The Relationship Between Exchange Rates and Interest Rates: Evidence from the U.S. and Europe

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Abstract. We analyze the relationship between exchange rates and interest rates for a sample of exchange rates between the U.S. dollar and the euro, as well as 10-year Treasury rates for both countries. The short-term relationship is modeled using ARMA, GARCH, and TGARCH models, and the study finds that the European 10-year Treasury bond has a significant effect on the exchange rate between the two countries while the U.S. 10-year Treasury bond does not have that effect. The long-term relationship is studied using the ARDL-ECM model, and the results show that between the U.S. 10-year Treasury rate, the European 10-year Treasury rate and the exchange rate during the financial crisis.

Keywords: Exchange rates; interest rates; GARCH; ARDL.

1. Introduction

The exchange rate is the external price of a country's currency and is one of the core variables of an open economy. Changes in the exchange rate act directly on commodity prices internally, causing price fluctuations and affecting domestic prices, employment conditions and financial market stability; externally, it affects import and export trade, foreign exchange reserves and capital flows. Interest rates, as one of the important tools of macroeconomic regulation, are usually controlled by the country's central bank, and in the United States by the Federal Reserve Board. Countries around the world frequently use interest rate leverage to implement macro-control, and interest rate policy has become the main tool for central banks to regulate money supply and demand and thus the economy. As the price of money, interest rate plays a very important role in the operation of the market economy and social credit.

The relationship between interest rate differential (IRD) and foreign exchange rate (FXR) is of great importance not only to market practitioners but also to policymakers. Previous studies have investigated this issue in various theoretical models: the Mundell–Fleming model, for instance, illustrates a negative relationship between exchange rates and interest rate spreads [1, 2]. Another well-known theoretical model – the uncovered interest rate parity condition – shows that the positive interest rate spreads can increase the attractiveness of domestic financial assets, which then encourages capital inflow, and thereby limits exchange rate depreciation [3].

Additionally, Furman et al. [4] suggested that the surge in interest rates is likely to be responsible for depreciation of exchange rates. Likewise, Baxter [5] documented strong correlations between real exchange rates and real interest differentials. By means of VAR model and impulse response analysis, Pattanaik and Mitra [6] observed that an interest rate shock appreciates the Rupee in the second month. Exploring the linkage between IRDs and exchange rates at different timescales, Hacker et al. [7] found negative relationships in the short term and positive ones in the long term. Using wavelet analysis and a Romanian sample, Andrieş et al. [8] confirmed these results. Engel and West [9] utilized causality tests and report causality from exchange rates to interest rates in the U.S. Si et al. [10] noticed that co-movement and causality between IRDs and exchange rates vary across frequencies and evolve over time in the BRICS economies (Brazil, Russia, India, China and South Africa).

Therefore, this study attempts to fill these gaps in the extant academic research and include the euro sample. Overall, this study examines the short- and long-term relationships between interest rates and exchange rate differentials, using the U.S. dollar and the euro as examples.

The remainder of the paper is organized as follows. Section 2 shows the data and preliminary analysis. Sections 3 illustrates the main methods. Sections 4 reports empirical results, respectively. Finally, section 5 Section concludes the study.

2. Data

In order to study the relationship between Eurodollar interest rate, European interest rate, and U.S. interest rate and the volatility of Eurodollar interest rate, the time interval selected is from April 2002 to October 2022, with 241 time points. Interest rate is the capital market interest rate is the main economic indicator to measure the cost of borrowing in the capital market, is one of the price signals to determine the allocation of resources in the capital market, the interest rate selected in this paper refers to the European region 10-year government bond yield and the U.S. 10-year Treasury rate, the data from the "U.S. Federal Reserve Economic Data Base". The exchange rate is the price signal that determines the allocation of resources in the foreign exchange market. Since international investors judge the appreciation or depreciation of a country's currency mainly through the nominal exchange rate, the nominal exchange rate of U.S. dollar to euro direct markup method is used as the variable in this paper. The original data is obtained from the Federal Reserve Economic Database. Considering that the financial crisis in 2008 had a big impact on the financial markets of Europe and the U.S. dollar, this paper chooses May 2007 to May 2009 as the structual point. The descriptive statistics of the data are as follows.

Table 1: Summary statistics.

