A Description of Game Theory

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Abstract. Game theory implies the logic of problematic situations with solutions through mathematical calculation. Since 1944, the game theory has been applied to military decisions, educational materials, and to find methods for complicated circumstances. The equilibria, different strategies to maximize surplus, and analysis of games could be inferred to assist players. The dominant strategy is the esse.ntial portion of the research to the game where the choice results in the best payoffs not affected by the other players' decisions. Closely connected to the dominant strategy, the Nash equilibrium represents the idea of the stable point where there is no profitable deviation for any player. Moreover, through calculation, the mixed strategy could be acquired, in situations when the probability of choices of all players is random, in the probability distribution. Finally, studies of shading the bid in auctions could maximize the surplus of bidders in partial auctions. The research could accommodate deliberate determinations in various games.

Keywords: Game Theory, Dominant strategy. nash equilibrium.

1. Introduction

Game theory is a set of theories that claim and structure how individuals make decisions strategically based on mathematic calculation and the referring logic. It is closely associated with microeconomics and macroeconomics and performs as the core. The representing games execute equilibriums solutions that give the strategies best for each player, to the most extent for both players. The theory provides conclude oligopoly game, bilateral monopoly, and duopoly games and emerges the analytical solution considering the prevailing circumstance. Individuals and groups interested in the game have difficulties with their determinations, where areas game theory makes assumptions to each of them[1].

1.1. Connections between game theory and economics

In terms of microeconomics, game theory assists the decision of the pricing of some firms and households. Whether airlines should rise, decrease, or keep the charge the same. One of the factors that affect that is the other airline’s decision to boost its competitiveness, the original firm’s decision to ignore it or make some shifts. Furthermore, some businesses like International Business Machine Corporation are highly competitive and have a significant market share due to their strategic development of products, which boosts the speed of innovation and growth of the business and increases market capitalization.

Game theory is also connected to macroeconomics. The “Entry Game” involves the situation of war. It is a sequential game where the first player has to choose whether to enter the game or to quit. If they quit, they get a payoff of 0, and the other player receives the payoff of 2; if they enter, the second player chooses to fight or accommodate. If they choose to fight, they both get a payoff of 0. If they choose to accommodate, they both get the payoff of 1. This game requires both players to think strategically. The interest of the participation of player 1 is to get the payoff higher than one due to the payoff of 0. There is a risk of entering if the second player chooses to fight. Strategies in this game are various. This game can be assimilated into other games when countries make strategic and decisive choices.

2. History

Game theory was initially originated in 1944 by John von Neumann and Oscar Morgenstern, creating the first strategic games where multiple participants are playing a game analyzing each other's optimal strategy and the appearance of the equilibrium[2]. The earliest game that the founders created was *The Theory of Games and Behavior*. Both creators assembled the advances and the basics of the literature of strategic games. There are limitations to the earliest advances, which are applicable for narrow and particular circumstances. As society develops and the specialization becomes more advanced, professors and researchers in universities generalize the game establishing *The Theory of Games and Behavior*. In 1970, game theory was established and recognized as an appropriate and beneficial tool[3].

Before John V. Neumann and O. Morgenstern invented game theory, decisions of strategies, communication, and analysis in the market for economists were competitive. It was a handful for economists to operate corporations and industries in businesses with the theory of demand and supply progressed by Alfred Marshall [4].

2.1. Development of equilibria

After when the game theory was first mentioned and appeared by John con Neumann, they came up with the concept of 'equilibria', which was further developed by John Nash. John Nash contributed to the field of game theory massively and expanded the possible outcomes and ways of calculating different kinds of solutions. The knowledge of studies of pure mathematics at Princeton's University commences his interests in game theory and therefore made progress. In a specific class of games, he presents that there are outcomes that haven't been explored. In 1950, he converted his thoughts to theories – Nash equilibria.

The algebraic geometry impressed the fellow developers and participants who applied the theory. This has made a decisive change to the field of game theory[4]. Nash equilibrium is the idea of the stable point where there is no profitable deviation for any player. This equilibrium sets the stable situation where all players have the best responding payoffs. The game's innovation provoked new extends of the equal and steady case for players and created another domain of how strategies can be made[3].

