Research on the Option Pricing of Tesla's Stock Based on B-S Model and Binomial Tree Method

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Abstract. With the announcement of price drop of Tesla's multiple vehicles during 2022 and 2023, its stock also entered a very unstable age which cause many investors meet significant loss, so did its option prices. In this article, B-S model, Binomial Tree, and Monte Carlo Simulation are implemented to do research on Tesla's stock price and its call options. The results shows that B-S model has a much lower value of option prices when Binomial Tree does not have multiple division of time period. On the other hand, Monte Carlo Simulation indicates that it would be hard to predict the actual price trend since actual price went to a very low percentile among all simulations. The meaning of this research is to prove Binomial Tree is complementary to B-S model with introduce the concept of time whereas B-S model only uses actual stock prices and calculate the average. Monte Carlo Simulation also warns the investors to continue to pay attention to the news and incidents happened around the companies they invested to.

Keywords: Tesla Stock, Call Option, Black-Scholes Model, Binomial Tree, Monte Carlo Simulation.

1. Introduction

Investments have been favored by a number of people all over the world when in history, the concept of investment could be traced back to prehistoric Middle East in terms of objects like sheep and obsidian instead of money between 9500 and 8500 BCE. It was the earliest form of investment. After thousand years development, the early derivatives emerged in Mesopotamia using coins made of gold and silver to trade [1]. After years of development of financial derivatives, people found that they typically have unexpected jumps with stochastic volatilities, therefore some mathematicians started to make models in order to predict or calculate the expected returns over a period of time no matter how long it is. The initial process they come up with is Brownian motion who forms unpredictable movements based on historical movements and make the general distribution of the random fluctuation be foreseeable and predictable [2]. Among all derivatives, options make up an important part of all financial derivatives. The concept of options was used around 600 BCE in ancient Greece which stated that people could buy the option for the rights to buy the asset at a agreed price level after a given period of time [1]. For options, Black-Scholes equation is one of the most famous theories in recent age to calculate the option prices which can also derive into several distinct approaches to work through it. After the appearance of B-S model, more theories regarding the calculation of the option prices came into the sight of the public. One of the representative theories is the Implied Tree Method in which the risk-neutral probability and volatility smiles are utilized. It was researched and developed by Rubinstein, Derman, and Kani in 1994. Another example of the theories working on option prices is The Fokker-Planck Equation. All of these were developed during the decade of 1990s [3]. B-S model was developed by two professors, Robert Merton and Myron Sholes in 1970s. They posted similar articles regarding the similar equation separately which is a interesting coincidence. Merton even expended the original model which made it suitable for other financial derivatives [4]. In 1979, Binomial Tree method was brought up which was mentioned to be a simple method to calculate option prices. It was thought to be a complementary method to the B-S model which only has two options to go up or go down with a certain probability. However, it gave the term of time period which could be divided into numbers of equal pieces which made it more suitable for complex options [5].

Tesla Motors was founded in 2003 by Martin Eberhard and Marc Tarpenning. In 2010, Tesla firstly had a public offering at $17 per share. Tesla is also a vehicle company focusing on innovation of electric cars partner with traditional manufacturer companies [6]. Recently, Tesla as a well-known vehicle company has been one of the favored choices for investors. However, during mid-2022 to early 2023, Tesla's stock met a unprecedented situation of stock price drop. The underlying reason is the price drop in several models of Tesla vehicles [7]. Although the products sold has a significant increase indeed, it does not help Tesla to a higher level in terms of stocks on the other hand which is not expected by the investor. Therefore, in this article, B-S model and Binomial Tree methods will be implemented to calculate the call option prices for Tesla stock using the stock data of one month. At the same time, Monte Carlo Simulation will be used to predict the trend of stock prices to see if the price drop phenomenon could be predicted or not.

The article will have four parts including introduction, data, results and discussion, and conclusion.

2. Data and Method

2.1. Data

The price data of the Tesla Stock is collected from Yahoo Finance of dates from November 30th, 2022 to January 19th, 2023. During this period, the drop in terms of prices is significant in response to the price drop of Tesla's vehicle products which is shown in figure 1.

