

The Impact of CPI and Unemployment Rate on Economic Growth: Based on TVP-VAR Model

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Abstract. Inflation has a significant impact on economic growth and monetary policy. In recent times, the US economy is facing the threat of high inflation, and the Federal Reserve has been continuously adopting interest rate hikes, which have an impact on the economy and finance. And the relationship between inflation and unemployment rate is a trade-off, so this article studies the impact of CPI and unemployment rate on the US economy. This artic le uses the US CPI and unemployment rate from 1948 to 2009 to construct a VAR model, predicts the final ten periods, and compares them with actual values. Using US CPI, unemployment rate, and GDP data from 1948 to 2009, construct a TVP-VAR model to analyze the impact of inflation and unemployment rate on economic growth.

Keywords: CPI, unemployment, GDP, TVP-VAR model.

1. Introduction

Against the backdrop of the epidemic, the United States is once again facing the threat of high inflation. The Federal Reserve raised interest rates in its recent interest rate meeting. On May 4th, after the Federal Reserve raised interest rates by 50 basis points, including market interest rates, it is already expected to raise interest rates by 50 basis points in June and July respectively. This round of the Federal Reserve's interest rate meeting has seen three consecutive interest rate hikes, each by 50 basis points. This is already the most aggressive rate hike by the Federal Reserve in the past 40 years. If US inflation data continues to spiral out of control and remain above 8% or even higher, the Federal Reserve may adopt hawkish interest rate hikes, continuing to raise interest rates by 50 basis points in September and possibly raising them to 3.5% this year. Inflation is the Achilles' heel of the current US economy. If the US continues to experience high inflation, it will force the Federal Reserve to continue aggressively raising interest rates; If the US inflation data falls back to around 5% in September, the Federal Reserve may adopt a dove like interest rate hike. It can be seen that inflation has a huge impact on economic growth and monetary policy. And there is a trade-off between inflation and unemployment, so this article focuses on studying the impact of CPI and unemployment on the US economy. This article uses the US CPI and unemployment rate from 1948 to 2009 to construct a VAR model, predicts the final ten periods, and compares them with actual values. Using US CPI, unemployment rate, and GDP data from 1948 to 2009, construct a TVP-VAR model to analyze the impact of inflation and unemployment rate on economic growth. Discuss how CPI and unemployment rate affect economic growth.

2. Descriptives

This paper selects quarterly data on US CPI, unemployment rate, and GDP from 1948 to 2009. In the VAR model, raw data is directly used. In order to better characterize the impact of CPI and unemployment rate on GDP, the TVP-VAR model is constructed by taking logarithmic returns on CPI, unemployment rate, and GDP data, that is, the impact of CPI inflation rate and unemployment rate UNE inflation rate on economic growth.

$$cpi = \ln(CPI_t / CPI_{t-1}) \times 100\%$$

Same treatment as GDP for UNE.

3. Data check

ADF unit root test. To avoid the phenomenon of "pseudo regression" in time series data, it is necessary to conduct stationarity tests on the data before conducting empirical research. The results of the variable unit root test are as follows:

Augmented Dickey-Fuller Unit Root Test on CPI		
Null Hypothesis: CPI has a unit root		
Exogenous: Constant		
Lag Length: 2 (Automatic - based on SIC, maxlag=14)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.357425	0.0135
Test critical values: 1% level	-3.457400	
5% level	-2.873339	
10% level	-2.573133	

*MacKinnon (1996) one-sided p-values.

Fig. 1 ADF unit root test

Augmented Dickey-Fuller Unit Root Test on UNE		
Null Hypothesis: UNE has a unit root		
Exogenous: Constant		
Lag Length: 3 (Automatic - based on SIC, maxlag=14)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-8.603751	0.0000
Test critical values: 1% level	-3.457515	
5% level	-2.873390	
10% level	-2.573160	

*MacKinnon (1996) one-sided p-values.

Fig. 2 ADF unit root test(2)

Augmented Dickey-Fuller Unit Root Test on GDP		
Null Hypothesis: GDP has a unit root		
Exogenous: Constant		
Lag Length: 0 (Automatic - based on SIC, maxlag=14)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.074099	0.0000
Test critical values: 1% level	-3.457173	
5% level	-2.873240	
10% level	-2.573080	

*MacKinnon (1996) one-sided p-values.

