

## Research on the problem of comprehensive evaluation

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**Abstract.** Comprehensive evaluation is a systematic and complex work. It is one of the important means for people to know things, understand things and influence things. It is a management cognitive process and a management decision-making process. It is widely used in the fields of economy, society, science and technology, education, management and engineering practice. Some mature theories and methods have been formed around the purpose and process of evaluation, the construction of evaluation index system, the determination of evaluation index weight and value, the source and processing of data, the integration of evaluation information and the application of evaluation results. This paper mainly introduces the background of comprehensive evaluation, commonly used comprehensive evaluation methods and their advantages and disadvantages, discusses the most critical step in comprehensive evaluation - determination of weights, mainly introduces several methods to determine weights, and analyzes their advantages and disadvantages. In view of the limitations of subjective and objective methods to determine weights in the process of comprehensive evaluation, we use the maximum deviation model to solve the proportion of subjective and objective weighting methods, Make the combination weighting more effective.

**Keywords:** Comprehensive evaluation, Weight, Combination weighting.

### 1. Introduction

Comprehensive evaluation is a general term for evaluating the objects of single objective or multi objective system, also called multiple indicator comprehensive evaluation. Comprehensive evaluation is not only a method, but also a system. It is a general term for multiple indicator comprehensive evaluation methods. Comprehensive evaluation generally refers to selecting the elements or indicators that can affect the evaluation object after people have determined the final purpose of evaluation, using appropriate evaluation methods to extract the information of the selected elements or indicators, ultimately reflecting the overall characteristics of the evaluation object, and then evaluating, and finally screening or comprehensive ranking according to the evaluation results. Comprehensive evaluation plays a very important role in the fields of economy, society, science and technology, education, management and engineering practice. It can not only be used as the decision-making basis for the ranking and optimization of evaluation objects, but also promote the improvement of practice process and the optimization of management measures, and make each process more reasonable.

As an important content in the field of management and decision-making, comprehensive evaluation has formed a relatively mature theory and method system with the research and development of philosophy, sociology, economics, statistics, operations research, computer science, systems engineering, management, decision science and other related disciplines. Moreover, due to the extensive and in-depth application of comprehensive evaluation in the fields of economy, society, science and technology, education, management and engineering practice, it has not only achieved rich practical application results, but also further developed the comprehensive evaluation theory and method system. This paper mainly introduces several common methods of comprehensive evaluation and the method of determining the weight of each index in the comprehensive evaluation, and analyzes the advantages and disadvantages of the method of determining the weight in order to find a more reasonable method of determining the weight.

## 2. Several common methods of comprehensive evaluation

### 2.1. Analytic hierarchy process

Analytic Hierarchy Process (AHP) was proposed by Sati, a famous mathematical scientist at the University of Pittsburgh in the 1970s. Analytic hierarchy process is essentially a decision-making thinking mode, which decomposes a complex problem into several factors, which are grouped according to membership and form an orderly hierarchical structure. The importance of each indicator is compared in pairs through expert experience to determine the order of importance of each factor to the decision-making objective (weight). The classical analytic hierarchy process is divided into five steps, namely, establishing the hierarchical structure, constructing the importance judgment matrix, hierarchical arrangement of individual indicators, consistency test and overall ranking of indicators.

There are obvious advantages and disadvantages of AHP. The advantage is that the combination of qualitative and quantitative analysis has brought into full play the advantages of quantitative analysis and qualitative analysis, and the quantity data needed is less; The disadvantage is that the subjective randomness is strong, persuasive power is not enough, sometimes too many elements are prone to inconsistent problems.

### 2.2. Principal component analysis

The principal component analysis method was proposed by Karl and Pearson in 1901. The principal component analysis method uses the correlation between indicators and the idea of dimension reduction to simplify the original multiple indicators into several principal components, and these principal components contain most of the information of the original indicators.

The purpose of principal component analysis is to reduce dimensions, that is, to convert multiple variables with certain correlation into several unrelated principal components, and replace the original variables with these principal components. The principle of principal component analysis is to linearly combine the original variables. Each combination contains the information of the original variables, and the variance is used to measure the amount of information. When the variance is greater than a certain threshold, the combination is retained; otherwise, it is deleted. The combinations are sorted according to the amount of information. The combination with the largest amount of information is recorded as the comprehensive variable F1, which contains the largest amount of information. If F1 cannot contain all information, then continue to select F2, which is recorded as the second principal component. The amount of information of F2 is not included in F1, which means  $cov(F1, F2) = 0$ . Similarly, P principal components are selected.

The advantage of principal component analysis is that it can eliminate the correlation between indicators, enhance the stability of the model, reduce the workload of screening indicators, and when there are many rating indicators, it can also use a few comprehensive indicators to replace the original indicators while retaining most of the information; The disadvantage of principal component analysis is that there is a strong correlation between the indicators, and sometimes the results are difficult to explain economically.

## 3. Several methods of determining weight

### 3.1. AHP method

Firstly, the comparison judgment matrix A is constructed. Then,  $a_{ij}$  is classified as level 9(1,2,...,9 and theirs reciprocal). The maximum eigenvalues  $\lambda_{max}$  and eigenvectors are obtained from  $|A - \lambda I| = 0$ . And the feature vector is the weight vector. From the  $\lambda_{max}$ , we can estimate the consistency of comparative judgment.

$$CI = (\lambda_{max} - M)/(M - 1) \quad (1)$$

Where M is the number of indexes.

According to the above formula and looking up the table, the random consistency index RI can be obtained, and the satisfactory consistency test can be carried out.

$$CR = CI/RI \leq 0.1 \quad (2)$$

### 3.2. Principal component analysis

Principal component analysis is based on statistical data, which is a typical method to screen and simplify the index system.