	N	Mean	Std. Dev.	min	max	skewness	kurtosis
EX	241	1.24	0.13	0.9	1.58	0.26	2.67
US10	241	2.93	1.16	0.55	5.15	0.1	1.98
EU10	241	2.71	1.58	0	5.3	-0.33	1.56

The table above shows that the average exchange rate is 1.24, the minimum exchange rate is 0.9 and the maximum exchange rate is 1.58 over the 10-year period. The average interest rate on the US dollar 10-year Treasury bond is 2.93%, the minimum interest rate on the US dollar 10-year Treasury bond is 0.55% and the maximum interest rate on the US dollar 10-year Treasury bond is 5.15%. The average interest rate on the European 10-year Treasury bond is 2.71%, the minimum value of interest rate on the European 10-year Treasury bond is 0% and the maximum interest rate on the European 10-year Treasury bond is 5.3%. In addition, in terms of the kurtosis and skewness of the data they are not normally distributed. The three time series plots are as follows.

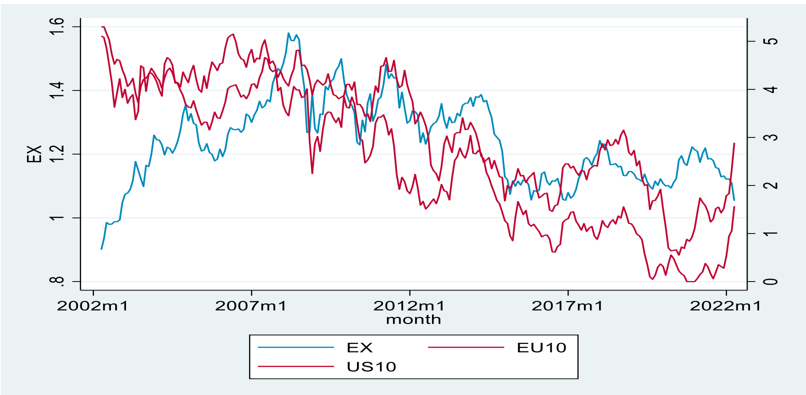

Figure 1: Time series of Variables.

It is obvious from the above figure that the exchange rate fluctuates up until 2008 and has been fluctuating down since then. In contrast, the U.S. 10-year Treasury bond rate and the European 10-

year Treasury bond rate fluctuate down until 2020 and rise rapidly thereafter. In addition, the exchange rate, the US 10-year Treasury bond rate and the European 10-year Treasury bond rate have declined significantly from May 2007 to May 2009, the period chosen in this paper to represent the structure time of the financial crisis.

Table 2: Unit root test.

	P-value			
	ADF	Stationary	First Difference ADF	Stationary
EX	0.0838	No	0. 0000	Yes
US10	0.2016	No	0. 0000	Yes
EU10	0.5311	No	0. 0000	Yes

From the ADF test, it can be seen that the original time series of all three variables do not satisfy stationary, while the time series satisfy stationary after the first-order difference, so this paper will establish a GARCH model for D.EX. And for the ARDL model, since each variable belongs to the I(1) time series, so it will be modeled with the original time series.

3. Method

In this paper, the ARMA model, GARCH model, and TGARCH model are used to study exchange rate volatility, and the best-fit model is selected based on the AIC criterion. For the interaction relationship between exchange rate and interest rate, the short-term relationship is fitted by ARDL model in this paper, while the long-term relationship is fitted by ARDL-ECM model.

3.1. Augmented Dicky-Fuller (ADF) unit root test

The Augmented Dicky-Fuller (ADF) test is usually used to test the stationary of variables, and the ADF is an extension of the DF test. It includes an additional term for the lagging difference in the dependent variable to eliminate autocorrelation between residuals. The number of the lag period is decided by the Schwartz Bayesian Criterion (SBC) or the Akaike Information Criterion (AIC) [11]. There are three forms of Augmented Dicky- Fuller (ADF) unit root test is provided below. In practice, we will choose the optimal model based on the time trend graph of the variables.

Model 1: Model with no intercept term and no time trend term

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \mu_t \tag{1}$$

Model 2: A model with an intercept term but without time trend

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \mu_t \tag{2}$$

Model 3: Model with intercept and time trend

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \mu_t \tag{3}$$

Where, Δ is the difference operator; p is the most recent period to make the residual term match the white noise; μ is the error term; t is the time subscript; α_0 is the intercept term; $\alpha_2 t$ is the model incorporation time potential term. The null hypothesis is that the variables under estimation have unit roots, and the alternative hypothesis is that they do not. In practical, if t-statistics > ADF critical value, then we accept the null hypothesis$(H_0: \gamma = 0)$, which describes the series as non-stationary. If the t-statistic < the ADF threshold, then the null hypothesis is rejected, which indicates that the series is stationary.

