Research on the Application of Var Model in Shanghai Interbank Lending Market

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Abstract. Based on the Shibor data of China's Shanghai Interbank offered rate from May 15, 2017 to May 15, 2022, this paper comprehensively discusses the construction methods of various GARCH family models under different residual difference distribution, and adopts the out-of-sample forecasting method of rolling time window. The forecast value of interest rate volatility under different model assumptions is calculated empirically, and the VaR is calculated based on it. Through empirical research, we draw the following conclusions: (1) t distribution can better describe the distribution of the series of Shanghai Interbank offered rate; (2) The leverage effect of Shanghai Interbank Offered rate series is significant; (3) At present, there are certain interest rate risks in Shanghai inter-bank lending market.

Keywords: VaR model; The interbank lending market; Interest rate risk; GARCH model.

1. Introduction

Along with our country financial opening gradually enlarging, the interest rate marketization process is accelerating unceasingly. Interest rates have become more frequent and increasingly volatile. The sound operation of commercial banks will further face the challenge of interest rate risk. Especially after the liberalization of the interbank offered rate in China, with the surge of trading volume, the performance of interest rate risk in commercial banks is more prominent. Therefore, seeking an effective quantization method of our country commercial bank interest rate risk becomes urgent affairs. This paper uses VaR model based on GARCH model to measure the risk of Shanghai interbank offered rate Shibor, and tests the accuracy of VaR model.

2. Theoretical analysis and model introduction

2.1. Introduction of GARCH model

There are many kinds of market risk measurement models, among which the VaR value calculated by GARCH model is the mainstream method for global financial institutions to measure market risk. This model is used to predict and explain the dynamic impact of random disturbance terms on variables in the system, including the degree, direction and duration of the impact. Thus, the mutual impact of variables in the economic system is solved. Compared with other risk measurement models, first of all, this model can be applied to various sectors in various fields and various asset portfolios, and can make objective evaluation of the market, which has a wide range of applicability. Secondly, the model can deal with the nonlinear relationship between multiple variables, which solves the problem that the traditional risk quantitative model is difficult to match the nonlinear variable relationship with financial derivatives, and there is a deviation in the market risk of quantitative securities portfolio. Third, the model quantifies the value of risk specifically, so that relevant regulatory authorities, financial institutions and investors can control risks through digital analysis and make correct regulatory measures or investment judgments. Based on the above points, the VaR value of GARCH model is used to calculate the market risk in the empirical analysis of this paper.

2.2. Basic form of GARCH model

In order to overcome the influence of conditional heteroscedasticity on empirical results in time series, T. Bllerslev (1986) proposed GARCH model, which is a regression model specifically for financial data. In other words, the variance is changing over time. When modeling the empirical analysis of carbon futures yield series and simulating the characteristics of carbon price, it is necessary to test the sample data, that is, whether there is unit root, heteroscedasticity and ARCH effect.

GARCH family models include: GARCH model, TARCH model and EGARCH model. They are all developed from ARCH models and are suitable for estimating time series in the presence of autoregressive conditional heteroscedasticity.

The GARCH family of models studied in this paper consists of two equations, one is a mean equation and the other is a conditional heteroskedastic equation. The mean equation is expressed as follows:

$$y_t = \mu + \varepsilon_t$$

Where: μ is the mean and ε_t is the residual of the mean equation.

Due to the differences in the conditional heteroskedastic equations, the GARCH (1,1) family of models can be divided into at least three models, namely, GARCH (1,1), TARCH (1,1) and EGARCH (1,1) models.

2.3. Residual distribution hypothesis of GARCH family models

In this paper, when using parametric method to calculate VaR, the residuals in GARCH family models are assumed to follow normal distribution and t-distribution respectively.

Normal distribution is an important probability distribution, and its good statistical properties, such as symmetry and easy addition, determine its important role in financial analysis.

