

Some fundamental model from Game Theory

Miaoqin Xu*

Syracuse University, Syracuse, United States

*Corresponding email: mxu126@syr.edu

Abstract. Game is an important way of thinking, as long as you are a person in society, there will be games around you in daily life. Game theory covers the fields of economics, diplomacy, politics, law, finance and so on. Learning game theory can enrich your understanding of life and explain the causes of economic and diplomatic phenomena. It helps us make predictions and solves problems. In short, game theory is the study of how to choose an equilibrium among many strategies.

Keywords: Game theory, Strategies, Equilibrium.

1. Introduction

Game theory is an important way of thinking in daily life, and various games may happen at any time in life[1]. Like the game you play with the teacher about grades, the game you play with the vendor about product prices. You are playing a game with your classmates about what movie to choose. Game theory is also widely used in economics, business, politics, law, diplomacy, traffic and other fields.[2] How two firms decide output, how two countries decide trade tariffs. How the principal motivates the agent to work hard. In our lives, we've actually mastered a lot of game skills. Through the study of game theory, we can further systematically organize and improve these skills. Game theory is also an economic research method. Since 1994, more than 20 economists and mathematicians have won the Nobel Prize in Economics for their work in game theory and related fields.

Game theory is the study of interactions. It is the theory of how to adopt strategies when the benefits of the player depend not only on his own actions but also on the actions of others. And the game theory we're studying here has a basic assumption: that all the players in the game are perfectly rational, on the one hand, they hope to get max payoff. On the other hand, they know how to choose the strategy that maximizes the payoff[3].

2. The basic fundamentals of Game

Game theory is the study of how to achieve equilibrium in interactions. The equilibrium results can often be used as a prediction of the game. A game consists of three basic elements: a set of players, a set of strategies for each player, the payoffs to each player for every possible choice of strategies by the players.

There are many categories of games. (1) From the number of players, Two-player game and multi-player game. (2) The choice of action, simultaneous play game and sequential play game. (3) The nature of strategy, pure strategy and mixed strategy. (4) whether know other play's payoff, complete information games and incomplete information games. (5) Based on the number of strategies, finite strategy game and continuous strategy game. (6) Based on the payoff, constant sum games and variable sum game[4]. There may be crossover between games of different classifications. Understanding the classification of different games helps us to understand the characteristics of different games and find different solutions accordingly.

In addition, we learned some concepts about player preferences. Players' attitude towards risk can be divided into risk preference, risk neutrality and risk aversion. For the most part, we consider players here to be risk-averse, who prefers to receiving with certainty rather than facing the uncertainty of the lottery. The utility function of risk-averse players is convex, i.e. the second derivative of the utility function is less than zero. This assumption can ensure that in the continuous strategy game, the

extreme value obtained by taking the first derivative of the utility function must be the maximum point. So, in the future, we just need to take the first derivative to make sure that it's a maximum solution. Sometimes we use diagrams to calculate corresponding equilibrium. The graphing method can strengthen our understanding of the idea of best response of Nash equilibrium.

3. Main content of Game theory

3.1 The simultaneous play game

We first study the games in which there are two players. Both the players chose their strategies simultaneously. Or they don't know what each other's choices are when they act. The two players have no private information, so they know each other's payoff. The same discussion applies to games with multiple players. However, the more players involved in the game, the more difficult it is to analyze.

We can use a payoff matrix to express the finite strategy game [5]. For example, the Prisoner's Dilemma game, arms race problem, chicken game, hawk-dove game and so on. Of course, if the number of players participating in the game becomes more, or the number of players' strategies becomes more, the income matrix will not be convenient. So let's consider the simple two-player game, each of whom can choose between only two actions, we can use the payoff matrix to find the strategy combination which is best reply to each other. The strategy combination are called Nash equilibrium, NE. Nash equilibrium is stable and can be used as a prediction of a game [6]. Nash equilibrium is a basic concept in non-cooperative games. According to the choice of strategies, it can also be divided into pure strategy Nash equilibrium and mixed strategy Nash equilibrium. A game may have no pure strategy Nash equilibrium, or have a pure strategy Nash equilibrium, or have multiple pure strategy NE, but it must have at least one mixed strategy Nash equilibrium.