3.2. ARMA model, GARCH model and TGARCH model

First, define the exchange rate as an ARMA (p,q) model as follows:

$$\Delta y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \Delta y_{t-1} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \hat{\varepsilon}_t \tag{4}$$

In this paper, the AIC (Akaike information criterion) criterion is used to determine the appropriate number of backward periods p and q. Since the GARCH model allows the conditional variance to be affected not only by the squared past error term, but also by the conditional variance of the self-lagging period, for simplicity, the GARCH (1, 1) model is as follow [12, 13]:

$$\begin{gathered} \Delta y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \Delta y_{t-1} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \varepsilon_t \\ \varepsilon_t \mid I_{t-1} \sim N(0, \sigma_t^2) \\ \sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \end{gathered} \tag{5}$$

Where $I_{t-1}$the set of all relevant information before t is, σ_t^2 is the exchange rate error ε_t to t period all the relevant information under the number of conditional variations, the error ε_t is a white noise (white noise). If there is a large fluctuation in the current period, the next period will also produce large fluctuations in the same direction, which means that the average of the exchange rate and its variance will change over time and is no longer a fixed constant, so the GARCH model can be used to observe the phenomenon of exchange rate fluctuations over time.

The TGARCH model is another volatility model that captures the leverage effect [14]:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} (\alpha_i + \gamma_i N_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{6}$$

And N_{t-i} is one if$a_{t-i} < 0$,

$$N_{t-i} = \begin{cases} 1, & a_{t-i} < 0 \\ 0, & a_{t-i} \geq 0 \end{cases} \tag{7}$$

$\alpha_i, \gamma_i, \beta_j$ Is positive. The impact on σ_t^2 of positive ε_{t-i} is$\alpha_i \varepsilon_{t-i}^2$, and the impact on σ_t^2 of negative ε_{t-i} is $\alpha_i a_{t-i}^2$ $(\alpha_i + \gamma_i) \varepsilon_{t-i}^2$

3.3. Cointegration test-ARDL Model Boundary Test

In this paper, the Autoregressive Distributed Lag (ARDL) boundary test is used to test the cointegration relationship between interest rate and exchange rate [15, 16]. And the model needs to satisfy the variables statistics have a non-standard distribution depending on whether the variable is I(0) or I(1) alone. The advantage of the ARDL method is that although other cointegration methods require the regressions to be integrated in the same specification, it can be applied regardless of their integration order, and cointegration tests under this constraint include a comparison of the thresholds and F-statistics.

The boundary test consists of two asymptotic critical value boundaries, which is depend on whether the variables are I(0) or I(1) or a mixture of both. If the test statistic exceeds their respective upper thresholds, then there is evidence of a long-run relationship, and if the F-statistic is below the threshold, we cannot reject the null hypothesis of no cointegration if the F-statistic lies between the two variables.

In this paper, we model the relationship between changes in interest rates and exchange rates.

$$\boldsymbol{y}_t = \gamma_{0j} + \sum_{i=1}^{p} \delta_j \boldsymbol{y}_{t-i} + \sum_{i=0}^{q} \beta_j' \boldsymbol{x}_{t-i} + \boldsymbol{\varepsilon}_{jt} \tag{8}$$

In the above equation, y_t is the exchange rate, x_t is interest rate, and ε_t is the error term. A linear ARDL-ECM model is developed as follows [17, 18, 19, and 20]:

$$\Delta\ y_t = \alpha_0 + \sum_{i=1}^{k} \alpha_{1i}\Delta\ y_{t-i} + \sum_{i=0}^{k} \alpha_{2i}\Delta\ x_{t-i} + \beta_0\ y_{t-1} + \beta_1 x_{t-1} + \varepsilon_t \tag{9}$$

All algebras in the boundary test are tested with the F-statistic $\ln\ y_t$ is the logarithm of exchange rate; $\ln\ x_t$ is exchange rate; Δ is the first difference; and ε_t is the error term. The null hypothesis (indicating no cointegration) in the above equation is $(H_0: \beta_0 = \beta_1 = 0)$ versus the contrarian hypothesis$(H_1: \beta_0 \neq \beta_1 \neq 0)$.

3.4. Akaike Information Criterion

Akaike's (1973) model selection criterion, the Akaike Information Criterion (AIC), has had a great impact on how to select the optimal statistical model. This section is included in a series of very important papers including Akaike (1973, 1974, 1977, and 1981a).The AIC criterion is used to identify the best or simplest model in data analysis of multiple competing models.

AIC (Akaike Information Criterion): The most appropriate lag period is the one with the smallest AIC value.

$$\text{AIC} = \text{T} * \ln{(SSE)} + 2K \tag{10}$$

Where T is the number of train points, K is the number of coeffecient to be estimated, and SSE is the sum of squared residuals.