Fig. 3 ADF unit root test(3)

From the table, it can be seen that all variables reject the original hypothesis at a significance level of 1%. The first-order stability of each sequence ensures the validity of the data and can be used to construct VAR models.

Draw an image of the original data.

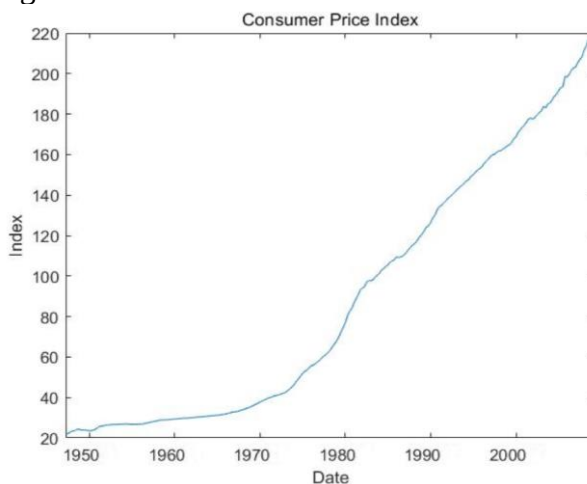


Fig. 4 Consumer price index

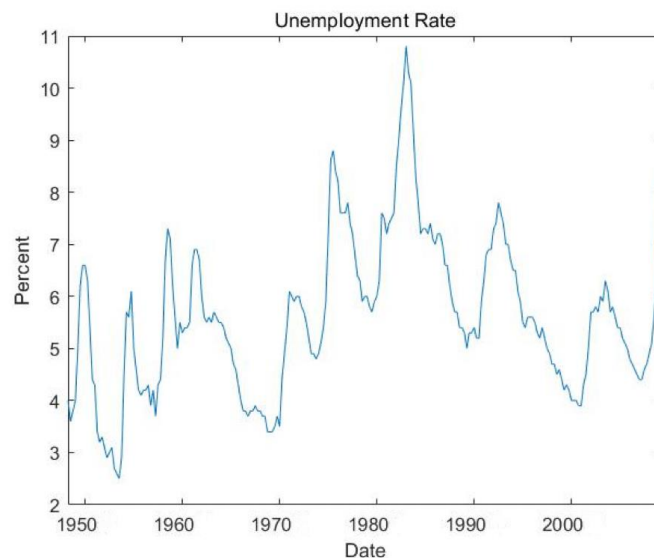


Fig. 5 Unemployment rate

Build a VAR model and estimate it. The estimated results of each parameter are as follows:

Effective Sample Size: 241
Number of Estimated Parameters: 18
LogLikelihood: 811.361
AIC: -1586.72
BIC: -1524

	Value	StandardError	TStatistic	PValue
Constant (1)	0.0017164	0.0015988	1.0735	0.28303
Constant (2)	0.31626	0.091961	3.439	0.0005838
AR (1) (1, 1)	0.30899	0.063356	4.877	1.0772e-06
AR (1) (2, 1)	-4.4834	3.6441	-1.2303	0.21857
AR (1) (1, 2)	-0.0031796	0.0011306	-2.8122	0.004921
AR (1) (2, 2)	1.3433	0.065032	20.656	8.546e-95
AR (2) (1, 1)	0.22433	0.069631	3.2217	0.0012741
AR (2) (2, 1)	7.1896	4.005	1.7951	0.072631
AR (2) (1, 2)	0.0012375	0.0018631	0.6642	0.50656
AR (2) (2, 2)	-0.26817	0.10716	-2.5025	0.012331
AR (3) (1, 1)	0.35333	0.068287	5.1742	2.2887e-07
AR (3) (2, 1)	1.487	3.9277	0.37858	0.705
AR (3) (1, 2)	0.0028594	0.0018621	1.5355	0.12465
AR (3) (2, 2)	-0.22709	0.1071	-2.1202	0.033986
AR (4) (1, 1)	-0.047563	0.069026	-0.68906	0.49079
AR (4) (2, 1)	8.6379	3.9702	2.1757	0.029579
AR (4) (1, 2)	-0.00096323	0.0011142	-0.86448	0.38733
AR (4) (2, 2)	0.076725	0.064088	1.1972	0.23123

Fig. 6 estimated results

Innovations Covariance Matrix:

0.0000 -0.0002
-0.0002 0.1167

Innovations Correlation Matrix:

1.0000 -0.0925
-0.0925 1.0000

Fig. 7 estimated results(2)

3.1. Prediction of VAR (4) model

Create and estimate a VAR (4) model for CPI growth rate and unemployment rate. Consider the last ten periods as the predicted range.

