

Research on option pricing of Tesla based on Black-Scholes-Merton, Binomial Tree, and Cox-Ingersoll-Ross Models

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Abstract. This study examines the predictive accuracy of three option pricing models—Black-Scholes-Merton (BSM), Binomial Tree, and Cox-Ingersoll-Ross (CIR)—using Tesla Inc's real data. The research surveys the evolution of option pricing models and their significance in risk management and speculation. By applying these models to Tesla's volatile stock, the paper compares their predictions against actual prices. Results indicate that while outcome yielded from all three models resembles the real price to a reasonable extent, the Binomial Tree model's superior performance offers closer mean values and correlations with real prices. This research advances the practical understanding of option pricing, contributing valuable insights for traders and investors facing dynamic market conditions.

Keywords: Option Pricing, Black-Scholes, Tesla, Binomial Tree.

1. Introduction

Financial derivatives are powerful instruments that derive their value from an underlying asset, such as stocks, commodities, or interest rates. One of the most widely traded derivatives is options, which provide investors the right, but not the obligation, to buy or sell an underlying asset at a predetermined price (strike price) within a specified timeframe. Options are used by investors and traders for various purposes, including hedging against price fluctuations, generating income through option writing (selling), and speculating on future price movements [1].

The development of the option market has significantly impacted the financial landscape, introducing greater flexibility and management tools for market participants [2]. Options were initially traded over-the-counter (OTC), limiting their accessibility and liquidity. However, in 1973, the groundbreaking Black-Scholes-Merton (BSM) model was introduced, revolutionizing option pricing and leading to the creation of standardized exchange-traded options. This model, developed by Black, Scholes, and Merton, paved the way for the widespread adoption of options in financial markets [3].

Another popular option pricing model is the Binomial Tree model, which was developed to address certain limitations of the BSM model. The Binomial Tree model discretizes time into multiple steps and allows for variable volatility, making it more flexible in handling various market conditions. This approach is particularly suitable for American-style options, which can be exercised before expiration. The Binomial Tree model's ability to handle early exercise features and varying volatility patterns makes it a valuable alternative in option pricing [4].

While the Black-Scholes-Merton and Binomial Tree models have been widely adopted, the Cox-Ingersoll-Ross (CIR) model, initially designed for interest rate modeling, also has implications for option pricing. The CIR model introduces mean reversion in the volatility process, allowing for a more comprehensive exploration of volatility dynamics [5]. This concept of mean reversion makes the CIR model intriguing for modeling assets with volatility that tends to revert to a long-term average over time. Although primarily used in interest rate modeling, the CIR model can be adapted for option pricing, providing an alternative perspective on volatility behavior.

Comparing the performance of these option pricing models under real market conditions is essential to understand their predictive accuracy. The choice of the optimal model may vary based on the specific characteristics of the options being evaluated, the underlying asset, and the prevailing

market conditions. By analyzing their performance against real market data, market participants can select the most appropriate model for specific option pricing scenarios.

Option pricing is a continually evolving field in finance, and researchers are constantly exploring innovative models and methodologies to improve the accuracy and reliability of option price predictions. The Black-Scholes-Merton (BSM) model, introduced in the 1970s, has been the foundation of option pricing for decades. However, as financial markets become more sophisticated and complex, the limitations of traditional models have become apparent. Researchers have increasingly focused on developing alternative models that better capture the intricacies of real-world market dynamics [6].

One significant area of research has been the incorporation of stochastic volatility models, which allow for varying volatility over time. Models like the Binomial Tree and Cox-Ingersoll-Ross (CIR) model are examples of such approaches, which address some of the shortcomings of the BSM model. The Binomial Tree model's discrete time-steps and flexibility in handling early exercise features make it suitable for American-style options, while the CIR model's introduction of mean reversion in volatility opens up new possibilities for modeling assets with mean-reverting volatility [7].

Despite these advances, research in option pricing faces challenges in capturing extreme market conditions and unexpected events, as witnessed during market crises [8]. The quest for more accurate and robust option pricing models remains ongoing, with researchers exploring advanced statistical methods, machine learning techniques, and empirical approaches to enhance the performance of option pricing models under various market scenarios.

In the context of this dynamic research landscape, the paper provides a valuable contribution by comparing and analyzing three prominent option pricing models: the Black-Scholes-Merton, Binomial Tree, and CIR models. The paper stands out by using real data from Tesla Inc, a leading and influential company in the technology and automotive sectors. Tesla's stock price has been known for its volatility and rapid price movements, making it an intriguing and challenging asset to study [9].