The advantage of AIC is that the minimized AIC is equivalent to the Mean Squared Error (MSE) of the minimized One-Step Prediction for large samples, and it is also selected for variables. It is of great practical importance, again demonstrating its importance in statistical inference.

4. Results

4.1. ARCH\GARCH\T-GARCH model

Before fitting the ARCH model and comparing the prediction accuracy of each model, this paper first performs autocorrelation and partial autocorrelation tests. In this paper, the model selection is done by AIC criterion and BIC criterion, and from Table 2, it can be seen that ARMA (2, 2) has the smallest AIC and BIC. The regression equation is as follows.

$$\Delta y_t = 1.6740 * \Delta y_{t-1} - 0.9217 * \Delta y_{t-2} - 1.7237 * \varepsilon_{t-1} + \varepsilon_{t-2} + \hat{\varepsilon}_t \tag{11}$$

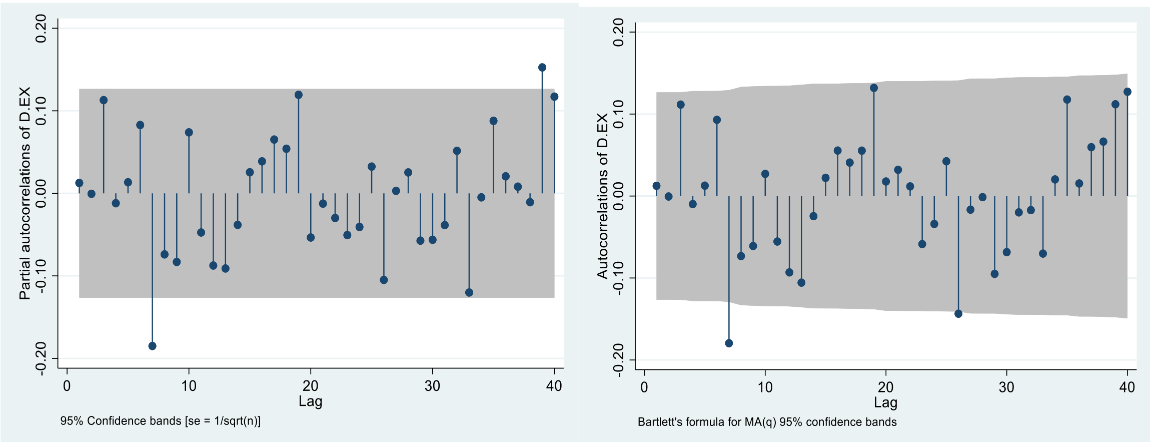

Figure 2: ACF and PACF of D.ex.

From the ACF and PACF plots, there is an obvious autocorrelation and partial autocorrelation for D.ex.

Table 3: ARMA regression.

Variables	1 ARMA	2 ARMA	3 ARMA	4 ARMA
L.ar	0.6455	-0.4629	-0.4896	1.6740***
	(0.6986)	(1.1644)	(0.9575)	(0.0364)
L.ma	-0.6168	0.4800	0.5034	-1.7237***
	(0.7185)	(1.1735)	(0.9662)	(0.0138)
L2.ma		-0.0252		1.0000***
		(0.0561)		(0.0000)
L2.ar			-0.0321	-0.9217***
			(0.0516)	(0.0318)
Constant	0.0006	0.0006	0.0006	0.0006
	(0.0026)	(0.0023)	(0.0023)	(0.0025)
Observations	240	240	240	240
AIC	-930.9	-929	-929.1	-937.3
BIC	-917	-911.6	-911.7	-919.9

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In this paper, we continue modeling by GARCH model, Tgarch model, and choose ARMA (2, 2)-GARCH (1, 1) as the best-fit model according to the AIC criterion and BIC criterion.

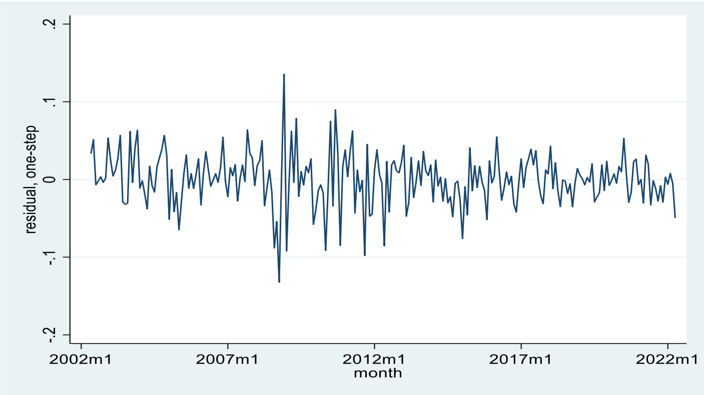

Figure 3: Residual of ARMA (2, 2) model.