What we found is that Nash equilibrium are not necessarily pareto efficient, for example the Prisoner's dilemma game. In the Prisoner's dilemma game, two players has two actions, confess (C) to a crime or stay silent (S). Confession by both results in 5 years each in jail. Silence by both results in 2 years each in jail. If A confesses and B stays silent then A gets no penalty and B gets 10 years in jail (and conversely). The Prisoner's dilemma game has a unique pure strategy Nash equilibrium, where both players confess, and the payoff is (-5, -5). But (Silence, Silence) is better for both the two players. So the Nash equilibrium of the Prisoner's dilemma is not pareto optimal [7]. Because the both prisoners choose their dominant strategy based on their perfectly rational human assumption.

The inefficiency of Nash equilibrium in prisoner's dilemma is caused by the conflict between individual rationality and collective rationality. Cournot model, Bettland model, common resources problem are all prisoner's dilemma type games [8]. There are conflicts between individual rationality and collective rationality in these games, so their only pure strategy Nash equilibrium is not pareto optimal.

Finding mixed strategy Nash equilibrium is an interesting process. First, assume the probabilities that the player chooses various strategies. Then, in such a probability setting, the expected payoff of each strategy chosen by the opponent is equal. For example, A has two actions, U, D. B has two actions, L, R. A selects a probability distribution PU and PD. It's meaning that with probability PU Player A will play U and with probability PD will play D. The player A want to make sure has the right probability PU and PD, so that the expected payoff of B choosing L is equal to the expected payoff of B choosing R. Equal is very important here, and if it's greater than, B will choose strategy L. Otherwise, if it is less than, B will choose strategy R. Anyway, if it's inequality, B can make a deterministic choice because of the probability that A sets. Perfectly rational A would not allow that to happen. Another way to find Nash equilibrium is to plot the best response function.

"Constant (payoff) sum" games is a special kind of game in the simultaneous play game, in which any increase in the payoff to one player is exactly the decrease in the payoff to the other Player. So the players in the game are a strictly antagonistic relationships. It is impossible to have the best response strategies combination of both people at the same time. There must be no pure strategic Nash equilibrium in this type of games. But this game must have at least one mixed strategy Nash

equilibrium. But sometimes in a game with multiple pure strategy Nash equilibrium, the mixed strategy Nash equilibrium may not have an advantage.

An important example is the hawk-dove game, which has two pure strategies Nash equilibrium.

.(Hawk, Dove) and (Dove, Hawk), the payoff are (8, 0) and (0,8). But there is a NE in mixed strategies. For each player, the expected value of the mixed-strategy NE is between the other two payoffs.

For the continuous strategy game, we need to use functions, equations and so on to solve the best response function, and then to solve the Nash equilibrium. For example Cournot model, Bertrand model, Common resource problem. At the same time, the discussion of the two-player game can be extended to the multi-player game, such as Cournot model have N firms, Bertrand's model have N firms.

3.2 The sequential play game

In this two-player games, one player chooses an action before the other player chooses an action. The first player's action is both irreversible and observable by the second player. The first player knows that his action is seen by the second player, for example, commitment games, bargaining games[9].

When the sequential play game has finite strategies, we can use the extensive form to rewrite the game. The first challenge is whether the Nash equilibrium of the simultaneous games applies to dynamic games. Because of the opportunism of the post-acting player in dynamic games, there are unbelievable elements in Nash equilibrium of the simultaneous games. So Nash equilibrium of the simultaneous game can't be used directly here, so you have to find a new equilibrium. Credible NE of this sequential play game are called subgame perfect Nash equilibrium, SPNE. The subgame perfect Nash equilibrium is more rigorous and feasible than NE. If there is more than one NE, subgame perfect Nash equilibrium can help we argue that which one is more likely to occur than another.

The way to find the subgame perfect Nash equilibrium is to go from the last stage of the dynamic game to the first stage to look for the Nash equilibrium at each stages. For example, in the principal-agent model, the principal first selects the strategy of entrusting or not entrusting. After the principal chooses, the agent chooses the strategy of working hard or not working hard[10]. When calculating the subgame Nash equilibrium of this game, we should first analyze the behavior of the agent who acts later, and then analyze the behavior of the principal who acts first.

