A Comparison of the BS-Model, the V-Model and the G-Model in Estimating Call Option Prices

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Abstract. This research used Monte Carlo Simulation (MCS) to simulate stock paths, which is used in estimating the Call Option Prices for the Black Scholes-Model (BS-Model), the Vasicek-BS-Model (write the V-Model for short) and the GARCH-BS-Model (write the G-Model for short). Then, the accuracies of the three models are compared using the MSE test (MSE-test), based on the estimated Call Option Prices and the actual Call Option Prices of the three models. The results revealed that the G-Model is the most accurate model, and that V-Model doesn't improve the BS-Model. The significance of this research is to realize that improving the interest rate component won't improve the accuracy of the BS-Model in estimating Call Option Prices whereas improving the volatility component.

Keywords: Black Scholes-Model, Vasicek-Model, GARCH-Model, Monte Carlo Simulation, Call Option Pricing.

1. Introduction

The worth of financial derivative, one of the financial instruments, is dependent on an underlying asset. The origin of financial derivatives should be traced back to when Aristotle made a profit in the trade with Greek philosopher Thales about signing contract for the transaction of olives. This concept of using contract for trades was spread globally ever since. There are several important uses of financial derivatives. Firstly, hedging, keeping the risk of unfavorable flows in price of assets at their minimum. Secondly, speculation, making profits under both cases when prices rise/fall. Financial derivatives options contracts are a derivative, offering owners the opportunity to buy or sell (depending on whether it is Call/Put Option) an underlying asset at a constant price at a specific time. Another form of option, provided by Black and Scholes, namely warrant, was much more complicated when analyzed comparing to simple options. Factors contributing this increasing sophistication were to do with the period of measuring a warrant and the fluidity of the exercise price of a warrant [1]. The first Option Pricing Models was the BS-Model, proposed by US economists Myron Scholes and Fischer Black. This Option Pricing Model had aroused debates in the academic fields.

About Option Pricing Models, the existing literature reviews mainly focused on three key aspects: the BS-Model, the V-Model (write the V-Model for short) and the G-Model (write the G-Model for short). Among the researches, several drawbacks of the BS-Model were mentioned by Shobhit, including: (1) risk-free rate of return and volatility were assumed to be constants. (2) the assumption that paths of stock prices follow a Brownian motion, thereby neglecting the impacts of large price perturbation in reality [2]. In Thomas's research, the Black Sholes Model, together with a piecewise linear weighting function for calculating implied volatilities, are used to estimate Call Option Prices. Then, sensitivity analysis was used to compare estimated Call Option Prices using the BS-Model with the implied volatility to Call Option Prices under simulated stochastic volatility. The results of the analysis revealed that the unsophisticated model was even able to generate a similar estimation to the simulated prices [3]. Aduda addressed a particular G-Model, namely N-G-Model (write non-linear GARCH in mean Model for short) for supplanting the BS-Model in estimating foreign currency pricing options for the Croatian Market. This replacement serves primarily to improve the BS-Model by considering the risk premium in underlying asset rather than a standard preference-free option pricing [4]. Chaudhury and Jason had carried out Empirical Martingale Simulation in addition to MCS, in order to reduce sampling error, on stock paths, which is used in estimating option prices for

BS-Model, Modified BS-Model and Pseudo-G-Model (write P-G-Model for short). One of the simulation results suggest that Modified BS-Model does not offer particular benefits over BS-Model, whereas P-G-Model improves BS-Model [5]. Another author mentioned advantages of Exponential-G-Model (write E-G-Model for short) over G-Model in estimating volatility. These include: the soothing of parameters' positive constraints, namely logarithmic specification and E-G-Model is more capable in obtaining the continuous volatility shocks [6]. Hsieh and Ritchken had made comparisons in the performances of both the G-Model and the Non-linear G-Model (write N-G-Model for short). Results showed that the N-G-Model revealed a better performance than the G-Model as the N-G-Model is more capable in omitting the biases from pricing residuals. Even if the re-estimation of the modelling parameters had not been carried out for a long time, the remarkable performance can still be maintained under the N-G-Model [7]. An option analysis result revealed, by Steven and Saikat, the better performance of the G-Model comparing to the BS-Model as the single lag version of the G-Model has the out-of-sample valuation errors significantly lower than that of the BS-Model. Another advantage of the G-Model over the BS-Model is that the G-Model performed well even when the parameters are fixed and the volatility being removed from the history of asset prices whereas the BS-Model need to be rejuvenated after a period of time. This is because the information about path dependence in volatility and about the volatility with spots returns' correlation can be obtained by the G-Model [8]. Zehra did the sensitivity analysis on both BS-Model and V-Model, the sensitivity of the Call Option prices with respect to other Model variables. The sensitivity analysis was based on the value of the correlation between two correlated Q-Wiener processes in BS-Model and V-Model. The result suggested that when the correlation is zero, Option Price estimated are the same for both models. It is also noticeable that interest rate will reach an equilibrium level in the long run [9]. Duan developed both an Option Pricing Model and the delta formula for that Option Pricing Model under the GARCH asset return process. The conclusion is that the systematic biases of the BS-Model can be addressed by the G-Model, was supported by empirical analysis [10].