From the residual plots, it can be seen that there is an obvious variance aggregation effect, so there is an ARCH effect in this time series,

Table 4: Regression results.

Variables	1 ARMA	2 GARCH	3 TGARCH
L.ar	1.6740***	0.0996***	-0.1372
	(0.0364)	(0.0320)	(0.3590)
L2.ar	-0.9217***	-0.9210***	0.6838*
	(0.0318)	(0.0296)	(0.3563)
L.ma	-1.7237***	-0.0940***	0.1845
	(0.0138)	(0.0080)	(0.3948)
L2.ma	1.0000***	0.9903***	-0.5999
	(0.0000)	(0.0073)	(0.3839)
L.arch		0.0984***	0.1271***
		(0.0313)	(0.0409)
L.garch		0.8739***	0.8534***
		(0.0410)	(0.0468)
L.tarch			-0.0560
			(0.0581)

Constant-arma	0.0006	0.0007	0.0005
	(0.0025)	(0.0021)	(0.0028)
Constant-Garch		0.0000	0.0001*
		(0.0000)	(0.0000)
Observations	240	240	240

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

From the model fitting results above, we can see that the coefficient of L.arch is 0.0984, which is significant at the 5% level, and the coefficient of L.garch is 0.8739, which is significant at the 5% level, and these two fitted coefficients satisfy the basic hypothesis of non-negative, summing up to less than 1. As for the TGARCH model, although the coefficients of L.arch and L.garch remain significant at the 5% level, the coefficient of tarch is insignificant at the 5% level. The above results suggest that exchange rate volatility has a significant garch effect, but not a leverage effect. This implies that the volatility of the exchange rate has greater feedback on negative news and a smaller feedback on positive news. In addition, the AIC criterion shows that the ARMA-GARCH model has a smaller AIC compared to the ARMA model, so for the volatility of the exchange rate, the ARMA-GARCH model is the best-fitting model.The regression equation is as follows.

$$\begin{aligned} &\Delta y_t = 1.6740 * \Delta y_{t-1} - 0.9217 * \Delta y_{t-2} - 1.7237 * \varepsilon_{t-1} + \varepsilon_{t-2} + \hat{\varepsilon}_t \\ &\varepsilon_t \mid I_{t-1} \sim N(0, \sigma_t^2) \\ &\sigma_t^2 = 0.0984_1 * \varepsilon_{t-1}^2 + 0.8739 * \sigma_{t-1}^2 \end{aligned} \tag{12}$$

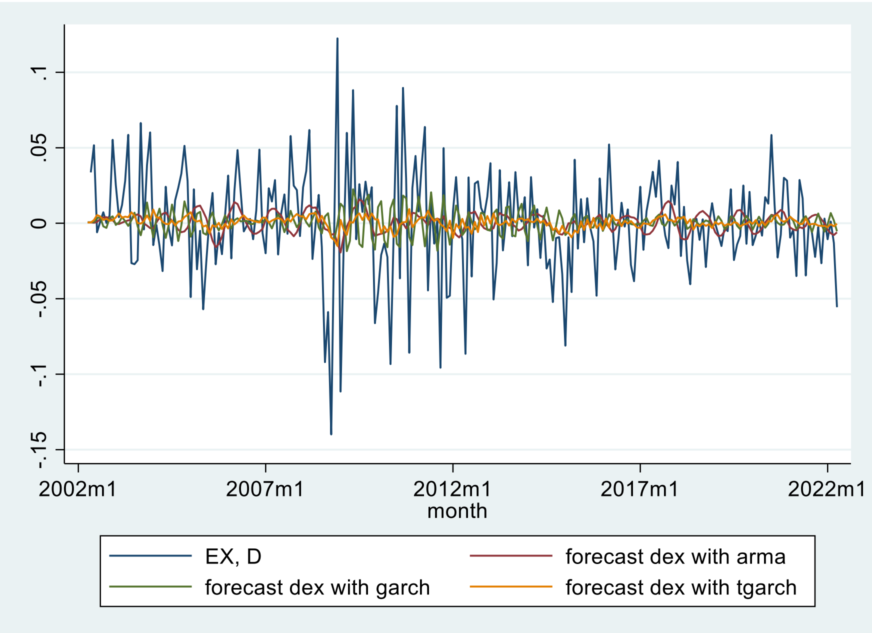

Figure 4: Prediction of ARMA\GARCH\TGARCH Model.