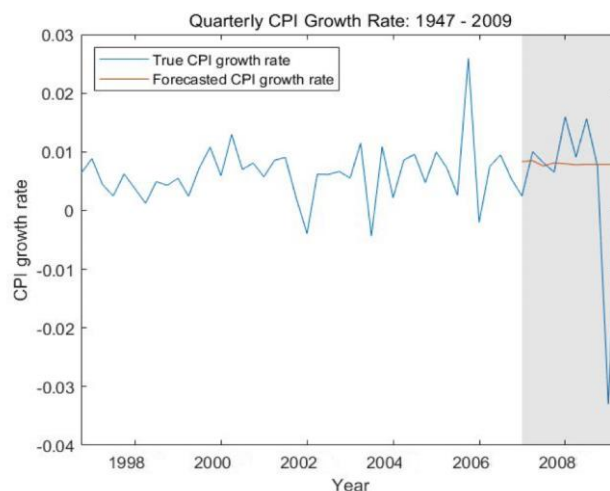


Fig. 8 VAR (4) model for CPI growth rate

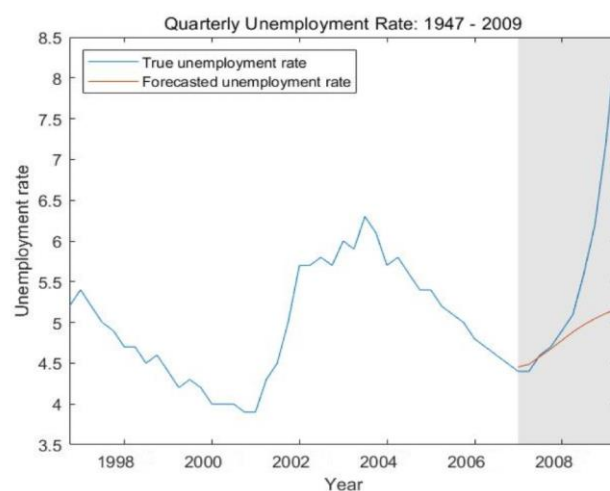


Fig. 9 VAR (4) model for unemployment rate

3.2. Using the TVP-VAR model to characterize the impact of CPI and unemployment rate on GDP growth

VAR Lag Order Selection Criteria
Endogenous variables: NUE GDP CPI
Exogenous variables: C
Date: 05/30/22 Time: 15:17
Sample: 1 244
Included observations: 236

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1890.468	NA	2.27e-11	-15.99549	-15.95146	-15.97774
1	1987.353	190.4844	1.08e-11	-16.74028	-16.56415	-16.66928
2	2012.566	48.93200	9.39e-12	-16.87768	-16.56946*	-16.75343
3	2029.384	32.21029	8.79e-12*	-16.94393*	-16.50362	-16.76644*
4	2038.118	16.50475	8.81e-12	-16.94168	-16.36926	-16.71093
5	2041.425	6.166620	9.25e-12	-16.89343	-16.18893	-16.60944
6	2048.530	13.06562	9.40e-12	-16.87737	-16.04077	-16.54013
7	2051.029	4.532252	9.94e-12	-16.82228	-15.85358	-16.43179
8	2061.200	18.18622*	9.85e-12	-16.83220	-15.73141	-16.38846

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error

Fig. 10 VAR Lag Order Selection

Firstly, using Eviews to determine the order of the model, it can be seen that the optimal order of the model is 3 orders. Based on this, a 3-order TVP-VAR model is constructed.