Original index vector (component)

$$X = (X_1, X_2, \dots, X_M)^T \quad (3)$$

New index vector (component)

$$Y = (Y_1, Y_2, \dots, Y_M)^T \quad (4)$$

is a linear combination of X., that is

$$Y = CX^T \quad (5)$$

Where  $Y_1$  is the largest variance component in linear combination, called the first principal component.  $Y_2$  is the second principal component of the second largest variance, etc. And C is the eigenvector corresponding to the M eigenvalues ( $\lambda_1 > \lambda_2 > \dots > \lambda_M$ ) of the eigenequation  $|R - \lambda I| = 0$ . If the contribution rate of the first principal component  $a_1 = \lambda_1 / \sum_{i=1}^M \lambda_i$  is greater than 0.85, the corresponding eigenvector can be roughly regarded as the weight vector of the original index. If  $a_1$  is not large enough, we can take the product combination of the contribution rate of the first several principal components and the corresponding eigenvector value, and get the weight problem after normalization.

### 3.3. Entropy weight method

In the process of comprehensive evaluation, the selected indicators contain different amounts of information, which leads to their different roles in the process of comprehensive evaluation. When an indicator has a large difference among the evaluation objects, it shows that it has strong discrimination ability and contains more information, so the weight should also be large. Entropy weight method is an objective weighting method based on this principle.

Let the initial data matrix of t evaluation indicators of s samples be  $X = \{x_{ij}\}_{s \times t}$ , where  $x_{ij}$  represents the jth index value of the ith evaluation object. The specific steps of weighting by entropy weight method are as follows:

(1) Calculate the proportion  $p_{ij}$  of the index value of the ith evaluation object under the j index:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^s x_{ij}} \quad (6)$$

(2) Calculate the entropy value  $e_j$  of index j:

$$e_j = \frac{1}{\ln s} \sum_{i=1}^s p_{ij} \ln \frac{1}{p_{ij}} \quad (7)$$

(3) Calculate the difference coefficient  $g_j$  of index j:

$$g_j = 1 - e_j \quad (8)$$

(4) Calculate the weight of indicators:

$$w_j = \frac{g_j}{\sum_{j=1}^t g_j} \quad (9)$$

### 3.4. Combination weighting method

Due to the defects of subjective and objective weighting methods, people began to seek more reasonable weighting methods, which is why the combination weighting method was born. The combination weighting method that is widely used is to linearly combine subjective and objective weighting methods. The key to this method is to reasonably allocate the proportion of subjective and objective weighting methods in the combination weighting method:

$$W = tW_{subjective} + (1 - t)W_{objective} \quad (10)$$

The value principle of  $t$  is that the value of  $t$  should be larger When the difference of each component is large of expert experience method.

It is a reasonable method to use the maximum deviation model to calculate the proportion of subjective and objective weighting methods. In statistics, deviation is an important indicator reflecting the degree of difference. For attribute  $P_j$ , the deviation between scheme  $s_j$  and all other schemes can be defined as:

$$V_{ij}(w) = \sum_{l=1}^n |r_{ij}w_j - r_{lj}w_j| \quad (R = (r_{ij})_{n \times m} \text{ is the normalized decision matrix})$$

Let  $V_j = \sum_{i=1}^n V_{ij}(w) = \sum_{i=1}^n \sum_{l=1}^n |r_{ij} - r_{lj}|w_j$ , Then  $V_j$  represents the total deviation of all schemes from other schemes for attribute  $P_j$ . Based on this principle, solving the weight vector is equivalent to solving the following optimization model:

$$\max V_w = \sum_{j=1}^m \sum_{i=1}^n \sum_{l=1}^n |r_{ij} - r_{lj}|w_j \quad (11)$$

$$s. t. \begin{cases} w_j \geq 0, j \in M = \{1, 2, \dots, m\} \\ \sum_{j=1}^m w_j^2 = 1 \end{cases} \quad (12)$$

Solve the optimization model and construct the Lagrange function:

$$L(w_j, \lambda) = \sum_{j=1}^m \sum_{i=1}^n \sum_{l=1}^n |r_{ij} - r_{lj}|w_j + \frac{1}{2}\lambda(\sum_{j=1}^m w_j^2 - 1) \quad (13)$$

Solve the model and normalize it , the following results can be obtained:

$$w_j = \frac{\sum_{i=1}^n \sum_{l=1}^n |r_{ij} - r_{lj}|}{\sum_{j=1}^m \sum_{i=1}^n \sum_{l=1}^n |r_{ij} - r_{lj}|}, j \in M \quad (14)$$

## 4. Conclusion

Both subjective weight and objective weight have their defects. The subjective weight is mainly assigned by people's cognition of the reality, while ignoring the problems reflected by the objective data; The objective weight only depends on the obtained data to analyze and obtain the specific weight of each indicator, ignoring the important role played by the real application. The combination weighting method is a combination of the two, which can not only explain the real situation, but also make good use of the real data, so the final result is more reliable. When using the combination weighting method to calculate the weight, a single weighting method should be selected as few as possible for combination, which can reduce the error of the final results. Therefore, this paper uses the subjective and objective weighting methods to select one for combination, and uses the deviation maximization model to calculate the proportion of the two, making the combined weight more reasonable. The compatibility test of the combined weight further verifies its rationality.

Comprehensive evaluation has been applied in various fields, and it is particularly important to evaluate it objectively and reasonably. As the most important link in the evaluation process, the method of weighting has also become the focus of research. The use of weighting methods sometimes needs to be selected and judged according to the problems studied. Different evaluation problems can have multiple weighting methods and models to choose from, but we should choose the most appropriate and reliable method for evaluation, so that the evaluation results can be more meaningful.

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