A large number of financial empirical studies have shown that the distribution of market factors (such as prices and interest rates) in financial markets has the characteristics of sharp peaks and fat tails, which is quite different from the normal distribution. The T-distribution is a distribution with a fatter tail than the probability density function of the normal distribution, which is suitable for analyzing distributions with fat-tailed properties.

2.4. Advantages and disadvantages of GARCH model

GARCH model considers the volatility clustering characteristics of financial time series, GARCH model can simulate and measure the VaR value well, and explain the fat tail of volatility. However, any model has certain limitations, and GARCH model is no exception. The biggest limitation of the model is that the parameter estimation is poor and has strong non-negative constraints. Meanwhile, the dynamics of the conditional variance is limited and there is a certain deviation from reality.

2.5. VaR model

Based on the definition of VaR, we express the VAR of interest rate volatility by the formula as follows:

$$VaR_{t+1} = w_t(E(r) - r^*) = w_t \alpha_c \sigma_{t+1}$$

Where $VaR_t \quad 1$ is the VAR predicted for the next day in period t; E(r) is the expected value of the interest rate; r* is the minimum interest rate; α_c is the quantile of the corresponding distribution at a given confidence level c; σ_{t+1} is the volatility of the next day's interbank lending rate predicted in period t; w_t is the asset value in period t.

The value of the quantile α_c depends on the probability distribution function of the interest rate. Standard deviation σ_{t+1} is divided into conditional standard deviation and unconditional standard deviation. The calculation of VaR in this paper is mainly achieved by establishing a dynamic model of the estimated conditional standard deviation.

3. Empirical analysis and model testing

3.1. Data source and processing

This paper selects 1237 sample data of the Shanghai Interbank offered rate Shibor (overnight O/N) from May 15, 2017 to May 15, 2022 (data from CSMAR database, EViews10 and Matlab2019a were used for data processing. Among them, 988 samples (the first two years) are used to estimate the model parameters and forecast the volatility of the next period, so as to measure the value of VaR, and 249 samples (the last year) are used to test the accuracy of VaR model. (This paper uses the rolling time window method to conduct out-of-sample prediction test).

In this paper, the logarithm of interest rate is used, that is, rt is set as shibor at time t, and shibor at time t − 1 is r_{t-1}, to transform the interest rate series into logarithmic rate of return:

$$R(t) = lnr_t - lnr_{t-1}$$

3.2. Empirical process

3.2.1 Analysis of statistical characteristics

The descriptive statistical characteristics of the series of Shanghai Interbank Offered Rate are shown in Figure 1:

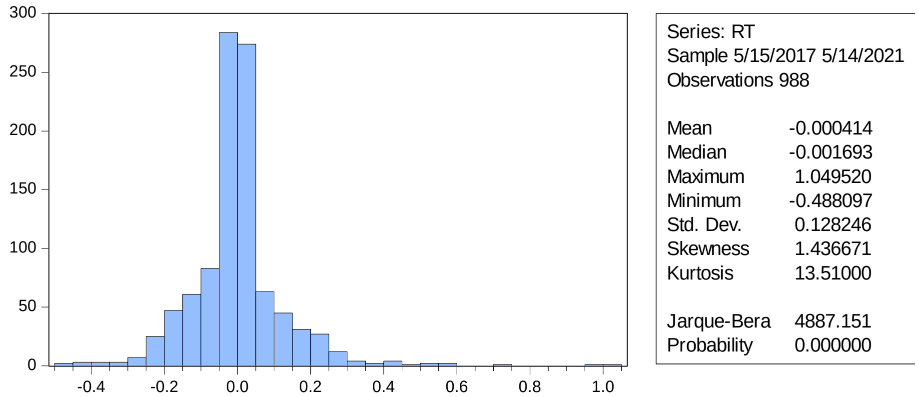


FIG. 1 Descriptive statistics of the series of the Shanghai Interbank Offered rate

Figure 1 shows that:

(1) The mean value of the interbank lending rate series is − 0.000414, less than 0;