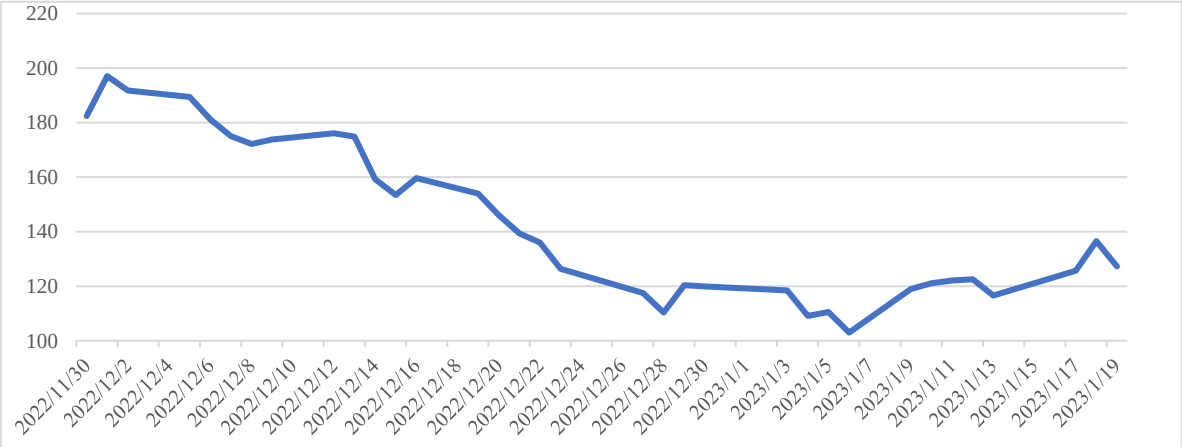

Figure 1. Stock Price During 2022.11.30 – 2023.01.19

For the B-S model, there are several additional elements required. The volatility value, σ, is chosen from alphaquery.com with the value of 180-day implied average during the selected period. Time period T is used in terms of years which means after n years, the call option will be able to be exercise, and the value of T is one in this case. Interest Rate needs to be risk-free, and it is 4.00% showing on finbox.com. The strike price is assumed in the case because there is seldom historical data of strike price available on the internet. Lastly, this paper used 1 to present that call option is what is asked for instead of put option.

For the Monte Carlo Simulation method, starting price of real stock data and the same value of volatility as in B-S model method are utilized. In addition, the daily volatity which is calculated from the implied volatility value is also necessary for the calculation.

As for the binomial tree method, the risk-free interest rate, 180-day implied volatility, time period T, and assumed strike price are requested to proceed.

2.2. Method

There are three methods used in this test: the Black-Scholes-Merton Model, Monte Carlo Simulation, and Binomial Tree Method. Among these models, the B-S model and Binomial Tree are used to calculate the average call option prices, and Monte Carlo Simulation is used to predict the final stock price on 2023.01.19 in comparison of the real stock price on this date.

2.2.1. The Black-Scholes-Merton Model

The Black-Scholes-Merton Model can be called in a simple form, B-S model, which gives a partial differential equation which is also known as "PDE" to calculate the corresponding call or put option prices under a given circumstance. Black-Scholes-Merton Model stands for the equation below:

$$C = N(d_1)S_t - N(d_2)Ke^{-rT} \tag{1}$$

where C stands for call option prices; N is a function which is cumulative distribution function; S_t stands for stock price at time stamp of t; K stands for strike price; r stands for risk-free interest rate; and T represents the time period until the maturity of the option. In this equation, d_1 and d_2 stands for separate equations below:

$$d_1 = \frac{ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{3}$$

Where σ is the volatility value in addition [8]. In this case, with the given value, a line with everyday call option prices will be calculated, and the final result would be the average value of the line.

