Research on the option pricing of Amazon and Google based on B-S model and Monte Carlo simulation

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Abstract. The emergence of financial derivatives has helped people to effectively hedge the risks present in the financial markets while giving them the ability to make leveraged investments in the financial markets. Although derivatives did not originate in the financial market, it still has a huge role in the financial market so far. In recent years, science and technology innovation is known as a growing concern for governments due to factors such as environmental issues. In this paper, we choose to study the financial derivative, European call option pricing problem of technology-based companies AMZN and GOOGL. In this paper, the European call options of these two companies are priced by using the BSM model, and the results predicted by the BSM model are improved by using Monte Carlo Simulation. Ultimately, the paper finds that Monte Carlo Simulation prices European call options on technology-based firms nearly three times higher than those priced by the BSM model. The results suggest that using a single model to price European call options on technology-based firms is inaccurate. The purpose of this paper is to help investors in pricing European call options on technology-based firms and to warn investors about the inaccuracy of a single option pricing model.

Keywords: Black-Scholes-Merton model, European call option, Monte Carlo Simulation, Derivatives.

1. Introduction

Finance, essentially the application of economic principles to decision-making, involves the allocation of funds under conditions of uncertainty. For investors, they allocate their funds to different financial assets to achieve investment goals, while companies and governments raise funds by issuing bonds and use the funds for operations. [1] Nowadays, in addition to the well-known stocks, there are also a variety of financial instruments for investment in the financial market that investors can choose from. Thus, how to categorize the various types of financial instruments appearing in the financial market has become a topic of concern for first-time investors in the financial market. According to the book "Introduction to Economics and Mathematics in Financial Markets" According to the economic classification, financial products can be categorized as bonds, stocks and derivatives. [2] Bonds imply fixed-income bonds, which are essentially risk-free, and both bonds and stocks can be characterized as securities.

What are derivatives? A derivative is a contract to obtain value from an underlying asset at a specified time and can be linked to physical commodities, stock indices and interest rates. [3] The price of a derivative depends on the future or present value of the underlying asset, and in its most primitive form it is a forward commodity contract. The Mesopotamian civilization of grain lending fit this profile 4,000 years ago. After that, various civilizations have had contracts that fit that profile, such as the rice trade in Japan in the 17th century. This shows that derivatives were initially used in agriculture. Today, on the other hand, derivatives are widely used in the financial sector. In the financial field, investors can use financial derivatives to hedge the possible risks in their investments, while for companies, derivatives can be price hedged to ensure the company's interests. In 2008, Southwest Airlines used a derivatives strategy to hedge the price of fuel, and the company was still able to purchase fuel at a lower and fixed contract price when the price of oil became higher. [3] It follows that a profit for one party to a derivatives contract means a loss for the other party.

Financial derivatives can be broadly categorized as options, futures, and forwards, and there is another type of derivative called a swap, but it will not be described in this paper. Forwards refer to a contract where two investors can draw up a deal for the future that can be highly customized according to their needs. Futures evolved out of forwards, and compared to forwards, futures contracts are more standardized and standardized, and investors can match counterparties on an exchange. Options, on the other hand, purchase an exercisable right, which refers to the right to purchase the underlying asset.

Unlike futures and forwards, an investor who purchases an option can choose to exercise the right or forgo exercising it. At the end of the 17th century, the London call and put options exchanges were established. This is the first time in human history that both call and put options were traded on the exchange at the same time. [4] Assuming that the current price of a stock is S, and the contractual exercise price is K, the investor believes that the stock price S (t) will be higher than K at time t in the future, and therefore chooses to purchase a call option at price C. If the future stock price is the same as predicted, the investor can choose to exercise the option. If the future stock price is the same as predicted, then the investor chooses to exercise the option and thus earn a profit. If the future stock price is lower than K, then the investor may choose not to exercise the right. As for investors in futures and forwards, they cannot forego the purchase because they are purchasing a liability. This paper is mainly concerned with stock options and hence in this paper it is argued that the essence of an option is the prediction of future stock rise or fall.