(2) There is a large difference between the maximum value and the minimum value of the interest rate series, which shows that the fluctuation range of the Shanghai Interbank Offered rate is large;

(3) Kurtosis (K), also known as the kurtosis coefficient, represents the number of characteristics of the peak of the probability density distribution curve at the average value. Intuitively, kurtosis reflects the thickness of the tail. The standard normal distribution has a K value of 3, and the interest rate series exhibits Kurtosis (Kurtosis=13.51> 3), indicating that the distribution has a relatively obvious "sharp peak and fat tail" shape;

(4) Skewness (S) refers to the asymmetry and skewness of the curve in statistics. The skewness is measured by dividing the third central moment by the third power of the standard deviation. If the normal distribution is followed, S=0 and the tail length on both sides is symmetric. S< 0 means that the curve has a negative deviation. In this case, the curve has a trend to the right, which means that the mean value of the right side is greater than the mean value of the left side, and the tail of the left side is longer. S> 0 means that the distribution has a positive deviation, also known as right-skewness, which intuitively shows that the tail on the right is very long compared with the tail on the left. However, if S is close to 0, the distribution can be considered symmetric. The skewness of the interest rate series is 1.436671, which is skewed to the right, indicating that the distribution of the series has the phenomenon of right tailing.

(5) J-B statistics, namely Jarque-Bera statistics, is used to test whether the data are normal, and it tests the samples according to the least square residual. When the variable follows a normal

distribution, the value of the J-B statistic is 0, and if the variable is not normal, the JB statistic will be a gradually increasing value. The JB statistic of this interest rate series is 4887.151, which is greater than the critical value of 5.99 at the significance level of 5% and the P value is 0.000000, so the hypothesis of normal return distribution can be rejected.

The above analysis shows that the interest rate series presents a non-normal "peak and fat tail" distribution characteristics.

3.2.2 Stationarity test

In the analysis of economic phenomena, if the time series of the analyzed data is not stable, the regression equation analysis cannot be done, otherwise it will cause "spurious regression," that is, there is no correlation between different variables at all, but the regression results may draw a wrong conclusion that there is some correlation, so that the constructed model does not conform to the economic phenomenon at all and has no explanatory power. Only by smoothing the non-stationary time series in a certain way can we do regression. In this paper, the ADF unit root test, which is commonly used and general, is adopted to test the stationarity of the series. The test results are shown in Figure 2:

Null Hypothesis: RT has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-23.72958	0.0000
Test critical values:	1% level	-3.436769	
	5% level	-2.864263	
	10% level	-2.568272	

FIG. 2 Serial ADF unit root test

At the significance levels of 1%, 5% and 10%, the critical values of the unit root test are − 3.4437, − 2.8673 and − 2.5699 respectively, and the test statistic value is − 23.72958, which is far smaller than the corresponding critical values. It indicates that the logarithmic interest rate series from May 15, 2017 to May 15, 2021 is stationary and does not have unit root.

3.2.3 Trend analysis of Shanghai Interbank Offered rate

According to the daily fluctuation of shibor overnight lending rate, eviews10 is used to draw the fluctuation characteristics of Shanghai Interbank offered rate yield series, as shown in 3:

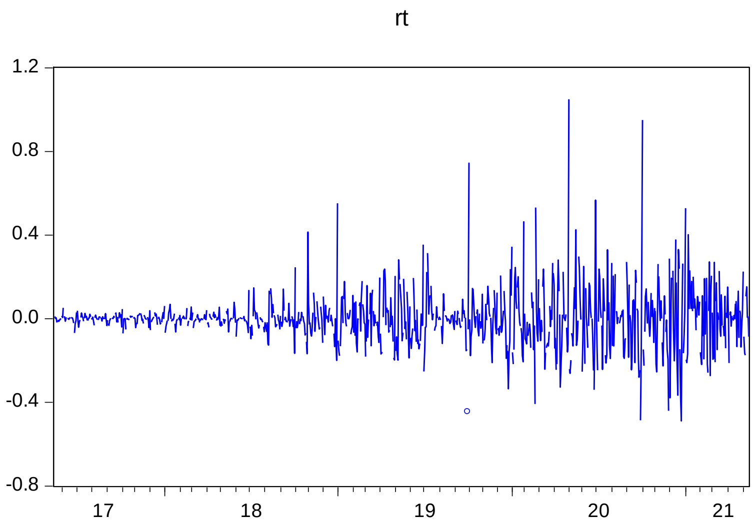


FIG. 3 Fluctuation of the yield series of Shanghai Interbank offered rate

First of all, it can be seen from the figure that the yield sequence of interest rate fluctuates up and down at 0. From the point of view of the stochastic process, it can be considered that the log return series belongs to the sequence of stochastic processes. In addition, the variance of Shibor yield series is constantly changing, that is, the variance is relatively small in one period, and the variance is relatively large in another period, showing a relatively obvious "volatility aggregation" phenomenon, which indicates that the volatility of Shibor yield series has autocorrelation and short-term memory. The market disturbance in the early stage will increase the volatility in the later stage, and this effect has a certain persistence. This suggests that there may be ARCH effects, which we examine in the next section.

3.2.4 ARCH effect test of the series of Shanghai Interbank Offered rate

In the ARCH effect test, ARCH-LM and residual square correlogram are used in this paper. As shown in the figure below, F-statistic is the omitted variable test for the joint significance of all squared lag terms of residuals: Obs*R2 is the LM test statistic, Obs is the number of data samples, R2 represents the coefficient of determination, and the test results are shown in Figure 4:

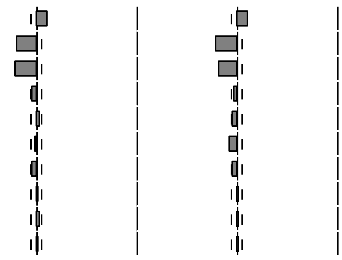
Heteroskedasticity Test: ARCH

F-statistic	33.45718	Prob. F(1,985)	0.0000
Obs*R-squared	32.42379	Prob. Chi-Square(1)	0.0000

FIG. 4 ARCH-LM test

At the 5% significance level, the p-values of the F-distribution and chi-squared distribution of the yield series of interest rates are both 0, which rejects the null hypothesis, indicating the existence of ARCH effect. Consistent conclusions can also be drawn by observing the autocorrelation (AC) and partial autocorrelation (PAC) coefficients of the square of the serial residual. The autocorrelation (AC) and partial autocorrelation (PAC) coefficients of the square of the serial residual are shown in Figure 5:

Date: 05/29/22 Time: 15:24
Sample: 5/15/2017 5/14/2021
Included observations: 988



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.121	0.121	14.629	0.000
		2	-0.189	-0.207	50.021	0.000
		3	-0.221	-0.178	98.426	0.000
		4	-0.035	-0.026	99.668	0.000
		5	0.035	-0.035	100.88	0.000
		6	-0.011	-0.068	101.01	0.000
		7	-0.041	-0.049	102.64	0.000
		8	0.007	0.000	102.69	0.000
		9	0.036	0.003	103.99	0.000
		10	0.031	0.009	104.94	0.000

FIG. 5 Residual square correlogram

It can be seen from Figure 5 that both the autocorrelation coefficient and the partial autocorrelation coefficient of the sum of squares of the serial residuals are significantly different from 0, indicating that there is autocorrelation in the sum of squares of the residuals, and ARCH effect can be determined.

3.2.5 Volatility prediction based on GARCH model

In the previous subsection, we established the ARMA (1,1) model. After the standardized residual analysis of the ARMA model, it was found that there was ARCH effect. The lag order of the mean equation is determined by judging the significance level of the model parameters and the information criterion. Since a high-order ARCH model can be replaced by a low-order GARCH model, we establish generalized autoregressive conditional heteroscedastic GARCH family models, namely GARCH (1,1), TARCH (1,1) and EGARCH (1,1) for modeling. To explain the volatility of the yield series of the interbank offered rate. At the same time, the residuals of each GARCH model are estimated by normal distribution and t-distribution respectively, resulting in a total of six different models.