2.2.2. Binomial Tree Method

The standard binomial tree is introduced by Cox, Roth, and Rubinstein in 1979 which generally divides the time periods into n times with corresponding level of calculation of asset value. Depends on the value of volatility, time-to-expiration, and risk-free-interest rates, the initial price can be divided into two branches with related probability and calculated asset value after a period. With the initial stock price S_0, the model with comes up with a probability p with the equation:

$$p = \frac{e^{rt/n}-d}{u-d} \tag{4}$$

$$u = e^{\sigma\sqrt{\frac{t}{n}}} \tag{5}$$

$$d = e^{-\sigma\sqrt{\frac{t}{n}}} \tag{6}$$

The probability of going up to the result of $u * S_0$ is p, and the probability of going down to $d * S_0$ is $(1-p)$[9]. In this matter, only the first branch of stock price changes and corresponding average call option prices will be calculated because of the limited technology.

2.2.3. Monte Carlo Simulation

Monte Carlo Simulation utilizes random numbers and comes up with signifant number of possible results based on the given condition. The number of the results differs from hunderds to billions. For the single trial, the results will be analyzed with statistical distribution and therefore obtain the related average value and variability [10]. In this case, the initial Tesla's stock price will be used as the beginning point of Monte Carlo Simulation and five hundred results will be analyzed to compare with the real stock in order to indicate the stability of Tesla's stock during an unstable market condition.

3. Results and Discussion

3.1. Results of B-S Model Calculation

Figure 2 below shows the everyday results of the option prices calculated from B-S model. The line is fluctuating with a dropping trend generally, and the lowest point is $11.00 per option on the 26th trading day which indicates that Tesla's stock has an unstable marketing condition. The average value of this graph is approximately $81.704.

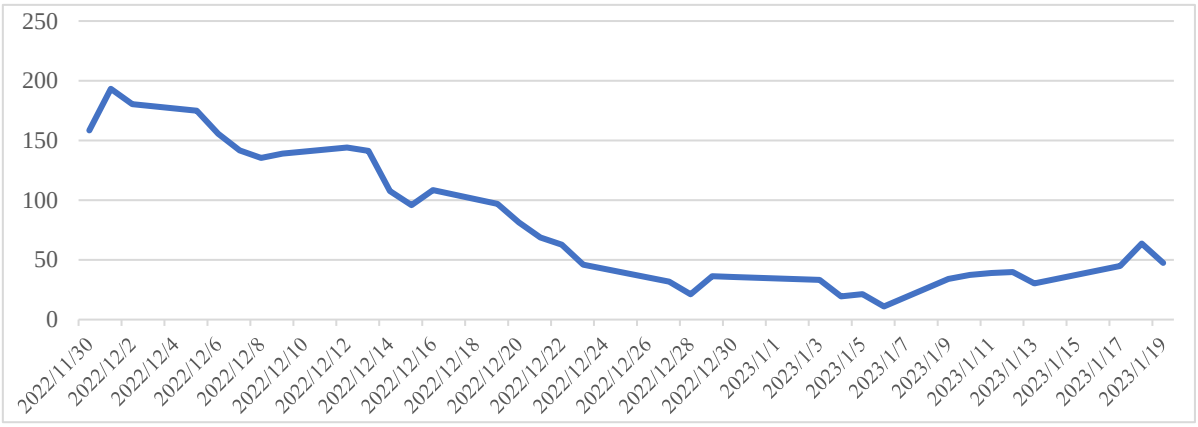

Figure 2. B-S Model Calculation

The explanation to this result is that based on the existing stock price trend of the selected period, overall average price level of a Tesla's call option which can be exercised after one year will be $81.704.

3.2. Results of Binomial Tree

Based on the information already provided, the model shows a increasing trend which is 114.89% of the intial price, and a decreasing trend which is 87.27% of the initial price. The result for this one period division is $209.60 for upward and $159.21 for downward. The probability to increase is 46.57% based on the formula, and it turns out to be 53.43% for decreasing on the other hand. Based on the probability and another formula for call option price calculation, Binomial Tree method comes up with the average call option price of $175.51.

3.3. Comparison between B-S model and Binomial Tree Method

The most obvious thing indicated from the results of two methods is that the average option prices calculated are significantly different from each other. The call option value calculated from B-S model is 93.806 lower than the result of Binomial Tree. The inferable explanation is that B-S model result is calculated from all the stock prices throughout the period whereas Binomial Tree only utilizes the initial stock price and available market condition variables. On the other hand, the general market condition of Tesla Stock is unstable, but Binomial Tree method in this case could not predict the fierce fall of the stock price. In this case, it is necessary to use Monte Carlo Simulation to verify if the trend of Tesla Stock is foreseeable or not.