Before game theory was used to analyze problems mathematically or use any other approaches, it was used in the military for some leaders and has dramatically affected the general condition of wars[5]. Hernan Cortes, a conqueror from Spain, comprised game theory when landing in Mexico leading a small troop and barely has the probability of surviving. Avoiding being beaten on land and by bombs and firearms, they landed successfully using vessels. He created an unfortunate effect when burning the ships flamboyantly, making the enemies conscious. They predicted that any commanders who have confidence in themselves would withdraw with decisions and commands once the situation is adverse to them to create extreme optimism. The enemies then retreated to the back hills creating an opportunity for Cortez to attack. "Mixed Strategies" has applied to the United States during the process of military planning in World War II. The commander selected a set of plans and actions for each part of the troop, predicting enemies' movements strategically due to their proficiency and potentialities[6].

3. Explanations and examples

Game theory is a theoretical structure that can be applied to specific situations where are multiple players make decisions that will affect the other players' payoff and punishment, mainly the process and the alignment of the circumstance. Therefore, decisions need to be made strategically considering what the outcome is despiting for whom. There must be one or more pure methods for each player in each game. In most situations, each player will be affected by each decision made by any player, only in some cases where the different effectiveness scale is extensive, such as one citizen and a city. Despite the difference in scale to some extent, all participants have positive and negative payoffs. All 'games' have a payoff matrix. The matrix presents each choice of each player's decision and the outcomes. The dominant strategy and Nash Equilibria could be found on most outcomes. A dominant strategy is a superior choice for a player regardless of how other players acts[7]. Nash equilibria are the idea of the stable point or outcome where every participant has no profitable deviation. On the other hand, it is when every player is best responding. It could appear more than once in some situations and in certain circumstances in which there is no Nash equilibrium. The purpose of a Nash equilibrium is to find the point where it is most stable, meaning a point where all players' payoffs are the same, making the game easier to analyze.

3.1. Prisoner Dilemma

Prisoners Dilemma is a classic and representative game of game theory.

Clyde / Bonnie	Silent	Confess
Silent	-5, -5	-30, 1
Confess	-1,-30	-10, -10

Figure 1. Payoff matrix of Prisoner's Dilemma

In this game, two players have been caught in the police station and being interrogated by the police. Suppose the participants are Clyde and Bonnie. They both have a choice to either stay silent or to confess. If both confess, they need to stay in accept their imprisonment for ten years. If one confesses and the other stays silent, the player who stays silent goes to jail for 30 years, and the one who confesses only has to stay at the prison for one year. If both of them stay silent, both of them need to go to jail for five years. For Clyde, if Bonnie chooses to stay silent, they have a choice of -5 and -1. -1>-5, so he should choose to confess in this situation. If bonnie chooses to confess, Clyde has the choice between -30 and -10. 10>-30, so he will choose to confess under this condition. Therefore, for Clyde, the dominant strategy is to confess. For Bonnie, if Clyde chooses to stay silent, she has the choice between -5 and -1. -1>-5; therefore, it is best for her to choose to confess. If Clyde chooses to confess, she has a choice of -30 and -10. -10>-30, in this case, it is best for her to choose to confess as well. Therefore, for Bonnie, the dominant strategy is to confess.

To confess is the dominant strategy for both players. It is the best response for both players, and every player's action choice is the best response to their opponent's action choices. This, therefore, meets the requirement of a Nash equilibrium.

3.2. Battle of the Sexes

A representing game of two – Battle of the Sexes.

Bob / Ann	Ballet	Football
Ballet	2, 1	0, 0
Football	-1, -1	1, 2

Figure 2. Payoff matrix of Battle of Sexes

In this game, there are two players, a male, and a female. Suppose they are Bob and Ann. They made an appointment to go out together. Bob likes watching football matches, and Ann enjoys watching the ballet. There are two Nash equilibria. One is that both Ann and Bob choose to go to the ballet. In this one, even though Bob cannot watch football as he wishes, he still stays together with Ann, as Ann is benefits by getting the best payoff – 2. The second Nash equilibrium is that they both choose football. Even though Ann cannot go watch ballet as she wishes, she could still stay together with Bob, as Bob is benefits by getting the best payoff. It is not required in this game that every player play a dominant strategy in Nah equilibrium. A dominant strategy is the best choice, not affecting others' choices. If Ann chooses b, Bob should choose b as well. However, if Ann chooses f, the best for bob is to choose f too. There is no dominant choice for both players. Therefore, all players don’t always have to play a dominant strategy in Nash equilibrium.