The heteroscedasticity trend represents the fluctuation difference of futures contract trading price and market risk. The heteroscedasticity trend of Shanghai Interbank Offered rate yield series is shown in Figure 6:

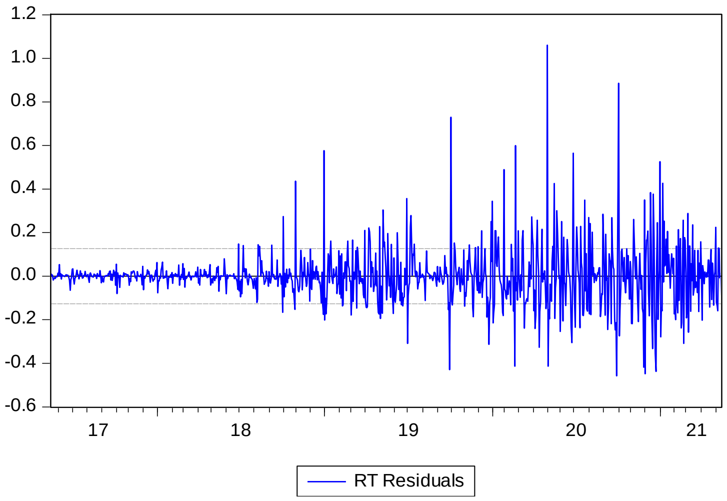


FIG. 6 Heteroscedasticity trend of interest rate yield series

It can be seen from the heteroscedasticity fluctuations of the above interest rate yield series that the fluctuation frequency of the interest rate yield series is high, the fluctuation range is large, the asymmetry is strong, and the change is obvious, indicating that extreme risks may occur in the Shanghai interbank lending overnight rate. The reason may be that the volatility of financial markets is affected by financial events and changes in government monetary and fiscal policies.

3.2.6 Prediction method and calculation of VaR

(1) Explanation of volatility forecasting methods

We perform out-of-sample forecast volatility with a rolling time window of length H for the six GARCH models discussed above (GARCH-Gaussian, GARCH-T, TARCH-Gaussian, TARCH-t, EGARCH-Gaussian, EGARCH-t). Specific methods are as follows:

The data sample population (t=1, 2..., N=1237) is divided into "estimation sample" and "prediction sample." Where, the estimation sample is the data of fixed length containing H=988 trading days, while the prediction sample contains the data of the last 249 trading days (i.e., t=H+1, H+2..., H+M, where M=249).

Step 1, we choose t=1, 2... As the first estimation sample, the data of H are used to estimate the parameters of the above volatility models, and then the volatility forecast of the next day is obtained based on the estimation, which is denoted as $\hat{\sigma}_{H+1}^2$. In other words, $\hat{\sigma}_{H+1}^2$ is the market volatility estimated on day 989 based on the previous 988 sample data.

The second step is to keep the length of the time interval of the estimation sample unchanged (H=988), move the time interval of the estimation sample backward by one day in parallel, that is, the data sample of t=2,3,···,H+1 is selected as the new estimation sample for the second time, and then re-estimate the parameters of the above volatility models. On the basis of this new estimation model, the volatility forecast of the last one day is obtained, which is denoted as $\hat{\sigma}_{H+2}^2$.

Similarly, repeating step by step, we can get $\hat{\sigma}_{H+3}^2, \hat{\sigma}_{H+4}^2, \hat{\sigma}_{H+5}^2 \cdots$. Until the last estimated sample interval at t = M, M + 1, H + M - 1, to get to the last day, that is, the market volatility forecast $\hat{\sigma}_{H+M}^2$ for t=N=H+M=1237.