For the continuous strategy dynamic game, the extensive form is invalid. And then we also need to use the functions, the equations, to find the best response function of the player. For example, Stackelberg model. In the Stackelberg model, we assume that the products decided by the firm can be subdivided infinitely, so the output is a continuous variable. The Stackelberg model can also be extended to the case of N firms.

3.3 Cournot Model, Bertrand Model and Stackelberg Model

Cournot model, Bertrand model and Stackelberg model, they are the decision-making problems of enterprises based on oligopoly market. And both they belongs to continuous strategy problems.

In the Cournot model, two firms producing the homogeneous product determine the output at the same time. In the Bertrand model, two firms producing the homogeneous but not exactly the same product simultaneously determine the price. In the Stackelberg model, two firms that produce the homogeneous product decide how much to produce in turn. For the sake of simplicity, let's assume that all businesses have no fixed costs and only marginal cost. Where necessary, it can be assumed that there are fixed costs, or that firms businesses have different costs.

Cournot model and Bertrand model, they're both continuous strategy simultaneous play games. But the firms in Cournot model produce exactly the homogeneous products. The firms in Bertrand model produce different but highly alternative products. The firms in Cournot model determines quantity; The firms in Bertrand model determines price. They both have a pure strategy Nash equilibrium. And in Cournot model the equilibrium outputs of two firms is equal. In Bertrand model the equilibrium

prices of two firms is equal, regardless of its size. Because the products produced by these two enterprises are homogeneous, if the prices are different, the enterprises with low prices will occupy all the markets. The Bertrand model can also be extended to the case of N firms.

In Cournot model and Stackelberg model, the firms produce exactly the homogeneous products and determine quantity. But firms in Cournot model make decisions simultaneously, and firms in Stackelberg model make decisions sequentially. So the Stackelberg model is also called the leader-follower model. Because the enterprises in Cournot model determine the number of products produced at the same time. Therefore, the best response function of two enterprises should be calculated at the same time, and then the intersection point of the reaction functions can be obtained. The intersection point is Nash equilibrium. So, in Cournot model the equilibrium output of the two firms is calculated simultaneously. However, in the Stackelberg model, the two enterprises determine the output in order. Therefore, we should first calculate follower's best response function, and then bring it into the profit function of the leader enterprise. At this time, the equilibrium output of leader enterprises is calculated first. Then the equilibrium output of the leader enterprise is brought into the response function to calculate the follower's equilibrium output [11]. Stackelberg model has only one pure strategy Nash equilibrium. The leader produces more in the Stackelberg model than the Cournot, and the follower produces less in the Stackelberg model than the Cournot. Because the leaders have the advantage of acting first. Of course, in a dynamic game, it is uncertain whether it is advantageous for leader or follower.

3.4 Incomplete Information Game

A game in which at least one player is not fully aware of the other players' payoff, the game can be called an incomplete information game. For example, there is the sealed bid auction game, in which two bidders write down their bids in sealed envelopes based on their own valuations of the object. The player with the highest price wins the object. We set the bidder's payoff equal to his valuation minus the final bid. Valuations are private information that only each participant knows himself, so we have no way of knowing what others are getting. For example, the principal-agent problem. When the principal is unaware of the efforts of the agent, the principal pays the agent according to the agent's output. The principal has no control over the agent's payoff.

In order to learn game theory better, we need to learn some basic knowledge of probability and statistics, and some knowledge of random variables. For example, when we are finding a mixed strategies Nash equilibrium, we need to assume the probabilities of a player choosing his strategies. And we also need to calculate the expected return in different strategies for player. For example, for a game with multiple players, the expected returns of multiple random variables are also required [12].

We need more statistical knowledge in the sealed bid auction game. For example, although the valuation of the object is our private information, we can still know the probability distribution of the valuation. That way, we can calculate our expected value when one bid higher than other. If we have N bidders, note the bidders's valuations are independent. We can calculate the joint probability and the joint expected payoff function for N bidders. The best strategy for each player in a first-price sealed bid auction game is to bid at half the price of the valuation. This bidding law reflects a basic contradiction faced by the game players in the bidding activities. The higher the bid price, the greater the chance of winning the bid, but the smaller the profit. The lower the price, although the chance of winning the bid is also low, but once win the bidder's benefit is greater [13]. The best choice for a player is to consider both the chance and profit of winning the bid.