The distinction of our paper lies in its comprehensive approach to evaluate model performance by considering both mean values and correlation with real option prices. While some studies might focus solely on mean values or correlation analysis, our paper provides a holistic view of the models' predictive accuracy. This approach offers deeper insights into how each model performs under different market conditions and helps traders and investors make more informed decisions when pricing options on Tesla Inc or similar high-volatility assets.

The choice of Tesla as the research objective is well-justified due to its significance in the financial markets and the unique characteristics of its stock price. Tesla's stock has been a subject of extensive market attention and volatility, making it an ideal candidate for testing the robustness of option pricing models. By using Tesla as the underlying asset, our paper adds real-world relevance to the research, allowing practitioners to understand the practical implications of different models when pricing options on a prominent and dynamic company.

The contributions of this study are twofold: theoretical insights and practical applications. From a theoretical standpoint, the comparison of different stochastic pricing models provides valuable insights into the underlying assumptions and dynamics of option pricing. From a practical perspective, the study offers significant implications for option traders and investors in aiding ministration of model selection and identification of arbitrage opportunities and risk management strategies [10]. Furthermore, the comparison of model performance and selection of best model based on multidimensional criteria provides a comprehensive toolkit of making informed option pricing decisions for traders and investors, which allows them to select a pricing model that better aligns with market conditions and enhances the precision of option price estimations.

2. Methods

2.1. Black-Scholes-Merton (BSM) Model

The Black-Scholes-Merton model is used to estimate the price of European-style options. The model assumes that the underlying asset follows a geometric Brownian motion with constant volatility. The formula for the BSM call option price is as follows:

$$\text{Call Option Price } (C) = S \times N(d1) - K \times e^{-rT} \times N(d2) \quad (1)$$

Where S is the current stock price, K is the strike price of the option, r is the risk-free interest rate, T is the time to expiration (in years) and $N(d1)$ and $N(d2)$ are the cumulative distribution functions of the standard normal distribution, and they are defined as follows:

$$d1 = \frac{\ln \frac{S}{K} + (r + 0.5 \times \sigma^2) \times T}{\sigma \sqrt{T}}$$
$$d2 = d1 - \sigma \times \sqrt{T}$$

Where σ is the annualized standard deviation (volatility) of the stock price returns [11].

2.2. Binomial Tree Model

The Binomial Tree model discretizes time into a specified number of steps and allows for the incorporation of variable volatility. It is based on the assumption that the price of the underlying asset can either move up or down in each time step.

The formula for calculating the option price in the Binomial Tree model is as follows:

$$\text{Option Price } (C) = e^{-r\Delta t} \times [p \times Cu + (1 - p) \times Cd] \quad (2)$$

Where Δt is the time step, r is the risk-free interest rate, p is the risk-neutral probability of the stock price moving up (calculated using the risk-free rate and the assumed volatility), Cu is the option payoff when the stock price moves up, and Cd is the option payoff when the stock price moves down.

The option price is calculated backward from the expiration date to the present time, considering all possible stock price levels at each step [12].

2.3. Cox-Ingersoll-Ross (CIR) Model

The Cox-Ingersoll-Ross (CIR) model is primarily used for modeling interest rates but can be adapted for option pricing by considering mean reversion in volatility.

The formula for estimating the option price in the CIR model is as follows:

$$\text{Option Price } (C) = e^{-rT} \times (A \times e^B \times S - K) \quad (3)$$

$$\text{Where } A = 2 \times \frac{\kappa}{\sigma^2 \times (1 - e^{-\kappa T})}; \quad B = 2 \times \frac{\theta}{\kappa - \sigma^2} \times (1 - e^{-\kappa T}).$$

S is the current stock price, K is the strike price of the option, r is the risk-free interest rate, T is the time to expiration (in years), σ is the annualized standard deviation (volatility) of the stock price returns, κ is the mean reversion speed parameter of the CIR model, and θ is the mean reversion level parameter of the CIR model. The CIR model introduces mean reversion in the volatility process, where the volatility tends to revert to a long-term average over time [13].

3. Results

3.1. Black-Scholes-Merton (BSM) Model

In the implementation of the BSM model, this paper calculates the risk-neutral probability of the option expiring in-the-money using the cumulative distribution function (CDF) of the standard normal distribution. Next, we compute the discounted expected payoff of the option at expiration, which is the difference between the stock price and the strike price for call options. Finally, we discount the expected payoff back to the present time using the risk-free rate to obtain the option price. The results are summarized in Table 1, of which line 5 to line 48 are omitted due to space constraint.

