Extended study about model in game theory

Ruizhe Li[1, †], Jingxiang Peng[2, *, †]

[1]New Oriental Education & Technology Group, Shanghai, China

[2]Guangdong Country Garden School, Guangzhou, China

*Corresponding author: 18409112@masu.edu.cn

†These authors contributed equally.

Abstract: The earliest examples of game thinking recorded in the literature can be traced back to the "King Tianji of Qiwei's horse racing" in China more than 2,000 years ago. Besides, the "marriage contract problem" was also crucial in the Babylonian Talmud of 1,500 years ago according to modern economics. The idea of game theory has aroused people's interest and attention. The theory began to have more and more followers. This article presents an extension of existing literature. The paper introduces the basic mainstream form of prisoner's dilemma in fundamental Symmetric forms based on earlier conclusions from other researchers. It examines the problem in the prisoner's dilemma. This work just solves the modal in an ideal condition and the exterior assumption introduces modal and consequence from other scholars.

Keywords: Prisoner’s dilemma, Nash equilibrium, Game theory, History

1. Introduction

Game theory is the study of mathematical models of rational actors' strategic interactions, and Game theory principles always include players, actions, order of player, information, strategies, payoffs, outcomes, equilibrium, etc. Game theory, in general, investigates conflict situations, player interactions, and decision-making.

In game theory, a game is defined as a collection of participants who interact according to predefined rules and formulate the correct strategy to enable a participant to address that challenge when confronted by a complex challenge and conflict. It was developed based on the premise that there exist methods that allow players to reach maximum profit for whatever circumstance.[1] The basic game type is included different types of forms, as shown in Table 1

Table 1. Different types of basic game

Symmetric	**Asymmetric**
The payoff and consequence are only affected by the decision of the rational participant, not affected by identity, status, and mindset.	The payoff and consequence are affected by both decision and identity of the participant.
Normal form	**Extensive form**
The payoff matrix usually expresses normal forms to represent the detail of the Nash equilibrium, dominant strategy, and payoff.	Extensive forms are usually expressed by the graphical game tree to represent the behavior, payoff and information during the game.
Zero-Sum	**Non-Zero-Sum**
Zero-sum forms have an inverse payout for each player. If one player triumphs, the other must suffer defeat.	Non-Zero-Sum forms typically represent the decision in a payoff matrix and allow for "one of each" or "each" benefit.
Cooperative	**Non-cooperative**
If participants can create externally imposed binding obligations, this game is cooperative.	If players are unable to create alliances or if all agreements must be self-enforcing, the game is non-cooperative.

Game theory is a subfield of applied mathematics that was developed to evaluate scenarios involving the interaction of parties with comparable, diametrically opposing or mixed interests. John von Neumann and Oscar Morgenstern first created a game theory in their book The Theory of Games and Economic Behavior. In a normal game or competition governed by established rules, "players" attempt to outwit one another by predicting their opponents' actions or movements. A game's solution specifies the ideal plan or tactics for each participant and forecasts the average or anticipated, result. Until 1967, when a highly constructed counterexample was created, it was believed that every contest had at least one answer. Table 2 shows early Precursors for Game theory

Table 2. Precursors for Game theory

Discoverer	Article	Discussion	Result
Pierre Rémond de Montmort	*Essay d'analyse sur les jeux de hazrd (1713)*	Probability, function, and Calculus included the card game "le Her" and "Problem of the pool"	Discussion regarding "le Her" but provided the same response with "greater than" and "less than symbols"
Nicolaus Bernoulli			
James Waldegrave	*Universitätsbibliothek Basel (1713 after)*		Provides a way for solving a two-person version of the "le Her" using a minimax mixed strategy solution, which is the first appearance of a mixed strategy solution in game theory.
Antoine Augustin Cournot	*Recherches sur les principes mathématiques de la théorie des richesses (1838)*	Mathematical modal about the theory of wealth, including monopoly, profit, tax, consumer, etc.	Analyze various economic conditions and provide mathematical modalities. Considered a duopoly and proposed a solution corresponding to the game's Nash equilibrium.
Ernst Zermelo Kalmar's König's	*Über eine Anwendung der Mengenlehre auf die Theorie des Schachspiels (1913)*	The Game of Chess theory discusses the infinite and finite position and strategy.	Demonstrated that the optimal chess strategy is deterministic.
Émile Borel	*Applications aux Jeux de Hasard and earlier notes (1938)*	Blotto game with probability and calculus	Proved a minimax theorem for two-person zero-sum matrix games only when the pay-off matrix is symmetric.