In the second-price sealed bid auction model, each player's optimal strategy is to make the bid price equal to their own valuation of the good. Or perhaps, honesty is the best bidding strategy at such times. Because in second-price sealed bid auction, when a bidder wins, the final bid price is independent of his valuation. In the absence of collusion, each bidder's optimal strategy is to choose a bid price based on his own valuation of the auction item. When the bidding price is lower than the valuation, it reduces the chances of winning the goods. When it is higher than the valuation, the bidder gets a no-profit deal, although it increases his chances of winning. Because he might have to pay more

than he valued the goods. Because of incomplete information, the Nash equilibrium in these two kinds of sealed bid auction models may not be the most efficient.

In real life, competitive firms often keep their production and sales confidential, making it difficult for other manufacturers to control the real situation. Therefore, we can also improve the previous Cournot model to become a Cournot model with incomplete information.

4. Conclusion

In the current course, we have learned a lot about games, including the research content of game, the definition of games, the classification of games, the equilibrium of games, as well as some specific and important games examples. We understand what a simultaneous play game is, what a sequential play game is, what a perfect information game is, what an incomplete information game is. We also know the concept of Nash equilibrium, mixed strategy Nash equilibrium, subgame perfect Nash Equilibrium. And the subgame perfect Nash equilibrium is more stringent than the Nash equilibrium, and the Bayesian Nash equilibrium are stronger than the conditions of subgame perfect equilibrium. At the same time, I also deeply felt the application of statistical probability knowledge in our study.

Learning such game theory and the concept of equilibrium can help us understand how players choose strategies to achieve equilibrium in an interactive relationship. Under the right conditions, this equilibrium can be used as a prediction of participants' eventual choices. Such predictions can then be used to explain what's happening, why it's happening, and how to fix it.

Learning game theory can also help us understand other subjects, Hawk-dove games, for example, can be used to model how members of a species act towards each other. Through classic game theory and interesting game cases, I have a brand new understanding of phenomena in daily life.

References

- [1] Larry Samuelson. Game Theory in Economics and Beyond[J]. The Journal of Economic Perspectives, 2016, 30(4) : 107-130.
- [2] Licai Yang et al. Route Choice Model Based on Game Theory for Commuters[J]. Promet - Traffic & Transport, 2016
- [3] QD Lã, Yong H C , Soong B H . An introduction to game theory[J]. Oxford University Press, 2005.
- [4] Gibbons R . Primer In Game Theory[M]. 1992.
- [5] Dean Patrick Vesperman and Chris H. Clark. Game Theory in the Social Studies Classroom[J]. The Social Studies, 2016, 107(6) : 194-199.
- [6] Chinchuluun A , Pardalos P M , Migdalas A , et al. Pareto Optimality, Game Theory And Equilibria[M]. Springer New York, 2008.
- [7] Rui Bai, Weiyu Pan, Chao Li. Game Theory Analysis about the Red Packet Money[C]//Proceedings of 2nd International Conference on Education, Management Science and Economics(ICEMSE 2017)(Advances in Economics, Business and Management Research (AEBMR), Vol.49), 2017:288-290.
- [8] Michael Finus and Bianca Rundshagen. Game Theory and Environmental and Resource Economics—In Honour of Alfred Endres, Part Two[J]. Environmental and Resource Economics, 2016, 65(1) : 1-4.
- [9] Colman, Andrew M . Rationality Assumptions of Game Theory[J]. Journal of Xian University of Architecture & Technology, 1998.
- [10] Chalkiadakis G , Elkind E , Wooldridge M . Computational Aspects of Cooperative Game Theory[J]. Springer Berlin Heidelberg, 2011.
- [11] Zhang M S . Dynamic analysis of Cournot model with multiple firms[J]. Journal of Management Sciences In China, 2002.
- [12] Ramalingam Shanmugam. Mathematical programming and game theory[J]. Journal of Statistical Computation and Simulation, 2020, 90(3) : 570-570.

- [13] Mathematics - Game Theory; New Game Theory Findings Has Been Reported by Investigators at University of Rosario (Informed entry in auctions) [J]. Journal of Technology & Science, 2018, : 631-.