The examples given above are simultaneous games, where both players make the decisions at approximately the same time. Both players' decisions are not based on how each other acts. A sequential game is a situation when certain participants make their decision before someone else does. There is an order of action. The examples given above are simultaneous games, where both players make the decisions at approximately the same time. Both players' decisions are not based on how each other acts. A sequential game is a situation when certain participants make their decision before someone else does. There is an order of action. The entry game has two versions, simultaneous and sequantial.

3.3. Entry Game

Firm 2 \ Firm 1	Out	Enter
Out	0, 2	0, 2
Enter	-1, -1	1, 1

Figure 3. Payoff matrix of Entry Game (simultaneous)

Figure 3 shows the payoff matrix of the simultaneous game of the entry game. There are two players in the game. Suppose they are firm 1 and firm 2. Both players choose to either enter our stay out of the competition. If firm 1 chooses to stay out, it is the same for firm 2 to choose either option. If firm 1 chooses to enter, firm 2 has the chance of -1 and 1. 1>-1; therefore, firm 2 will decide to enter under this condition. If firm 2 chooses to stay out, firm 1 has the option of 0, and -1, 0>-1; under this condition, firm 1 will choose to stay out. If firm 2 decides to enter, firm 1 has the option of 0 and 1, 1>0; firm 1 will choose to enter under this condition. Both players stay out, and both enter is the two Nash equilibriums for this game.

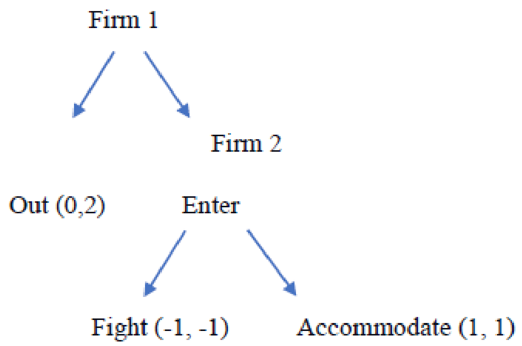

Figure 4. Payoff matrix of Entry Game (sequantial game)

Figure 4 shows payoff matrix of the sequential version of the entry game. Firm 1 will choose to either stay out or enter. Firm 2 will choose after when firm 1 chooses. Its choices will be made differently based on the first choice. If firm 1 stays out of the game, firm one will get the payoff of 0, and firm 2’s payoff is 2. If firm one chooses to enter, firm two have the choices between -1 and 1. Their payoffs will be the same if firm 1 joins. Understanding the rules and the payoff matrix, before firm 1 takes action, suppose they have a chance to communicate. To get the highest payoff – 2, firm 2 need to express that they are going to choose to fight to form a threat to firm 1, saying that they will choose to fight if firm 1 enters, and if it is as they wish, firm 1 stays out, and they get the payoff they want. However, firm 1 will not believe them because 1>-1, firm 2 will not choose -1. Therefore, firm 1 gets the highest payoff of themselves as firm 2 avoids the worse outcome for them.

An incredible threat is something that an individual or company lies that they will take some action. To get positive results, the referring players will not believe it. A game with incredible threats has a subgame perfect Nash equilibrium. The subgame perfect Nash equilibrium in this game is (0, 2).

When players randomize their choices and the game has no pure strategies, we need to use a mixed strategy to calculate the probability of each option to be chosen.

Players can adjust the probability distribution. This is a payoff matrix of a game of 2 players. Player A has the options of up and down; player B has the options of left and right. To form a formula, we suppose 'π_u' represents up and 'π_L' represents left.

3.4. Mixed strategy

Player B / Player A	Left : π_L	Right: $1-\pi_L$
Up: π_u	1, 2	0, 4
Down: $1-\pi_u$	0, 5	3, 2

Figure 5. Payoff matrix of an example game

Figure 6 shows the calculation of the expexted payoff for all the options in the matrix shown in figure 5. Calculating π_L requires information from π_u.