Throughout the essay, the Call Option Prices of each of the three Option Pricing Models, BS-Model, V-Model, G-Model, will be estimated by using MCS. The accuracy of the three Option Pricing Models will be illustrated by using MSE on the estimated Call Option Prices and market Call Option Prices. This research both substantiates the literature review, and provides a foundation for estimating Option Prices for innovating market options products.

The structure for the content beneath: Section2 is data and method, Section3 presents the results of estimated Call Option Prices under a fixed K, the results after using Mean Square Error, and the graphs of stock price-time graph and interest rate-time graph. Section4 discusses the results, and Section5 is the conclusion.

2. Data and Method

2.1. Data Source

This paper collects the daily Carnival's Equity Price from Yahoo Finance.

2.2. Method

2.1.1. BS-Model

The differential equation of stock price:

$$d\big(S(t)\big) = \mu S(t)d(t) + \sigma S(t)d\big(W(t)\big) \tag{1}$$

where $W(t)$ is a Brownian Motion and that $W(t) \sim N(0,1)$

Name the function: $C(T) = g(S(T))$, to be the Call Option Price at time maturity T, so that the Call Option Price at Time maturity T is a function of stock price at time T.

Assume that Call Option Price at any time is a function of time t and stock price at time t.

Then, by Ito's rule, the differential equation of Call Option Price is:

$$d\big(C(t,S)\big) = \left(C_t + \left(\frac{1}{2}\right)(\sigma^2)(S^2)C_{ss} + \mu S C_s\right)d(t) + \sigma S(C_s)d(W) \tag{2}$$

At time t, in the stock, the amount invested is denoted by $\pi(t)$, with the differential equation:

$$d\big(X(t)\big) = (rX + (\mu - r)\pi)d(t) + \sigma\pi d(W) \tag{3}$$

$C(t,S(t))$ can be computed from a partial differential equation, which can be resulted from the Replication process.

In the Replication process, $C(t) = X(t)$. Therefore, both the $d(t)$ and $d(W)$ on both sides on the equation should be equal.

By equating the $d(W)$ terms:

$$\left(\frac{\pi(t)}{S(t)}\right) = C_s\big(t,S(t)\big) \tag{4}$$

By equating the $d(t)$ terms together with the expression above, the BS-Model-partial differential equation can be written as:

$$C_t + \left(\frac{1}{2}\right)(\sigma^2)(S^2)(C_{ss}) + r(s(C_s) - C) = 0. \tag{5}$$

With the initial condition: $C(T,s) = g(s)$

By solving the partial differential equation together with the initial condition: $g(s) = (S-K)^+$, the Call Option Price is written as:

$$C\big(t,S(t)\big) = \big(S(t)\big)N(d_1) - (K)\big(e^{(-r)(T-t)}\big)N(d_2) \tag{6}$$

Where $N(x), d_1, d_2$ satisfies following:

$$N(x) = P(Z \le x) = \left(\frac{1}{2\pi}\right)\int_{-\infty}^{x} e^{-\frac{y^2}{2}} d(y) \tag{7}$$

$$d_1 = \left(\frac{1}{\sigma(T-t)^{\frac{1}{2}}}\right)\left(log\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right) \tag{8}$$

$$d_2 = d_1 - \sigma(T-t)^{\frac{1}{2}} \tag{9}$$

2.2.2. Combination of the V-Model and the BS-Model

The differential equation for interest rate component:

$$d\big(r(t)\big) = \kappa\big(\theta - r(t)\big)d(t) + \sigma d\big(W(t)\big) \tag{10}$$

The addition of V-Model to the traditional BS-Model improves the accuracy of the interest rate component, since the interest rate component of the traditional BS-Model is a constant value, whereas the V-Model considers the variation in the interest rate component by introducing the differential equation.

The formula for estimating the Call Option Price is just the estimation of Call Option Price using traditional BS-Model.

2.2.3. Combination of the G-Model and the BS-Model

The recursive relation of volatility:

Introducing the parameter: $a_t \sim GARCH(p,q)$ such that

$$a_t = (\sigma_t)(\varepsilon_t) \tag{11}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i)(a_{t-i})^2 + \sum_{j=1}^{q}(\beta_i)\left(\sigma_{t-j}\right)^2 \tag{12}$$

Where $\{\varepsilon_t\}$ is a sequence of independently identically distributed random variables with mean of 0 and variance of 1, $\alpha_0 > 0$, $\alpha_i \geq 0, \beta_j \geq 0$,

$$\left(\sum_{i=1}^{max(p,q)} \alpha_i + \beta_i\right) < 1. \tag{13}$$

And $\alpha_i \equiv 0$ for $i > p$ and $\beta_i \equiv 0$ for $i > q$

The Call Option Price is estimated using the formula in the traditional BS-Model.