4. Model settings:

The TVP-VAR model has two characteristics: time-varying and nonlinear, which can effectively characterize the dynamic relationships between variables. Firstly, consider the following model:

$$y_t = c_t + B_{1,t}y_{t-1} + \dots + B_{k,t-k}y_{t-k} + \mu_t, t = 1, 2, \dots, T \quad (1)$$

$$A_t \Omega_t A_t' = \Sigma_t \Sigma_t' \quad (2)$$

Where, y_t and c_t are $n \times 1$ dimensional vector, $B_{i,t}$ and k are $n \times n$ dimensional vector is the covariance matrix. The definition of A_t is a lower triangular matrix with all 1 main diagonals, and Σ_t is a symmetric matrix with all 0 except for the diagonal

$$A_t = \begin{pmatrix} 1 & 0 & \dots & 0 \\ \alpha_{21} & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & 1 \end{pmatrix}, \Sigma_t = \begin{pmatrix} \sigma_{1,t} & 0 & \dots & 0 \\ 0 & \sigma_{2,t} & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \sigma_{n,t} \end{pmatrix} \dots \dots \quad (3)$$

If $B_{i,t}$ is transformed into a vector with time-varying properties B_t , then equation (1) can be transformed into

$$y_t = X_t' \beta_t + A_t^{-1} \Sigma_t \varepsilon_t \quad (4)$$

$X_t' = I_n \otimes [1, y_{t-1}', \dots, y_{t-k}']$, Set α_i as a vector of a matrix A_t that is not 0 and not 1, and also set σ_t as a vector of the matrix Σ_t . The parameter model with time-varying properties is as follows

$$\beta_t = \beta_{t-1} + \mu_{\beta_t} \quad (5)$$

$$\alpha_t = \alpha_{t-1} + \mu_{\alpha_t} \quad (6)$$

$$h_t = \log \sigma_{t-1} + \mu_{h_t} \quad (7)$$

At the same time, it is necessary to set all parameters to follow random walks, i.e

$$\begin{pmatrix} \varepsilon_t \\ \mu_{\beta_t} \\ \mu_{\alpha_t} \\ \mu_{h_t} \end{pmatrix}_t \sim N \left(0, \begin{pmatrix} I_\varepsilon & 0 & 0 & 0 \\ 0 & \Sigma_\beta & 0 & 0 \\ 0 & 0 & \Sigma_\alpha & 0 \\ 0 & 0 & 0 & \Sigma_h \end{pmatrix} \right) \quad (8)$$

Finally, the Markov Chain Monte Carlo (MCMC) method is used to estimate the posterior values of the parameters, ensuring the stability of the model's parameter estimation.

The data adopts quarterly data on US CPI, unemployment rate, and GDP from 1948 to 2009, and is processed by taking logarithmic returns. Represented by cpi, une, and GDP respectively. The unit root test has already been done in the previous section, and it will not be repeated here.

4.1. Model estimation.

Perform a lag order test on the VAR model to confirm that the optimal lag order of the model is 3 orders. Before estimating the MCMC model, the initial parameters of the MCMC model were set according to the method proposed by Nakajima (2011).

$$\mu_\alpha = \mu_{\beta_0} = \mu_{h_0}, \Sigma_{\alpha_0} = \Sigma_{\beta_0} = \Sigma_{h_0} = 10 \times I, (\Sigma_\beta)_i^{-2} \sim \text{Gamma}(20, 10^{-4}), (\Sigma_\alpha)_i^{-2} \sim \text{Gamma}(4, 10^{-4}), (\Sigma_h)_i^{-2} \sim \text{Gamma}(4, 10^{-4}).$$

This article uses the MCMC algorithm to extract $M=10000$ samples and discard the first 1000 sampling results to construct an effective sample set. As shown in Table 2, the mean of the posterior

parameters of the model is within the 95% confidence interval, the Geweke probability is within the 5% critical range, and the maximum value of the ineffective factor is 324.90, which is slightly greater than the reasonable interval. However, the invalid factors of other parameters are far less than 100, so we will continue to conduct MCMC model estimation and pulse response analysis.

[ESTIMATION RESULT]						
Parameter	Mean	Stdev	95%U	95%L	Geweke	Inef.
sb1	0.0023	0.0003	0.0018	0.0029	0.561	15.50
sb2	0.0023	0.0003	0.0018	0.0028	0.013	9.15
sa1	0.0053	0.0015	0.0033	0.0091	0.490	52.04
sa2	0.0055	0.0015	0.0033	0.0090	0.064	83.23
sh1	0.3908	0.0458	0.3103	0.4931	0.283	24.64
sh2	0.1290	0.0685	0.0317	0.2362	0.000	324.90

TVP-VAR model (Lag = 3)
Iteration: 10000
Sigma(b): Diagonal

Fig. 11 Estimation result

5. Analysis of pulse response results

The advantage of the TVP-VAR model is that it can accurately provide the pulse response relationships of various variables under different economic conditions and time points. During the sample period, we selected lag periods of 3, 6, and 12 to provide equal interval pulse response