Options are important for two main reasons, firstly, investors can use options for hedging to avoid risk. For example, investor A wants to invest in a certain stock, the current stock price S (0) is $100, and the option price is $2. The future stock price may fall to $80 or rise to $110. In order to avoid this risk, investor A chooses to purchase an option. To avoid this risk, A chooses to buy the option, because if he buys the stock directly, he will lose $20 per share if the stock price falls in the future, but if he chooses to buy the option, he will only lose $2. Secondly, an investor can use options to leverage his investment, which simply means that an investor can borrow against an option to invest in a stock. Let's say an investor wants to invest in a certain stock. If the investor is very optimistic about the future price of the stock, then he can invest the entire amount of money directly in the option. Let's say the investor has $200 in total assets, and the stock the investor wants to invest in has a current price of $150 and a European Call Option with a price of $2 and a strike price of $130. If the investor chooses to invest directly in the stock, he can only buy one stock with the money he has available to him. If the investor chooses to invest in options, then the investor can purchase 200 call options. If the stock price rises to $210 in the future, then the investor's return is ($210-$130) *200=$16,000. Since the nature of the option operation is that one party earns a profit is the other party's loss, in other words, this is the game between the investor and the option seller, so how to carry out the option pricing, which became the most concerned about the subject of this paper.

Options are more volatile than stock prices, and small fluctuations in stock prices will lead to large changes in option prices...Previously, most of the work on option price estimation has been used to estimate warrants. These valuation methods are Incomplete, usually with one or more arbitrary parameters. [5] Warrant is the most basic financial derivative, usually issued by a joint stock company, and the warrant is a call option that can be used to subscribe for its stock. Due to the greater volatility of option prices, it has been difficult to use formulas to price them until the BSM model was proposed in 1973, which made the mathematical method of option pricing perfect and became a complete pricing formula.

After experiencing the third industrial revolution, technology has become a topic of concern to every country and everyone. According to a report by PricewaterhouseCoopers (PwC) [6], seven of the top ten companies in the world by market capitalization are technology-based companies. At the same time, technology-based enterprises are of great significance to environmental protection and national development.

The main research content of this paper is whether the use of different models will affect the value of stock options in the option pricing problem of technology-type enterprises. The purpose of this kind of research is to help investors choose the option pricing model of technology-based enterprises in the future.

2. Data and Method

2.1. Data

2.1.1. Sub heading

This paper chooses to analyze the European call option prices of the technology-based companies Alphabet and Amazon.com, the stock prices for forty trading days between September 9, 2022, and November 4, 2022, are selected and the stock data is collected through Yahoo Finance, followed by Black-Scholes Model and Monte Carlo Simulation to predict the option prices of the two underlying companies. The Black-Scholes Model and Monte Carlo Simulation are used to predict the option prices of the two underlying companies. Finally, a conclusion is drawn by comparing the data calculated by the two models.

The figure 1 shows that the change in stock price over a forty-day period, both companies are technology-based companies, and the stock price is in a downward trend over the selected forty-day period, for both companies the stock price is at its lowest point for the year.