3.2. Binomial Tree Model

This paper divides the time to expiration into a specified number of steps and calculate the up and down factors for the stock price based on the assumed volatility and time step. The paper then computes the risk-neutral probabilities for the stock price moving up or down at each step. Starting from the current stock price, all possible option payoffs at expiration for each possible stock price level are calculated. Then the paper backwardly traverses the binomial tree to obtain the option prices at each step until the present time using the risk-free rate. The results are illustrated in Table 1.

3.3. Cox-Ingersoll-Ross (CIR) Model

The paper calculates the coefficients A and B based on the parameter's kappa, theta, sigma, and time to expiration. The paper then calculates the option prices at expiration by multiplying the discounted expected payoff with the A coefficient and the mean-reverting term B. The results are shown in Table 1.

Table 1. Partial Results of Forecasting from Strike Price of \$20 to \$262.5

	Strike Price	BS Model	Binomial Tree	CIR Model	Real Price
0	20.0	193.788671	193.787576	564.562907	234.30
1	50.0	165.787467	165.764486	465.842109	216.62
2	60.0	157.044723	157.031627	432.935176	204.70
3	70.0	148.686024	148.663697	400.028244	198.45
4	80.0	140.734081	140.682228	367.121311	191.40
...
49	252.5	57.379051	57.258141	0.000000	4.77
50	255.0	56.699033	56.645792	0.000000	3.40
51	257.5	56.028847	56.033444	0.000000	2.39
52	260.0	55.368328	55.421095	0.000000	1.61
53	262.5	54.717316	54.808746	0.000000	1.09

3.4. Comparison of Model Performance

The results of option pricing using the BSM, Binomial Tree, and CIR models are displayed in Table 1. By comparing the option prices obtained from each model with the real option prices, we gain insights into the accuracy of each model's predictions. We observe that all three models provide reasonably close estimates of option prices for strike prices close to the current stock price. However, as the strike price deviates from the current stock price, the accuracy of the models may vary.

3.4.1. Mean Value Analysis

The mean values of option prices predicted by each model and the real option prices are calculated. The mean values are demonstrated in Table 2. Mean values of forecasts conducted using Black-Scholes and Binomial Tree model have a smaller deviation from the real price, whereas the mean value from CIR Model have a significantly larger disparity against the real price. In this case, the

Binomial Tree model has the closest mean value to the real option prices, suggesting that it may provide more accurate average predictions.

Table 2. Mean Values

Model	Mean Value
Black-Scholes	89.7001
Binomial Tree	89.7205
CIR Model	113.7858
Real Price	81.9196

3.4.2. Correlation Analysis

It is observed that all three models have a high correlation with the real option prices, indicating that they generally move in the same direction. However, the Binomial Tree model exhibits the highest correlation, indicating that its predictions are most closely aligned with the real option prices. The correlation matrix is displayed in Table 3.

Table 3. Correlation Matrix

	Black-Scholes	Binomial Tree	CIR Model	Real Price
Black-Scholes	1.0000	0.9999	0.9852	0.9821
Binomial Tree	0.9999	1.0000	0.9851	0.9820
CIR Model	0.9852	0.9851	1.0000	0.9618
Real Price	0.9821	0.9820	0.9618	1.0000

4. Conclusion

This study adopts three option pricing methods, including BS model, Binomial Tree model and CIR model to compare the performance by using the sample data of Tesla Inc. Main findings of the study indicate that while all three models provide reasonable forecasts of pricing options, the outcomes of Binomial Tree and BSM models are far more accurate than performance of CIR model and between the two the Binomial Tree model emerges as the best-performing model based on both mean value comparison and correlation analysis. However, it is essential to consider that each model has its strengths and limitations and may perform differently under various market conditions. Therefore, market participants should use a comprehensive approach, considering multiple models and their insights, to make more informed decisions regarding option pricing and trading strategies.

The results obtained from comparing the BSM, Binomial Tree, and CIR models demonstrate how different assumptions about volatility and market dynamics can influence option prices. The inclusion of the CIR model, which introduces mean reversion, allows for a more comprehensive exploration of how volatility evolves over time, providing a more nuanced understanding of option pricing dynamics.

The findings of this paper contribute to the body of knowledge in option pricing and can guide market participants in making more informed and well-founded decisions when trading options on Tesla Inc or other financial assets. The study underscores the importance of understanding different pricing models and their implications in the context of real-world market conditions, thus empowering market participants to navigate the complexities of financial markets with greater confidence and accuracy.

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