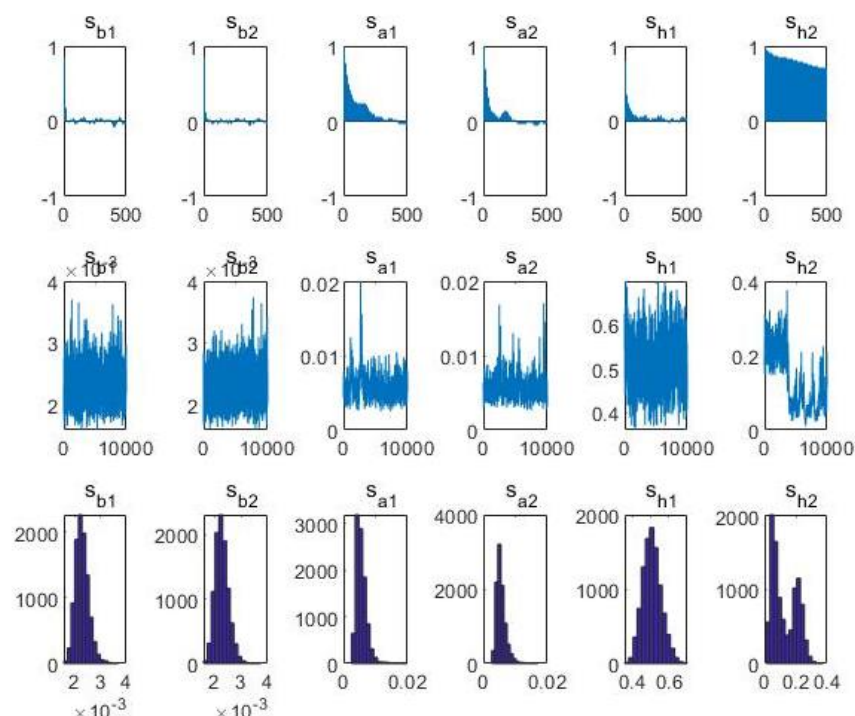


Fig. 12 pulse response results

Combining Figure 1 and Table 1, the first row of Figure 1 is an autocorrelation graph, where the images of the first five parameters are significantly truncated and have significant autocorrelation. The sixth parameter Sh2 is the same as the estimated result, with a large invalid factor and no significant autocorrelation within the image display range. The second line shows the stationarity of the value path, which fluctuates up and down with relatively few extreme values. The value path is

stable, and the Sh2 image is the same as before. The third line shows that the characteristics of the first five parameters in the graph are similar to normal distribution, indicating that value sampling is effective, while the Sh2 image does not conform to normal distribution. The invalidity factor in the data in Table 1 is generally lower than 100, which is better. The other five parameters here are all lower than 100, and only Sh2 exceeding 100 is acceptable.