As can be seen from the above figure, although we demonstrate that the volatility of the exchange rate has a significant garch effect, the forecasts are not very good, mainly because the information set on which the forecast is based is only the historical data of the exchange rate itself. The next analysis shows that the model forecasts better with the addition of the US 10-year Treasury rate and the European 10-year Treasury rate.

4.2. ARDL model

This paper investigates the short-run relationship between the Eurodollar exchange rate and the interest rates of both countries by using the ARDL model, before confirming the existence of a 2nd lags between the Eurodollar exchange rate and the interest rates of both countries by using the AIC criterion. And the lags changes to 4 after we add the structure (2007m5-2009m5).

Table 5: Lags selection.

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-481.09	0.011933				4.08515	4.10284	4.12905
1	560.996	2084.2	9	0	2.00E-06	-4.63288	-4.5621	-4.45728*
2	578.098	34.204*	9	0	1.8e-06*	-4.70125*	-4.57739*	-4.39395
3	584.871	13.547	9	0.139	1.90E-06	-4.68246	-4.50552	-4.24346
4	592.002	14.262	9	0.113	1.90E-06	-4.66669	-4.43666	-4.09599

Table 6: Regression of ARDL model and ARDL model with structure.

	(1)	(2)
Variables	ARDL	ARDL
L.EX	0.923***	0.913***
	(0.0206)	(0.0220)
US10	-0.00248	-0.00194
	(0.00316)	(0.00299)
EU10	0.00629**	0.00566**
	(0.00263)	(0.00250)
z		0.697***
		(0.201)
L.z		0.102
		(0.253)
L2.z		0.273
		(0.258)
L3.z		-0.358
		(0.271)
L4.z		1.021***
		(0.214)
z_us10		-0.0983***
		(0.0250)
L.z_us10		0.158***
		(0.0250)
z_euro10		-0.0597
		(0.0569)
L.z_euro10		-0.177**
		(0.0703)
L2.z_euro10		-0.0585
		(0.0612)
L3.z_euro10		0.0795
		(0.0648)
L4.z_euro10		-0.233***
		(0.0509)
Constant	0.0869***	0.0977***
	(0.0250)	(0.0267)
Observations	239	237
R-squared	0.932	0.947

The analysis of the ARDL model for the exchange rate, the U.S. 10-year Treasury rate and the European 10-year Treasury rate shows that there is a short-term relationship between them, where the coefficient of the lagged term of the exchange rate is 0.923, which is significant at the 5% level, indicating that for every 1% increase in the exchange rate in the lagged period, the exchange rate in the current period increases by 0.923%. In addition, the coefficient of European 10-year Treasury rate is 0.0063, which is significant at the 5% level, indicating that an increase in the current European 10-year Treasury rate has a positive effect on the exchange rate, and a 1% increase in the current

European 10-year Treasury rate will increase the exchange rate by 0.0063% in the current period. The regression equation is as follows:

$$\boldsymbol{EX_t} = 0.0869 + 0.0923 * \boldsymbol{EX_{t-1}} + 0.0063 * \boldsymbol{EU10} + \boldsymbol{\varepsilon_{jt}} \quad (13)$$

And through the ARDL boundary test, this paper can find that the F-value of the boundary test is 4.78, which is smaller than the critical value of 4.85 at the 5% level, indicating that although there is a significant short-term relationship between the exchange rate and the European 10-year Treasury rate, there is not a significant long-term relationship.

It can be seen from the figure below that the model is less stable and shows a large deviation during the financial crisis.

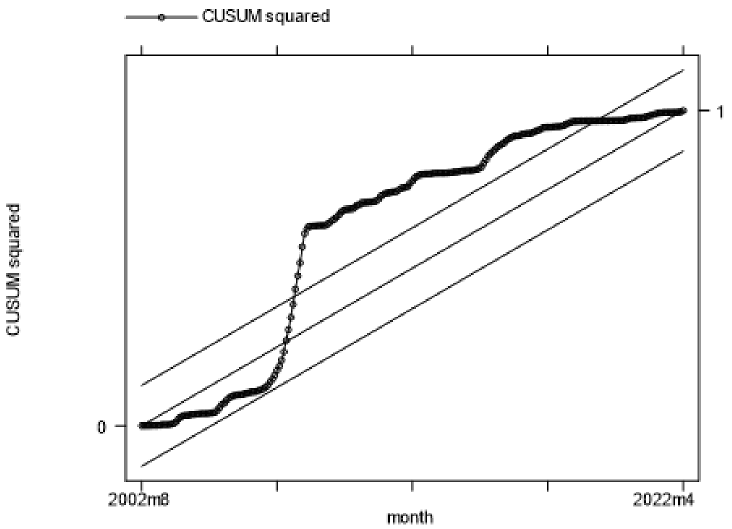

Figure 5: Cumulative sums (CUSUM) of the recursive residuals and their squares from the regression without structure.