This study will present the basic mainstream form of prisoner's dilemma in fundamental Symmetric forms based on earlier conclusions from other researchers. It will examine the problem in the prisoner's dilemma in the first half. Then, include trustworthy threat and reward systems in your list of hypotheses and methods from other experts. Finally, summarize the known modalities and make additional recommendations and findings on the prisoner's dilemma.

2. Research framework

2.1 prisoner's dilemma

The prisoner's dilemma is a typical example of a Non-Zero-Sum, Non-Corporate Symmetric (Basic form) game. "The Prisoner's Dilemma was devised in 1950 by mathematicians Merrill Flood and Melvin Dresher for a class of Stanford University psychology majors in military and strategic settings." [2][3].

Merlin Freud, Melvin Drescher, and Albert Tucker created the prisoner's dilemma in 1950. The prisoner's dilemma is frequently employed in business games, conflict, and rivalry.

Consider the following situation: The police apprehended two suspects, A and B are accomplices. The suspects were separated, and each was offered one of two options: remain silent or alert the authorities. If one chooses to stay quiet and prosecute the other, the charged individual faces ten years in prison, while the other is acquitted. Both suspects would face a two-year prison sentence if they kept silent. Both suspects would face a five-year prison sentence if they chose to cooperate against one another. Each suspect in the ideal mode is rational and, according to game theory, strives to maximize profit. Finding Nash equilibrium is straightforward when using a pure approach. Create a direct two-by-two matrix graph.

Table 3. Payoff Matrix when using a pure approach

Decision	Silence (B)	Betrayal (B)
Silence (A)	(-2, -2)	(-10, 0)
Betrayal (A)	(0, -10)	(-5, -5)

One player has a dominating strategy: betray, which suggests that in the case of an apparent Pure Nash equilibrium (-5, -5), one person is unable to find another lucrative deviation while the other player does not—as a consequence of this paradigm, it is advantageous to inform against the other player. This is, however, Pareto inefficient. "The -dilemma" happens when an individual's logic of pursuing their own best interests reveals that the actual advantages to both parties are not equal. Only the combination (-2, -2) is Pareto optimum." The game outcome is essentially the strategy that results in the lowest collective interest due to the participants' rationality; nevertheless, this does not mean that individuals who are not rational will not raise the collective revenue."

3. Solve prisoner's dilemma

3.1 Cooperative form and repeat from

Many competitions are similar to the Prisoner's dilemma in economic activities. Their models can be converted to the prisoner's dilemma model, such as price competition between companies, market competition, the trade war between countries, etc. In these competitions, both sides have different choices. In most cases, cooperation can bring the best benefits, but only unilateral interests will be considered, so the optimal situation cannot be achieved.

For example, countries A and B are engaged in a trade war. Both countries can choose to increase or reduce their import taxes. If countries A and B both increase, each country's economic level will decrease by 1. If both countries decrease their import taxes, the financial status of each country is increased by 5. Suppose one country increases and the other country decreases the taxes. In that case, the economic level of the increased country is increased by 10, and the financial status of the reduced country is reduced by 10.

Table 4. Payoff Matrix

Decision	Increase (B)	Decrease (B)
Increase (A)	(-1, -1)	(10, -10)
Decrease (A)	(-10, 10)	(5, 5)

Therefore, when countries a and b are making decisions, they will just consider the maximization of their own interests, so the final result is the only Nash equilibrium (-1, -1). However, such a result does not maximize the collective interests.

In these "prisoner's dilemma" models, both sides of the decision are rational and unemotional. In this case, "cooperation" becomes difficult, because when one side keeps its promise, the other side will breaks its credibility to maximize its own interests, thus reducing the level of cooperation.

It can be seen that the rational market has no way to solve such a dilemma, so how to get out of the "prisoner's dilemma" has become a necessary topic to discuss.

In the rational choice, market regulation and self-regulation cannot let both sides get out of the dilemma. Therefore, we refer to the following literatures, which add "reward" or "punishment" and other mechanisms on the basis of the original benefits, to increase the level of cooperation between the two sides of the game and maximize the collective interests.

If given a condition, two suspects have been chosen to cooperate before interrogation, and both of them choose to be silent. Suppose the suspect has been in jail several times, N is the number of repeat imprisonments

Payoff Matrix 2.2

Set Silence=S, Betrayal=B Satisfy T > R > P > S

If both of them choose to be silent, they will both get maximum profit, which is the best consequence to reach Pareto Optimality.