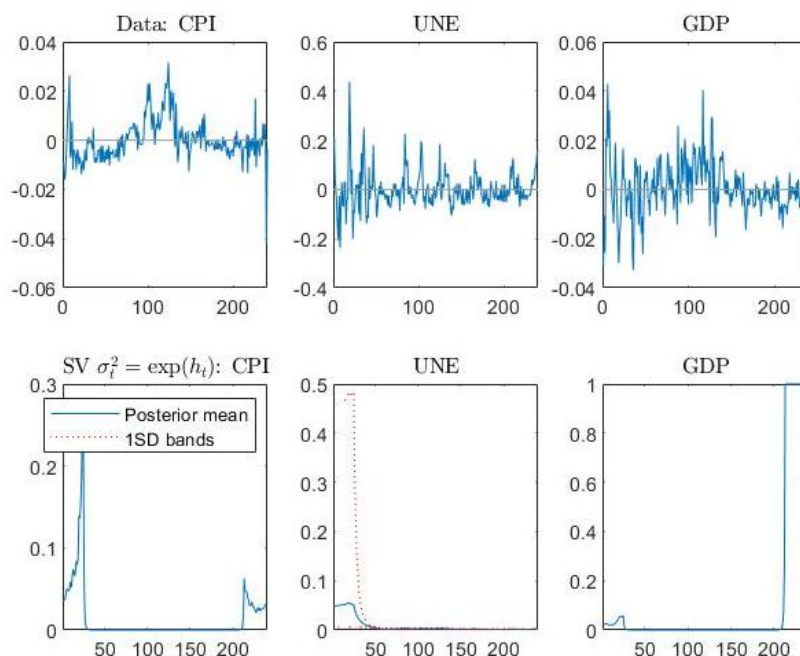


Fig. 13 time-varying characteristics of the random volatility

Figure 13 shows the time-varying characteristics of the random volatility between various variables. The last three figures in Figure 2, taking the variable GDP as an example, show that the random volatility of GDP reached its maximum in 2008. This volatility feature can to some extent reflect the actual situation of economic operation.

