e(k) = [r_3 + \lambda_l N_l(k-1) + \lambda_c N_c(k-1)] N_e(k-1) \\ N_c(k) = [r_4 + \alpha_c N_f(k-1) - \theta_c N_e(k-1) + \eta_c N_e(k-1)] N_c(k-1) \\ RA(k) = \frac{N_f(k)}{N_l(k) + N_{c1}(k)} \\ FM(k) = \begin{cases} 0.22, RA(k) \leq \mu_1 \\ 0.44 - \frac{0.22}{\mu_2 - \mu_1} [\mu_2 - RA(k)], \mu_1 \leq RA(k) \leq \mu_2 \\ 0.44, RA(k) \geq \mu_2 \end{cases} \end{cases} \quad (8)$$

The subscripts l and c indicate the relationship of different species to Sea Lamprey and competitor, respectively. We introduce a new parameter, η , denoting the coefficient of growth rate for a parasite hosted by an enemy.

4.1.2 Simplification and Analysis of the Model

Assuming identical impact coefficients for Sea Lamprey and its Competitor allows for a direct comparison of competitive advantages under uniform conditions. Although our model accommodates variable coefficients, we chose to standardize them for clarity. By setting a low initial population size for the Competitor, we mitigate the influence of its favorable sex ratio. Initially, the Sea Lamprey's role in system stability results in a moderate population growth rate. However, as the system evolves, the Competitor, benefiting from its sex ratio, surpasses the Sea Lamprey, eventually establishing a higher, stable population.

4.2. Simulated Annealing Algorithm

We employed a Simulated Annealing Algorithm to simplify the system's complexity and optimize parameters for stability, confirming our conclusions. The algorithm's energy function is based on the occurrence of system collapse over 200 years, with higher values for earlier collapses and zero for no collapse. After 200 iterations, a trend was discerned, showing the 'Enemy' population is greater than zero but is considered negligible.

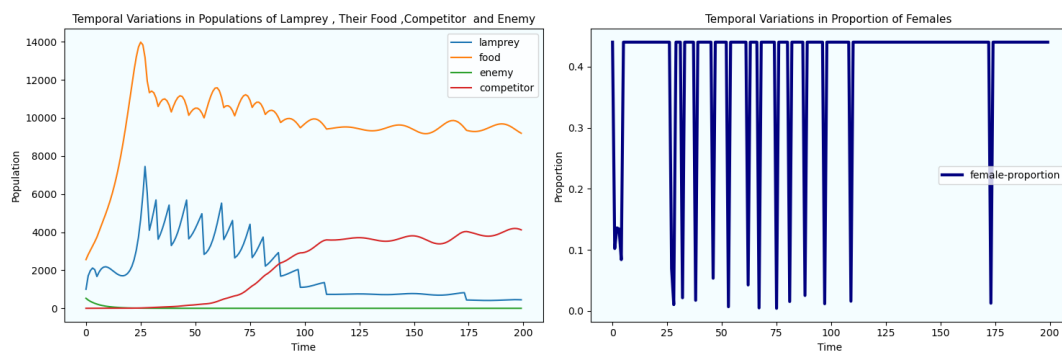


Fig. 8 Results of the Simulated Annealing Algorithm

Our analysis reveals that different parameter combinations have contributed to system stability over a two-century period, confirming our model's predictions: (1) Sea Lamprey plays a crucial role in early system stability, facilitating the Competitor's survival when population sizes are minimal.[7] (2) As the system progresses, the Competitor's population is likely to surpass that of Sea Lamprey, influenced by its advantageous sex ratio. (3) In the advanced stages, the Competitor is poised to outcompete Sea Lamprey, forming a stable triad with the Food source and itself, while the Sea Lamprey, hampered by its small population and variable sex ratio, is at a distinct competitive disadvantage for food. Furthermore, in extreme scenarios where resource availability remains low, the Sea Lamprey population is increasingly susceptible to extinction.

5. Summary

Through the development and application of complex mathematical models, this study delves into the ecological and evolutionary significance of sea lamprey sex ratio variability and its impact on population dynamics and ecosystem stability. Our research underscores the critical role of sex ratio variability in the sea lamprey's adaptation and its ecological consequences, suggesting more effective approaches for biodiversity conservation and ecosystem management.

Our findings reveal that sex ratio variability enhances the sea lamprey's survival in resource-limited environments, increases population stability, and contributes to food resource stability, indirectly fostering ecosystem stability. Additionally, changes in the sex ratio significantly affect sea lamprey predators and the food chain, influencing reproductive and predatory behaviors and potentially altering ecosystem species diversity and dynamics.

The establishment of a feedback loop mechanism shows that sex ratio adjustments mitigate environmental stress during resource scarcity, facilitate quicker ecosystem equilibrium, and improve ecosystem stability chances. This not only speeds up ecosystem recovery but may also positively

impact other species by enhancing food availability for predators and prey, thus supporting ecosystem stability [8].

However, in conditions of prolonged severe resource scarcity, the sea lamprey population is at a higher extinction risk. This highlights the necessity to consider the ecological effects of sex ratio changes and their long-term impact on ecosystem stability and resilience in sea lamprey population management.

In conclusion, this study offers valuable insights into sea lamprey sex ratio variability's effects on population dynamics and ecosystems, emphasizing the need to incorporate this phenomenon into conservation and management strategies. Future research could explore the broader implications of sex ratio changes on ecosystems and refine sea lamprey management to protect and restore ecosystems.

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