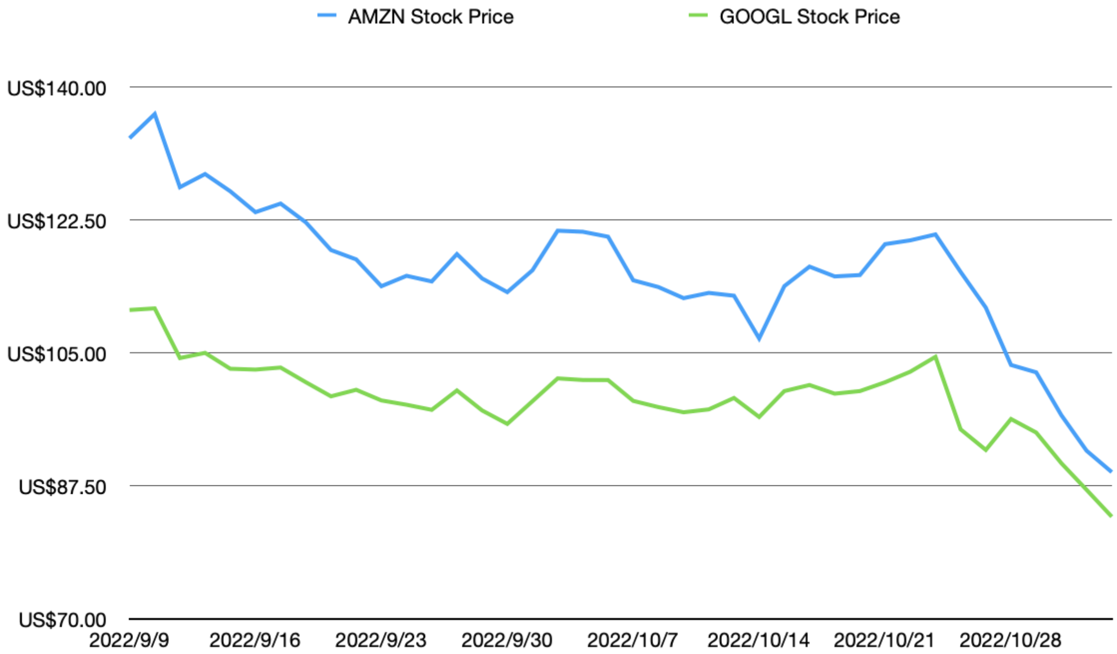

Figure 1. The 40 days Stock price of GOOGL and AMZN

2.2. Methods

2.2.1. Black-Sholes Model

Black Scholes Model, which can also be called the Black-Scholes-Merton Model. Fischer Black and Myron Scholes, along with Robert Merton, created this model, which was published in 1973. And it won the Nobel Prize in Economics in 1997. Until now, the model is still considered one of the best methods for pricing option contracts. [7] The main purpose of the BSM model can be considered as constructing a risk-free asset portfolio, which includes bonds (cash), options and underlying stocks. [8] Assuming that option prices and stock prices are uncertain, so are individual investment returns. Therefore, the risk existing in the investment can be covered by changing the asset allocation (replication) of the portfolio. [7], Simply to say, the BSM model can be understood as copying a portfolio composed of risk-free assets, options and stocks, and then applying the BSM model to copy this portfolio and then arbitrage.

In this model there are a total of five variables which will be assumed in this paper to be the following letters: Strike Price [K], Stock Price [S], Expiration Time [T], Risk-free Interest rate [r] and Volatility [σ].

Also, in this model, it is assumed that the model satisfies the following conditions.

(1): Purchases have no transaction costs, no additional expenses such as taxes, etc.

(2): The risk-free rate [r] and volatility [σ] are constant, and in this paper, we choose to use the data that can be collected in the network as r and σ.

(3): The model is only applicable to European options.

(4): The underlying asset of the option conforms to geometric Brownian motion, while the underlying asset return is normally distributed.

(5): The option does not pay a dividend during the life of the option.

The BS model originally looked like a partial differential equation, but since the derivation process is too cumbersome, this paper will not present the BS model in the partial differential equation style but will simply write out the equation for readers who want to investigate it. B-S PDE:

$$dC(t,S(t)) = \left[C_t + \frac{1}{2}\sigma^2 S^2 C_{SS} + \mu S C_s\right]dt + \sigma S C_s dw \tag{1}$$

Where, μ refers to the drift coefficient, σ refers to Volatility, and dw is randomly sampled through a normal distribution.

This paper uses the Normal Distribution method to explain the BS model and continue to use this method to calculate the option price of the underlying company in subsequent research.

B-S-M Model:

$$C = S * N(d1) - Ke^{-r(T-t)} * N(d2) \tag{2}$$

Where, t denotes Time to Maturity.

The $d1 = \frac{LN\left(\frac{S(t)}{K}\right)+(r+\frac{\sigma^2}{2})(T-t)}{\sigma*\sqrt{T-t}}$ and the d2 is equal to $d2 = d1 - \sigma * \sqrt{T-t}$

In the model, N(d1) means the factor that the present value of the contingent stock return exceeds the present value of the stock when the option is exercised. N(d2) means the risk-adjusted exercise probability in the BS model [9].

In short, the BSM model can be interpreted as option price = (stock price) * (probability that the present value of earnings exceeds the present value of the stock) - (payment for discounted options) * (probability of risk-adjusted strike).

2.2.2. Monte Carlo Simulation

Monte Carlo Simulation was proposed during World War II, and the model is named after a casino town in Monaco. Monte Carlo Simulation, originally developed by Stanislaw Ulam, a mathematician who was involved in the Manhattan Project, is a model of chance and random outcomes at its core. Like gambling games like roulette, dice and slot machines. [10] The Monte Carlo model is applied in a wide range of fields, and its most classic application is to find the value of π. Suppose a circle is placed in a square, and then the beans are continuously thrown in the square area. In this way, it can be obtained the area of the circle, because when there are enough beans, there is every chance to fill the entire circle, and then the Value of π can be obtained. When the number of beans is higher, it means that the approximation is more accurate, which is completely consistent with LLN. The most common usage in finance is stock option pricing. Unlike the BSM model, the option pricing performed by BSM is very dependent on the stock price, while Monte Carlo Simulation only needs to get the initial few variables, and first use these variables to calculate the stock price. Stochastic simulations are then simulated for option prices by simulated stock prices. In this paper, Monte Carlo Simulation exists with the following variables: Initial Stock Price S (0), Strike Price (K), Risk-free Interest rate (r), Volatility (σ), Daily Volatility (Daily_σ), Path length (Δt), Time (T). Due to the special calculation process of Monte Carlo Simulation, it is not a closed formula but a calculation framework. The framework is as follows:

1. Simulate the stock path: Since the stock price is a geometric Brownian motion, you need to use the following formula to simulate the stock price

$$S(t) = e^{(\left(r-\frac{\sigma^2}{2}\right)\Delta t+\sigma\sqrt{\Delta t}Z)} \tag{3}$$

It should be noted that Z in the above formula refers to a value randomly selected in the standard normal distribution, which can be written as: $NORM.S.INV(RAND())$

2. Use the simulated stock price to calculate the discounted option price by the following formula:

$$C(t) = e^{-rT} * max\ (S - K, 0) \tag{4}$$

In here, the e^{-rT} means the risk-free assert.

3. compute the average value of option price

$$AVERAGE(C(t)) \tag{5}$$

Therefore, Monte Carlo Simulation can be understood as estimating the expected value of a certain value by drawing many samples.

3. Results and Discussion

3.1. Results of BSM model

In the BS model, this paper use stock data from Yahoo finance selected from 40 trading days of Close to Close. At the same time, since r and sigma in the BSM model are constant values, this paper chooses the implied volatility data from website of Alpha query, and Interest rate data is from Fed. The Assumption of the parameters in BSM model is shown in table 1.

Table 1. Assumption of BSM model

\	**GOOGL**	**AMZN**
180 -days implied Volatility	0.2807	0.3386
T	1	1
r	2.95%	2.95%
K(assumed)	90	100
If call option	1	1

This paper compute the Call option price based on the above table's conditions and use this formula below.

$$C = S(t) * NORM.S.DIST(d1, TRUE) - K_assumed * EXP(-rT) * NORM.S.DIST(d2, TRUE) \tag{6}$$

In here, the value of d1 and d2 can calculate by these two formulae.

$$d1 = (LN(S(t)/K\_assumed) + (r\_ + (impilied\_Volatility^2)/2 * T)/(\_180\_days\_impilied\_Volatility * (SQRT(T))) \tag{7}$$

$$d2 = d1 - 180\_days\_impilied\_Volatility * (SQRT(T)) \tag{8}$$

It seems that the formula for calculating options is different from the expression in the second part of this article, because in this part, this article shows the calculation using the method of excel, so there will be differences.

After calculation using the above formula, the value of GOOGL and AMZN option prices can be obtained here. This article makes the line charts Figure 2 of option prices. The stock prices of companies can be compared through F1 and F2, and the option prices of F3 and F4 can be compared at the same time, which makes the data look more intuitive. It can be seen from the figure that the change path of the option price is basically the same as that of the stock price, except that the fluctuation range of the change of the option price is slightly larger, there is no difference.