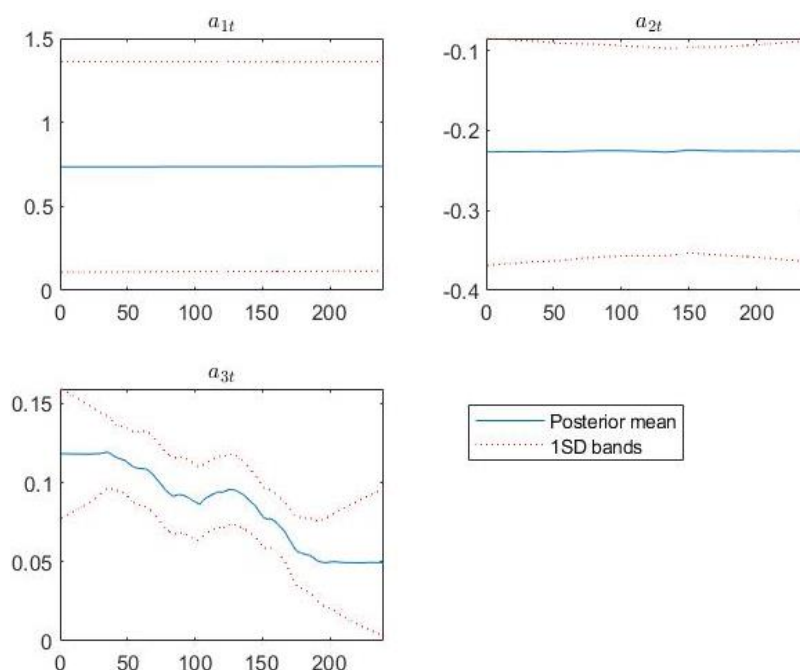


Fig. 14 the time-varying relationship

Figure 14 reflects the time-varying relationship between variables

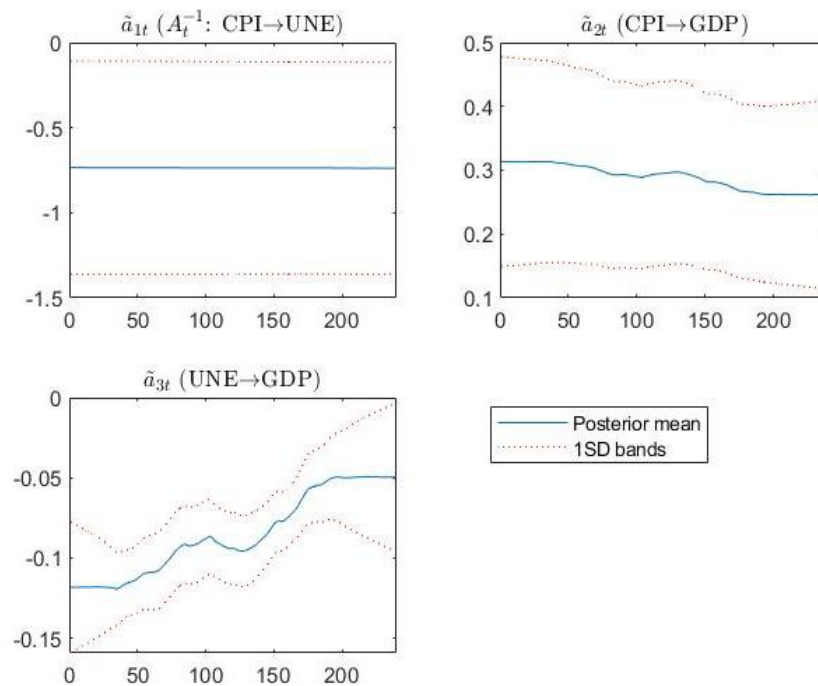


Fig. 15 the time-varying relationship

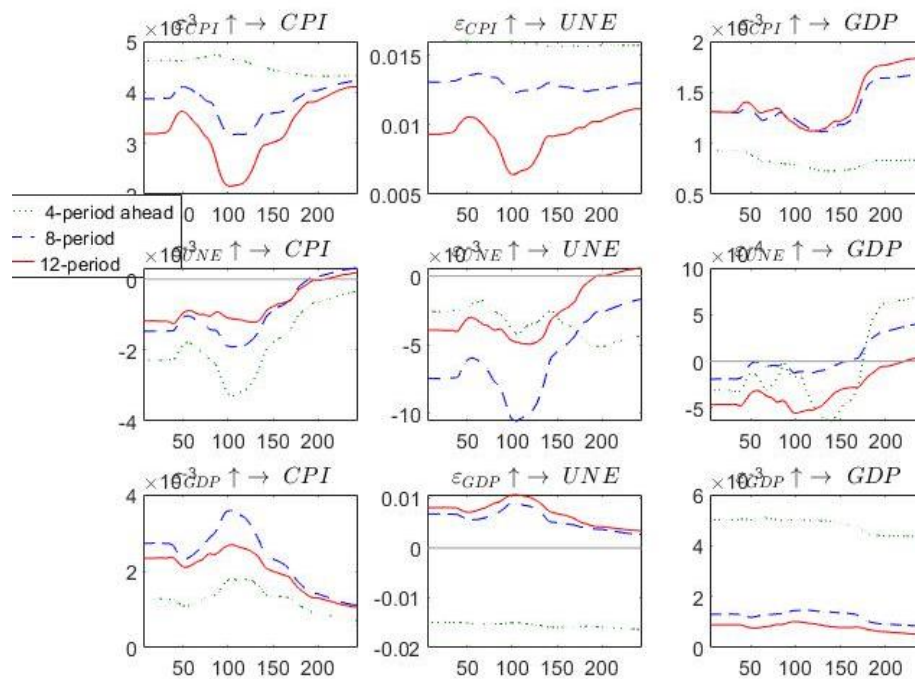


Fig. 16 the pulse response plots for different lead times

Figure 16 shows the pulse response plots for different lead times. Taking the first row and second column as an example, at the horizontal axis of 100, it represents the impact of applying one unit of CPI on the unemployment rate UNE. The pulse response plots for lead times 4, 8, and 12 are shown in the figure.

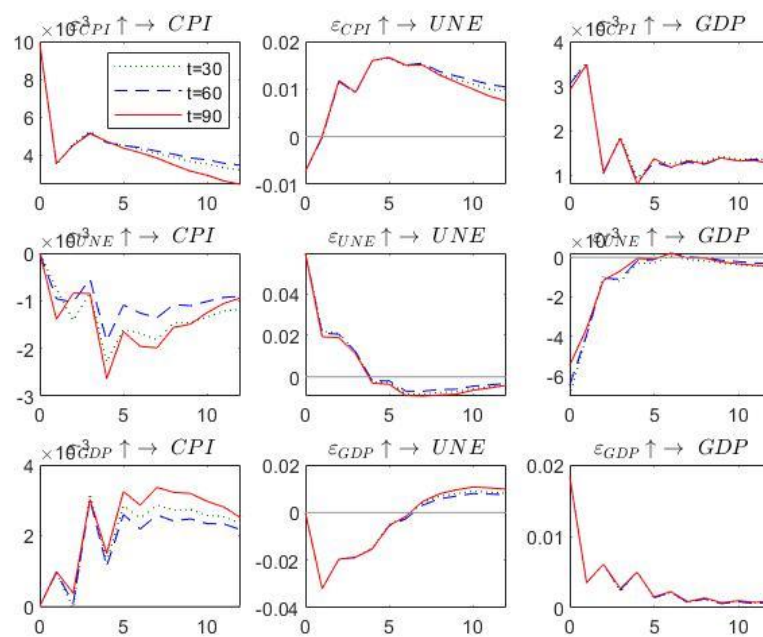


Fig. 17 the pulse response diagram at different time points

Figure 17 shows the pulse response diagram at different time points, setting the relationship between various economic variables at different time points.

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