3.4. Results of Monte Carlo Simulation

With starting price of 182.43 and volatility of 0.753, daily volatility is calculated to be 5.61%. Figure 3 shows five hundreds results after 34 days. For the whole simulation, the mean price after 30 days is 182.16 with standard deviation of 61.34. With all 500 simulations, 5% of predictions will be below 102.77, and the real stock price is 127.26 after 34 days with a percentage rank of 17.6%. All of the data above is based on ten times simulation and takes its average. The final percentage rank indicates that the there is a low chance that the stock will have such a drop during a month. It is rational to assume that during the period, the stock market for Tesla is quite unstable.

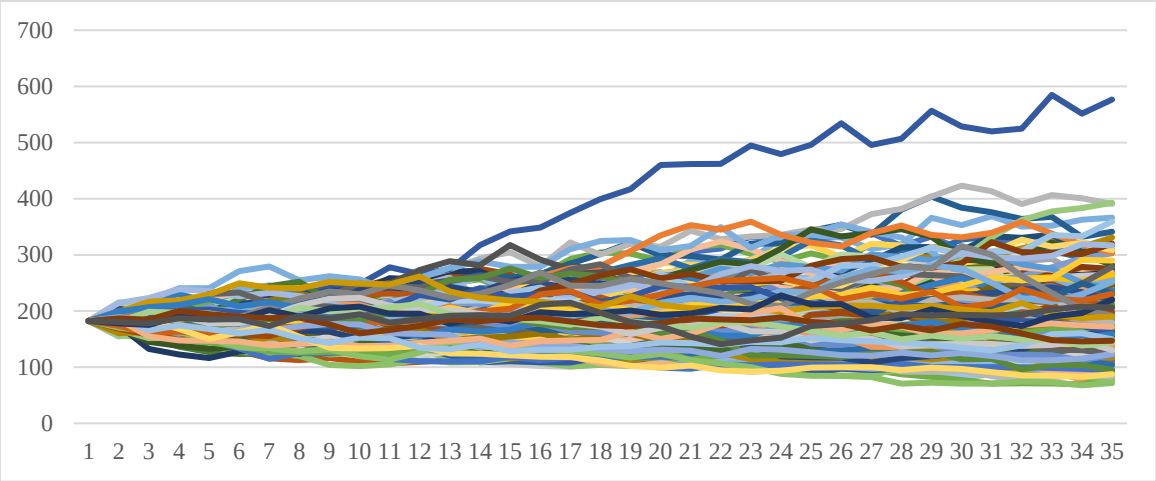

Figure 3. Monte Carlo Simulation

4. Conclusion

This article used B-S model and Binomial Tree methods to calculate the call option prices for the rights to buy Tesla stock at $100 per share after one year, but only one time period is used for Binomial Tree method because of the lack of programming background; at the same time, Monte Carlo Simulation is also utilized to predict the future trend of Tesla Stock. First research result is that B-S model's result is smaller than Binomial Tree method's because B-S model used the whole months' stock data and calculated the average, however Binomial Tree method only has one period which means the real trend is not considered as an element in the calculation, and the general decreasing trend causes the lower value of B-S model result. Second research result is that Monte Carlo Simulation indicates that general possibilities of Tesla's stock trend within a month has a relatively low chance to predict the real stock price drop happened in real world.

These results present the suitable situations of when to use B-S model result and when to use Binomial Tree method. They prove the theories state that B-S model and Tree Methods complement each other on certain sense. These Results also warns the investors to continue to watch the news and incidents happening related to the stock owned by these companies which could influence the stock market.

On the other hand, the limitation also existed for this research. The very first limitation is that Binomial Tree method only utilizes one period instead of dividing it into multiple periods which will indeed increase the accuracy of the calculation. Second limitation is that no actual option price or strike price data is available for the period selected which will make the prices calculated lack of comparison. Lastly, Monte Carlo Simulation may need more simulations to increase its accuracy which also due to the lack of programming skill.

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