After considering the financial crisis, this paper adds the dummy variable from May 2007 to May 2009 and adds the product of this dummy variable and the US 10-year Treasury rate and the UK 10-year Treasury rate. By fitting the ARDL model here, the regression equation is as follows.

$$\boldsymbol{EX_t} = 0.0977 + 0.9127 * \boldsymbol{EX_{t-1}} + 0.0057 * \boldsymbol{EU10} + \boldsymbol{0.6972} * \boldsymbol{z_t} + \boldsymbol{0.6972} * \boldsymbol{z_{-4}} - 0.0983 * (\mathbf{z} * \mathbf{US10})_{\mathbf{t}} + 0.1584 * (\mathbf{z} * \mathbf{US10})_{\mathbf{t-1}} - 0.1771 * (\mathbf{z} * \mathbf{EU10})_{\mathbf{t-1}} - 0.2332 * (\mathbf{z} * \mathbf{EU10})_{\mathbf{t-4}} + \boldsymbol{\varepsilon_{jt}}, \text{Which } z_t = 1 \text{ if } 2007.5 < t < 2009.5 \quad (14)$$

As can be seen from the above regression results, in terms of the short-term relationship, the coefficients of the lagged term of the exchange rate and the European 10-year Treasury rate remain significant at the 5% level, at 0.9127 and 0.0057, respectively, indicating that a 1% increase in the exchange rate in the previous period is associated with a 0.91% increase in the current exchange rate, and a 1% increase in the European 10-year Treasury rate is associated with a 0.0057% increase in the current exchange rate. For our new financial crisis variable, it has a significant coefficient of 0.6972, which is significant at the 5% level, indicating a significant increase in the exchange rate during the financial crisis compared to the average level. The coefficient of $(\mathbf{z} * \mathbf{US10})_{\mathbf{t}}$ is -0.0983, which is significant at the horizontal level, and the coefficient of $(\mathbf{z} * \mathbf{US10})_{\mathbf{t-1}}$ is 0.1584, which is significant at the 5% level, indicating that the current interest rate of the 10-year U.S. Treasury rate has a significant negative impact on the exchange rate during the financial crisis, and a 1% increase in the 10-year U.S. Treasury rate decreases the exchange rate by 0.0983%. However, the previous period's interest rate of the U.S. 10-year Treasury rate has a significant positive impact on the exchange rate, with a 1% increase in the U.S. 10-year Treasury rate and a 0.1584% decrease in the exchange rate.

For the European 10-year Treasury rate, the coefficient of $(\mathbf{z} * \mathbf{EU10})_{\mathbf{t-1}}$ is -0.1771, which is significant at the horizontal level, and the coefficient of $(\mathbf{z} * \mathbf{EU10})_{\mathbf{t-4}}$ is -0.2332, which is significant at the 5% level, indicating that both the one lagged interest rate and the four lagged interest rate of the European 10-year Treasury rate have a significant negative impact on the exchange rate during the financial crisis. A 1% increase in the one-period lagged interest rate of the European 10-year bond is associated with a 0.1771% decrease in the exchange rate, and a 1% increase in the four-period lagged interest rate of the European 10-year bond is associated with a 0.2332% decrease in the exchange rate.

And through the ARDL boundary test, this paper can find that the F-value of the boundary test is 12.147, which is smaller than the critical value of 3.79 at the 5% level, indicating that there is not a significant long-term relationship. The cointegration equation is as follows.

$$\Delta EX_t = 0.0873 * \Delta EX_{t-1} + 0.0649 * \mathbf{EU10} + 19.87 * \mathbf{z}_t + 0.688 * (\mathbf{z} * \mathbf{US10})_t - 5.144 * (\mathbf{z} * \mathbf{EU10})_t + \varepsilon_t \quad (15)$$

Table 7: ARDL-ECM model with structure.

	D.EX
ADJ	
L.EX	-0.0873***
	-0.022
LR	
US10	-0.0222
	-0.033
EU10	0.0649**
	-0.0245
z	19.87***
	-5.846
z_us10	0.688**
	-0.231
z_euro10	-5.144***
	-1.502

Specifically, in the long run, the European 10-year Treasury rate and the exchange rate have a significant positive relationship with a coefficient of 0.0649, while during the financial crisis, the European 10-year Treasury rate has a significant negative relationship with the exchange rate with a coefficient of -5.114, and the U.S. 10-year Treasury rate and the exchange rate have a significant positive relationship with a coefficient of 0.668.