Table 5. Payoff Matrix of maximum profit

N (X)	Decision(A/B)	A (Payoff)	B (Payoff)
N=1	S/S	R	R
N=2	S/S	R	R
N=X+1	S/S	R	R
Total payoff		N*R	N*R

If one of each chooses to betray during cooperation, the Nash equilibrium will switch from Pareto Optimality to Pareto inefficiency, leading the other side will choose revenge by betrayal.

Table 6. Payoff Matrix of solution

N (X)	Decision(A/B)	A (Payoff)	B (Payoff)
N=1	S/S	R	R
N=2	S/B or B/S	S/T or T/S	T/S or S/T
N=X+1	B/B	P	P
Total payoff		x*R+(S or T)+(N-x)*P	x*R+(T or S)+(N-x)*P

x=cooperation term. (N > 0)

Consequently, the ideal method to solve a prisoner's dilemma only relies on the Asymmetric form and external circumstances. But some previous existing research has given different modal to analyze the exterior effect.

3.2 Example of solution

3.2.1 Reward system

In the game, each individual will choose the choice that maximizes his own interests without any personal feelings. It also explains why it is so hard to get prisoners to cooperate when the choice is the best interest to do so. In order to improve the level of cooperation and let the overall interests become more, we cited literature [4] to add a reward mechanism into the game. Observe and summarize the influence of different reward methods on the adoption of cooperative strategies.In the original game, the individual's return after strategy selection is P (Nash equilibrium), but after the incentive mechanism is added, the return function becomes: Pn+1=(1+x)Pn , Pn is the individual's income at time t, Pn+1 is the individual's income at time n+1, and x is the reward factor. Under such a mechanism, literature [5] uses Matlab software to simulate the three incentive mechanisms. We can find that different reward mechanisms have different effects on the level of cooperation. When only the cooperator was rewarded, the cooperation level increased with the increase of the reward factor (X). When only the defector was rewarded, the overall level of cooperation decreased with the increase of the reward dactor (X); When both rewards are given, the level of cooperation increases with the increase of the reward factor (X). This indicates that the effective rate of reward factor (X) on cooperation is higher than that of betrayal. Therefore, it is concluded that the reward factor (X) can improve the overall cooperation level in the game.

3.2.2 Reliable threat

Assume there is a game that can be depicted using the same matrix modal to explain the interaction between two reasonable suspects [6-10], A and B, where B has extra prior criminal suspicions unknown to the authorities. A threatened B that if B chose to inform A and decided to remain silent, A would instruct police B to hire an additional prior criminal to wreak vengeance on B, which is asymmetric for A and B because decision making is affected by identity. A pose a severe threat to B, B will opt to avoid the possibility of betrayal. B chooses silence. When a decides to remain silent, B chooses to stay quiet, resulting in Pareto Optimality (-2, -2).

Table 7. Sub Payoff Matrix

Decision	B
Not Threat (A)	(-10, 0)
Threat (A)	(-10, -8)

If A and B are members of a criminal organization and one is acquitted of a charge, their boss will shoot the betrayer with a gun; this rule assumes that A and B are aware of the situation. (A negative payoff (-100) indicates that the player has "died."), Exterior conditions lead this game to be asymmetric; the risk of being punished by the boss is a credible threat in this case. Both A and B chose silence to avoid death in this game. As a result, Pareto Optimality is achieved.

Table 8. Payoff Matrix

Decision	Silence (B)	Betrayal (B)
Silence (A)	(-2, -2)	(-10, -100)
Betrayal (A)	(-100, -10)	(-5, -5)

3.2.3 Knowledge sharing in virtual communities

From literature [6], we know that the concept of "virtual community" was put forward by H.Rheingold in 1993. Virtual community means people in real society exchange and release their information in cyberspace, such as Zhihu, MicroBlog, and Facebook. However, with the rapid development of the virtual community, its problems and contradictions also appear [11-13].

The cooperation, competition, and conflict among virtual community members will affect the knowledge-sharing level in the virtual community. In practice, the members of the virtual community tend to be more interested in acquiring knowledge from others than in providing it to others. Therefore, the prisoner's dilemma exists in the process of knowledge sharing in virtual communities.

Assuming that there are only two members A and B in A virtual community, each of them hold different information Q of equal value, and sharing information will take time and energy P, then when both members choose to share information, the benefit of both parties is X=2Q. When only one member shares, the benefit of the non-sharing person is 2Q, and the benefit of the sharing person is -p. If neither member shares, the benefit of both parties is 0. See table (3).