Calculating π_u requires information from π_L.

$$E(L)=2\pi_u+5(1-\pi u)=5-3\pi_u \quad w \tag{1}$$
$$E(R)=4\pi_u+2-2\pi u=2+2\pi_u \tag{2}$$
$$5-3\pi_u=2+2\pi_u \tag{3}$$
$$\pi_u=60\% \tag{4}$$
$$E(u)= \pi_L+0*(1-\pi_u)= \pi_L \tag{5}$$
$$E(d)=0*\pi_L+3(1-\pi_L)=3-\pi_L \tag{6}$$
$$\pi_L=3-3\pi_L \tag{7}$$
$$\pi_L=3/4 \tag{8}$$
$$\pi_L=75\% \tag{9}$$

Figure 6. Calculaiton of the probability distrubution of the mixed strategy

After calculating each option (left, right, up and down), we draw an equal sign and calculate the value

Player B / Player A	Left : 75%	Right: 25%
Up: 60%	1, 2	0, 4
Down: 40%	0, 5	3, 2

Figure 7. Payoff matrix of the game shown in Figure. 5 after plugging in the probabiloty distributions.

Figure 7 shows the probability distrubution using percentages which is how likely the players might choose this choice in mixed strategy.

4. Auctions

Auctions are also representetive for game theory. In all auctions, there must be two or more players which set their valuation prices and bid. The product will be sold to the highest bidder which in most cases promots the competiveness of the game. We distinguish types of auctions in 4 main categories. The English auction, dutch auction, first price sealed-bid, and second price sealed-bid.

4.1. English auction

English auction is the most common type of auction due to the use of some global companies. Each parcitipant has their independent private values[8]. The auctioneer announces the lowest price of getting the product and the level of price will be increased as bidders and must not be reversed. There is a specified amount of cost to increas each time by the players. Until a bidder bids to a price when no players are willing to bid higher, the highest bidder will therefore win the auction and get the product, paying their valuation price. Shade their bid means bidding the price lower than they think how much the product worth. This strategy is suitable in the English auction because bidders aim is to get the product and pay between the max surplus and the secon dhighest valuation in order to win the game. Max surplus>payment>the second highest valuation. However, everyones' valuations are private, therefore, if the bids are shaded too much, the probability of winnig the auction might decrease. Moreover, it is the best advantage for the bidders to quit exactly at their valuation price to avoid the negative surplus of paying higher than their valuation.

4.2. Dutch Auction

Dutch auction is an auction in some ways opposite to the English auction. The price starts from very high which barely anyone would buy it at that point. Then, the price will gradually fall announcing by the auctioneer. When it reaches to the highest player's valuation, and when they raise their hands, the game ends immediately[9]. The bidder who wins will pay the second highest valuation.For example, the highest bidder's valuation price is 100, and the price arrives 100. The surplus will be higher for the player if they chooses to bid at 80. As they shade their bid, the probability of winning gradually decreases.
Max E [surplus]=Pr [win]*surplus. The maximise expected surplus is the probability times surplus.

4.3. First Price Sealed-Bid auction

In the first price sealed-bid (FPSB)auction, buyers records their bids and provides to the managers privately without anyone knowing[10]. The submition of valuation prices must be simultaneous. The highest bidder will get the object and pay their bid. In this auction, shading the bid is not suitable in the situation when the expectations of other participants are high. Oppositely, when the expectation value of other participants is high, shading the bid could increase the surplus, getting the product lower than the individual private surplus.

4.4. Second Price Sealed-Bid auction

Second price sealed-bid (SPSB) auction has great similarities with FPSB. Participants submits their valuation privately and simultaneously. The second highest bidder wins the auction and gets the product.

5. Conclusion

We have concluded the detailed definition of game theory in two parts, examples of games and auctions, which both include various strategies and forms. First, we zoomed into the inner structure of a game and identified types of games, characteristics, and rules. Then, we used numerical solutions to calculate some methods, such as the probability distribution of a mixed strategy. Comparing each payoff of all players, we can find the dominant strategy and Nash equilibria. Having the ability to spot the dominant strategy and Nash equilibria could assist participants in making decisions about which is the best response for them. Moreover, we also identified the four types of auctions and their strategies. By having an overview of the game's rules, we could find the strategy of shading the bid and either bid higher or lower than the valuation price of each bidder. Knowing the strategies could assist participants in winning the game and getting a suitable surplus.

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