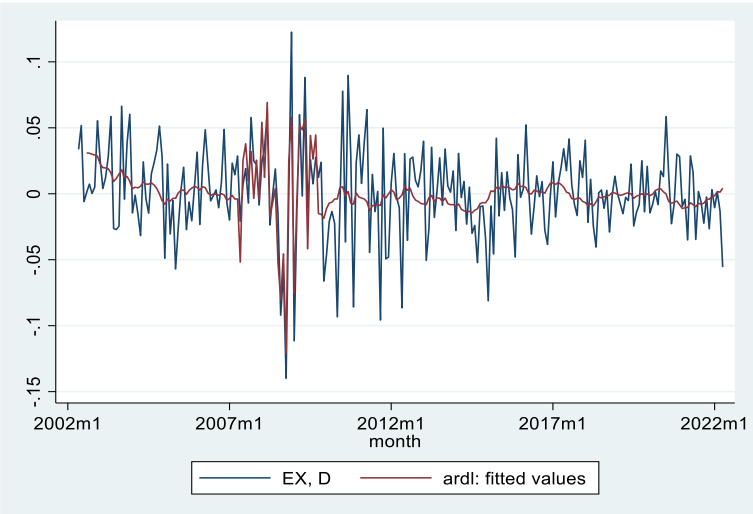

Figure 6: Prediction of ARDL-ECM model.

As can be seen from the figure, ARDL-ECM has a good fit for the exchange rate changes during the financial crisis.

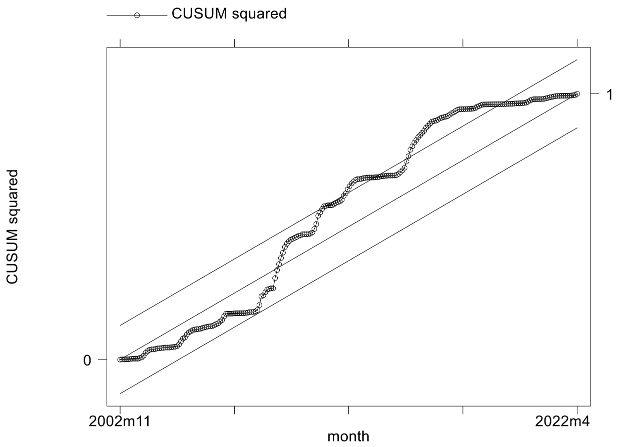

Figure 7: Cumulative sums (CUSUM) of the recursive residuals and their squares from the regression with structure.

Further, this paper also shows from the CUXUM plot that the stability of the model is greatly improved by adding the time structure of the financial crisis, which is basically within the 5% confidence level.

5. Conclusion

We focuses on the volatility of the Eurodollar exchange rate and whether there is a short-run and long-run relationship between the Eurodollar exchange rate and the U.S. 10-year Treasury rate and the European 10-year Treasury rate.

The volatility of the exchange rate has a significant ARCH effect, and for the model fitting of the exchange rate volatility, GARCH (1, 1) and TGARCH (1, 1) are used in this paper. The results show that for the volatility model of exchange rate, GARCH (1, 1) model has the smallest AIC and significant garch coefficient, which indicates that the fall of exchange rate will trigger a bigger fall crisis, which is also consistent with the characteristics of exchange rate market.

This paper analyzes the relationship between the Eurodollar exchange rate, the U.S. 10-year Treasury rate and the European 10-year Treasury rate using the ARDL model. In the short run, the European 10-year Treasury rate has a significant positive effect on the exchange rate, while the U.S. 10-year Treasury rate has no effect. However, during the financial crisis, the current period of the U.S. 10-year Treasury rate has a significant negative impact on the exchange rate, while the lagged period of the U.S. 10-year Treasury rate has a significant positive impact on the exchange rate. This is mainly because the 2008 financial crisis was mainly caused by the subprime mortgage crisis in the U.S., which was transmitted globally, and the U.S. 10-year Treasury rate, as an important financial indicator, had an important impact on the Eurodollar exchange rate during that period.

In the long run, we finds that there is a long-run cointegration relationship between the U.S. 10-year Treasury rate, the European 10-year Treasury rate and the exchange rate during the financial crisis. Specifically, there is an obvious positive correlation between the U.S. 10-year Treasury rate and the exchange rate change during the financial crisis, while there is a distinctive negative correlation between the European 10-year bond rates and the exchange rate changes during the financial crisis.

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