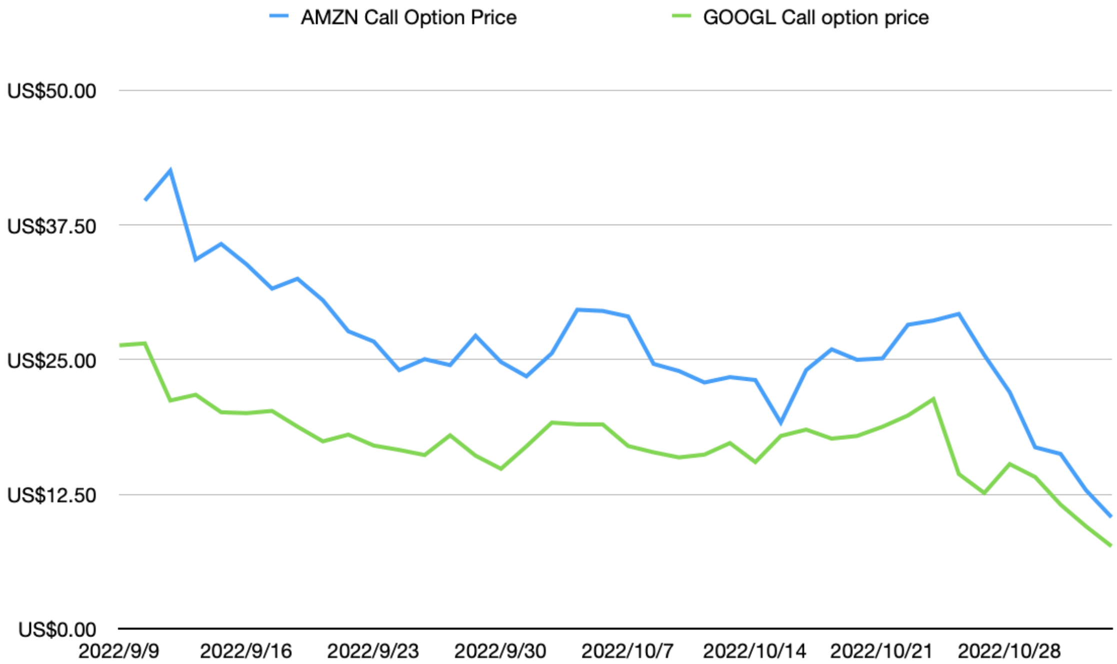

Figure 2. BSM Model pricing EU Call option price of AMZN and GOOGL

3.2. The Monte Carlo Simulation Improved BSM model

This paper uses the BS model to make the most basic prediction of option prices. Since the pricing of the BS model is path-dependent, this study uses random MC simulation to compare with the value of the BS model (table 2).

Table 2. Assumption of MC simulation

\	GOOGL	AMZN
S (0)	110.650002	133.270004
K	90	100
r	2.95%	2.95%
Historical Volatility	0.3259	0.3367
T	1	1
Daily Volatility	0.02052977	0.021210106
Δt	0.003968254	0.003968254
Mean (After Calculate)	111.1894963	134.0038825

Daily Volatility=$\frac{\sigma}{\sqrt{252}}$ “Sigma means Historical Volatility”

Δt=$\frac{1}{252}$ “Delta t means length of path”

After doing so, these data and formula below can be used to emulate the GOOGL and AMZN’s stock price.

$$S(1) = S(0) * EXP((\_r - 0.5 * Daily\_Volitality\sigma^2) * delta\_t + Daily\_Volitality * \sqrt{delta\_t}) * NORM.S.INV(RAND())) \quad (9)$$

After 401 simulations of S (1), this paper derives 252 values for S (1), and then for subsequent stock price calculations simply replace S (0) with S (1,2,3 ...., n) will yield 252 price simulations for the stock S (40) after 40 days.

After this it is only necessary to derive Mean for the data of S (40) to get the estimated stock price after the fortieth day Mean $_{\text{GOOGL}}$ = 111.1894963, Mean $_{\text{AMZN}}$ = 134.0038825

Now can simulate the option price using the following equation.

$$EXP(-\_r * T) * MAX(S(t) - K\_assumed, 0) \tag{10}$$

After simulating the complete path using Eq. the path diagram can be obtained [F3] and [F4].

The next step is to find the mean value of the 252 simulations of C (40), and the predicted value of the GOOGL's option price C_{GOOGL} (40) is 20.60125.

The Same step can be used to calculate the call option price of AMZN, C_{AMZN} (40) = 33.01541947

3.3. Comparison

This paper compares the option prices of GOOGL predicted by BS and MC as summarized in table 3. And found that there is a significant difference in the predicted values of the two models. The predicted value of S (40) by the BS model is 7.6845, and the predicted value by MC is 20.60125.

At the same time, this paper compared the option price of AMZN predicted by BS and MC, and there is also a significant difference. The S (40) predicted by the BS model is 9.0094, and the value predicted by MC is 33.0154.

Therefore, it can be considered that the option price result of Monte Carlo simulation of technology companies will be greater than the option price of BS model simulation.

Table 3. Comparing for two model

	AMZN	GOOGL
BSM model	9.0094	7.6845
MC simulation	20.60125	33.01541947

4. Conclusion

In this paper, BSM model and Monte Carlo Simulation are used to study the option prices of AMZN and GOOGL. The results obtained are that the option prices simulated by Monte Carlo Simulation are always higher than the option prices simulated by the BSM model for technology-based firms, even nearly three times higher.

In general, investors tend to use a single specific model to estimate the price of the underlying option that needs to be invested, which is usually inaccurate. In this article, this paper uses two models to calculate the option pricing of technology-based companies. Therefore, the research value of this paper is not only to calculate the European call option pricing of technology-based companies, but also to prove that only using a single pricing model is important for Option pricing research is not accurate enough, investors need to use another model to improve and compare the first model.

However, this paper still has some shortcomings, such as, the data precision of Monte Carlo simulation is not enough, and the prices of the selected stocks are all falling, and the situation of rising is unknown. Thus, the above issues are needed to be discussed in the future research.

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