Table 9. the benefit of both parties

Decision	Share (A)	Unshared (A)
Share(B)	(2Q, 2Q)	(-P, 2Q)
Unshared (B)	(2Q, -P)	(0, 0)

At this point we know that there is only a Nash equilibrium in which neither party shares.Static games of complete information and repeated games of complete information can solve the prisoner's dilemma in specific situations. In virtual communities, non-material ways are usually used to motivate members to participate in sharing, such as levels, achievements, tasks, etc., but the actual effect of such incentives is very low, Therefore, we refer to literature [7] to clarify the cooperative payment among virtual community members, as shown in Figure1, and adopt the static game of complete information to verify and analyze the constructed mode.

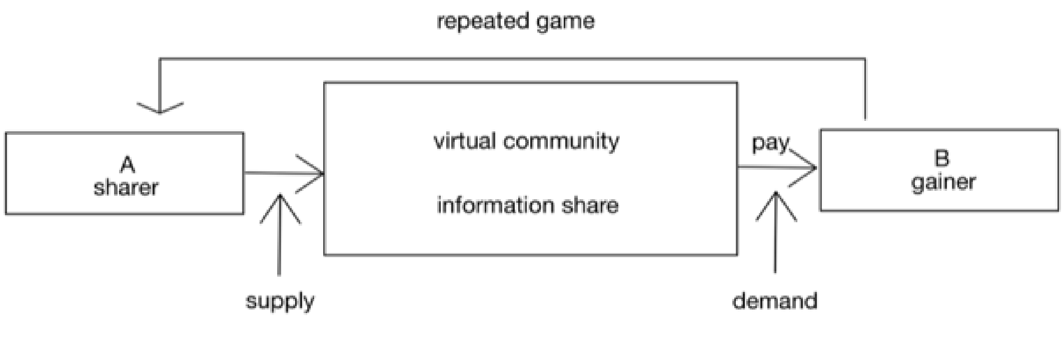

Figure 1. the static game of complete information

The model in the literature creates a virtual community, in this virtual community 1% of all users are knowledge providers, 90% are passive information seekers, and the remaining 9% are silent and contribute little to the community.

The model added rewards to the answering community to motivate members to actively share knowledge. In the case that the game is played only once, it is difficult to achieve the expected effect [14-15]. The members who buy knowledge are not satisfied with the knowledge provided, In the end, most members still chose not to share. Then the game is repeated for Limited times, which leads to the continuous improvement of the quality of knowledge produced in the answering community, and the optimal strategy is gradually changed to both sides sharing. However, when the quality of knowledge content is gradually reduced, the amount of sharing will also decrease, eventually, it became a situation where some people shared it but no one paid for it [16-17].

3.3 Existing research

In real life, humans will be affected by emotion and environment, the decision-making is always asymmetric, previous research constructs modal and given consequences. And It’s unpredictable during cooperation when change decision making without a complex hypothesis and abstract modal.

David John Butler develops a framework to illustrate probability, characterizes "A" as cooperative, and characterizes "B" as defecting. P denotes A’s except value, while (1-P) denotes B's except value. C represents participants in the collective rationality process.

Table 10. the value of both parties

Value	A	B
EV(A)	[p(1-c)+c]R +	[1- p(1-c)-c]S
EV(B)	[p(1-c)]T +	[1-p(1-c)]P

Butler set C* is a threshold value, EV(A)=EV(B). When C<C*, participants will choose defect. When C*<C, participants will choose to cooperate. Butler creates function C*(p) to give a consequence. The probability of selecting cooperation is C*(P). The part of the curve is a percentage of units squared divided by two.[8]

Alessandra Smerilli created a modal for the “We” mode and “I” mode; Individuals can think about one differently.[2] [9] Use the transition matrix to simulate the human decision-making process.

Smerilli used a transition matrix to illustrate the transformation in mind to calculate the utility of personal and collective. Matrix Q

$$=\begin{pmatrix} P(I,I) & P(I,W) \\ P(W,I) & P(W,W) \end{pmatrix}=\begin{pmatrix} 1-q & q \\ p & 1-p \end{pmatrix} \tag{1}$$

p is the probability of transition from we to me, q is the probability of change from I-We, and r is function earning.

4. Conclusions

Game theory develops quickly. The earliest reference in the literature to contain game ideas often cited in game theory, Cournot model of competition through output decisions can be seen as the beginning of early research on game theory. The influence of global political, military, and economic specific environmental conditions, as well as the needs of the development of the economic theory itself, have played an important role in the development of game theory.

Throughout the inquiry, this article demonstrates fundamental prisoner's dilemma rules, discusses concerns about prisoner's dilemmas, and identified asymmetric forms capable of resolving by the previous modal. Introduce previously conducted research about joint probability, and modal consequence. This work just solves the modal in an ideal condition and the exterior assumption introduces modal and consequence from other scholars.

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