For the six different types of volatility models mentioned above, the model estimation is repeated 249 times respectively, and each model obtains 249 market volatility estimates of the last one day, which are denoted as $\hat{\sigma}_{\mathrm{m}}^2$, $\mathrm{m} = \mathrm{H} + 1, \mathrm{H} + 2, \cdots, \mathrm{H} + \mathrm{M}$.

(2) Calculation of VaR

Based on the definition of VaR, we express the VAR of interest rate volatility by the formula as follows:

$$VaR_{t+1} = w_t(E(r)_{t+1} - r^*) = w_t \alpha_c \sigma_{t+1}$$

Where VaR_{t+1} is the predicted value-at-risk of the next day in period t (t =H, H+1,···,H+ m-1, where H=988,M=249); $E(r)_{t+1}$ is the expected value calculated from the interest rate that moves the window each day but the window length is fixed at H=988 (that is, 988 interest rate data per day); r* is the minimum interest rate; α_c is the quantile of the corresponding distribution at a given confidence level c; σ_{t+1} is the standard deviation of $\hat{\sigma}_m^2$ calculated in the previous section; w_t is the asset value in period t (t=H,H+1,···,H+ m-1, where H=988,M=249).

3.2.7 Analysis of empirical results

Table. 1 Estimation results under the GARCH (1, 1) equation

Parameters	Normal	Student-t
ω	0.00010*** (7.8916)	0.00009*** (5.9354)
α	0.28955*** (19.674)	0.35395*** (11.918)
β	0.71045*** (60.499)	0.64605*** (34.682)
Degree of freedom	-	12
Quantile (95%)	1.6449	1.782
Actual number of failed days	17	15
Daily average VaR(95%)	0.169136663Pt	0.178297511Pt

Table. 2 Estimation results under the TARCH (1, 1) equation

Parameters	Normal	Student-t
ω	0.00007*** (7.7145)	0.00006*** (5.3859)
α	0.01858* (1.653)	0.07411*** (3.3846)
β	0.81152*** (81.004)	0.73636*** (40.272)
γ	0.3398*** (16.977)	0.37905*** (9.2726)
Degree of freedom	-	12
Quantile (95%)	1.6449	1.782
Actual number of failed days	18	19
Daily average VaR(95%)	0.163648965Pt	0.174681732Pt

Table. 3 Estimation results under the EGARCH (1,1) equation

Parameters	Normal	Student-t
ω	-0.13428*** (-8.3172)	-0.21501*** (-6.6145)
α	0.33144*** (12.397)	0.49087*** (11.94)
β	0.97076*** (303.27)	0.95786*** (160.04)
γ	-0.2521*** (-17.817)	-0.22741*** (-9.2721)
Degree of freedom	-	12
Quantile (95%)	1.6449	1.782
Actual number of failed days	15	14
Daily average VaR(95%)	0.179391255Pt	0.189794392Pt

Note: *** represents significance at the 1% confidence level, ** represents significance at the 5% confidence level, and * represents significance at the 10% confidence level. The actual failure days are used in the model backtesting tests in the next section.

From Tables 1 to 3, it can be seen that all the estimated parameters of the three GARCH (1,1) family models are significant under normal distribution. The estimated parameters of the resulting GARCH (1,1), TARCH (1,1) and EGARCH (1,1) models are all significant at the 95% confidence level. The LM test of the heteroscedasticity effect on the estimated residuals shows that there is no significant heteroscedasticity, indicating that the above six models can basically describe the heteroscedasticity of the interbank offered rate series, and then estimate the fluctuation of interest rates. Comparing the VaR calculated under the two distributions, we can see that the VaR obtained under the student-t distribution is larger, so the actual number of failure days is also less.

3.2.8 Back-test test of VaR model

The above is the VaR result of our country's inter-bank lending market calculated by establishing GARCH family model.

VaR model backtest refers to the model testing method that systematically compares the estimated VaR value with the actual loss and analyzes the actual loss exceeding VaR. The backtest test method used in this paper is the likelihood ratio verification method based on failure rate proposed by Kupiec. The Kupiec backtest is expressed by the formula:

$$LR = -2\ln\left[(1-p)^{T-N}p^{N}\right] + 2\ln\left\{\left[1-\left(\frac{N}{T}\right)\right]^{T-N}\left(\frac{N}{T}\right)^{N}\right\}$$

Where: N is the actual number of failure days; T is the number of sample observations; p is the probability level.

If the initial hypothesis is true, that is, under the condition that p is the true probability level, LR approximately follows the chi-square distribution with 1 degree of freedom. If the confidence level is 95%, that is, p = 0.05, the quantile of the Chi-squared distribution with a degree of freedom of 1 at the 95% confidence level is 3.841. When LR> 3.841, the initial hypothesis is rejected and the model is rejected. In the following, we calculate the LR value and test it on the basis of the above estimation results, and the results are shown in Table 4.

Table. 4 Kupiec test results for VaR model estimation of the Shanghai Interbank Offered rate

Model	Expected days of failure	Actual number of failed days	Actual failure rate	LR statistic
GARCH(l,l)-N	12.45	17	0.068273	1.578837
GARCH(l,l)-t	12.45	15	0.060241	0.517476
TARCH(l,l)-N	12.45	18	0.072289	2.302686
TARCH(l,l)-t	12.45	19	0.076305	3.146362
EGARCH(l,l)-N	12.45	15	0.060241	0.517476
EGARCH(l,l)-t	12.45	14	0.056224	0.195606

Note: The confidence level is 95%, and the expected number of failure days is obtained by multiplying the sample observation value 249 by 0.05; The actual failure rate is the ratio of actual failure days to sample observations; The LR statistic is obtained from the Kupiec backtesting test formula.

Table 4 shows that GARCH(l,l) model, TGARCH(l,l) model and EGARCH(l,l) model

The estimated failure days are basically the same, and the LR statistics are all less than 3.841, which does not reject the null hypothesis, indicating that the six models are basically valid. In contrast, in the GARCH and EGARCH models, the LR value under normal distribution is large, resulting in more actual failure days and a large deviation from the expected failure days, while in the TARCH model, the LR value under normal distribution is small, resulting in a small deviation from the expected failure days.

4. Summary

(1) By comparing the estimation results of GARCH(l,l) model, TARCH(l,l) model and EGARCH(l, l) model, it is found that the VaR values calculated by the three GARCH family models under T-distribution are greater than those of normal distribution. The LR values under normal distribution are large in GARCH and EGARCH models, but small in TARCH models. Among the six accepted GARCH models, the number of failure days under the T-distribution is closest to the expected number of failure days, and the LR of the Kupiec backtest test is small, which indicates that the T-distribution can better describe the distribution of the Shanghai Interbank Offered rate series, indicating that the Shanghai Interbank Offered rate series is non-normal.

(2) Under the TARCH(l,l) and EGARCH(l,l) models, the estimated coefficient γ is significant, indicating that there is an obvious leverage effect in the series of Shanghai Interbank offered rate, and the impact of good and bad news of interest rate is significantly greater than that of good news.

(3) In GARCH(l,l)-t model and EGARCH(l,l)-t model with the best fitting effect, when the confidence level is 95%, The VaR calculated for the Shanghai Interbank Offered Rate from May 15, 2017 to May 15, 2022 is 0.178297511Pt and 0.189794392Pt, respectively. It indicates that the daily maximum possible loss of the interbank lending funds with the holding value of Pt will not exceed 17.8297511% or 18.9794392% of Pt, which indicates that there is a certain interest rate risk in the current Shanghai interbank lending market. Shanghai Interbank offered rate Shibor was established as a benchmark interest rate, which can well reflect the supply and demand of market funds, so its high risk needs to be paid attention to.

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