3. Results

3.1. Parameters setting

For BS-Model, both interest rate and volatility components are considered to be constants, with values $r(0)$ and $\sigma(0)$ respectively.

For G-Model, the interest rate component is considered to be constant, with value $r(0)$.

For V-Model, the volatility component is considered to be constant, with value $\sigma(0)$.

All the parameters used for the Call Option Pricing Models used in this research are listed in the table 1.

Table 1. List of Parameters

K	900
$S(0)$ stock price at time 0	15.6769
$\sigma(0)$ volatility at time 0	0.4966
$r(0)$ interest rate at time 0	0.047
T: time maturity	43/360
θ	0.000033
κ	0.7

3.2 Results of estimating Call Option Prices

In this research, for the BS-Model, MSC is carried out, via python, on the paths of stock prices. For the V-Model, MSC was carried out twice, first on the paths of the interest rates, then on the paths of the stock prices. For G-Model, firstly, update the volatility value by using the previous volatility value in each MCS step. Then, carry out MCS of the path of stock prices.

The estimated Call Option Prices of the BS-Model, the V-Model and the G-Model, for a constant $K = 900$ are listed in the table 2.

Table 2. Estimated Call Option Prices for a fixed $K = 900$

Call Option Pricing Models	Value
BS-Model	703.4227
V-Model	660.0569
G-Model	674.7461

The Values of K and market Call Option Prices for 20 exchange dates, for calculating the Mean Square Error values, are listed in the table 3.

Table 3. Values of K and market Call Option Prices(M)chosen for 20 exchange dates:

K	M
500	746.5
550	697.0
575	672.0
600	647.5
625	623.0
650	598.5
675	574.0
700	549.5
725	525.5
750	501.0
775	477.0
800	453.0
825	429.5
850	406.0
875	382.5
900	342.5
950	313.5
1000	269.5
1200	121.5
1600	10.5

The values for the calculated MSE, based on the data above, are listed in the table 4. It is observed that G-Model estimation of Call Option Price shows the smallest value under the MSE approach.

Table 4. Values of MSE Values

Call Option Pricing Models	MSEvalue
BS-Model	96240.5444
V-Model	99561.5168
G-Model	94767.6586

Figure 1 reveals the variation of stock prices in BS-Model.

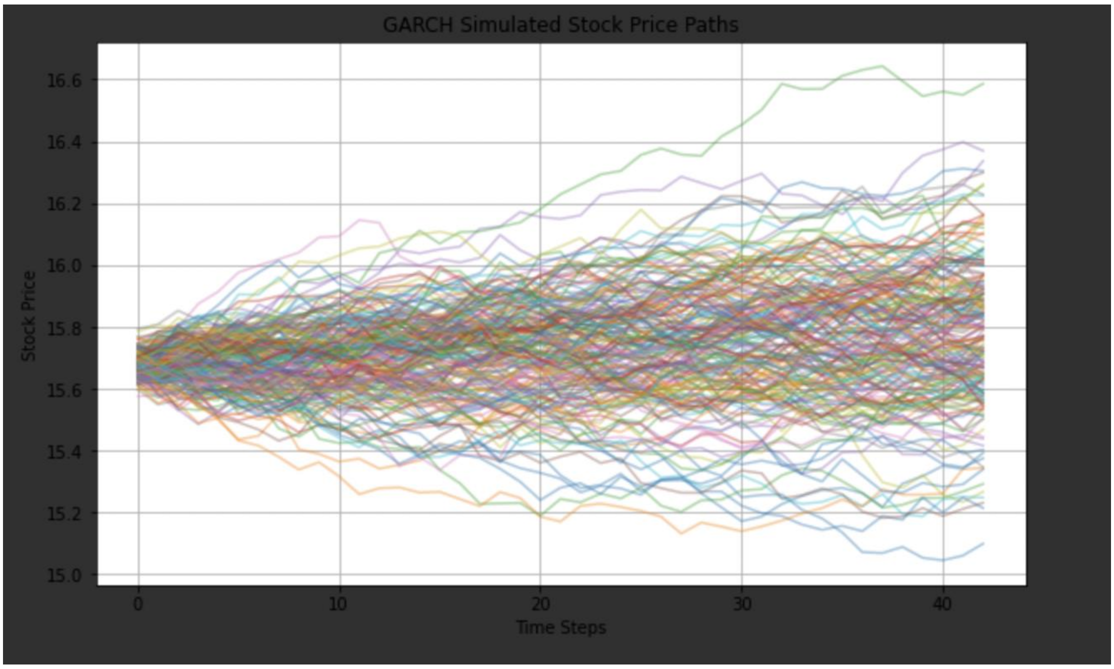

Figure 1. The BS-Model’s graph of Stock price over time

Figure 2 demonstrates the variation of the interest rate in V-Model.

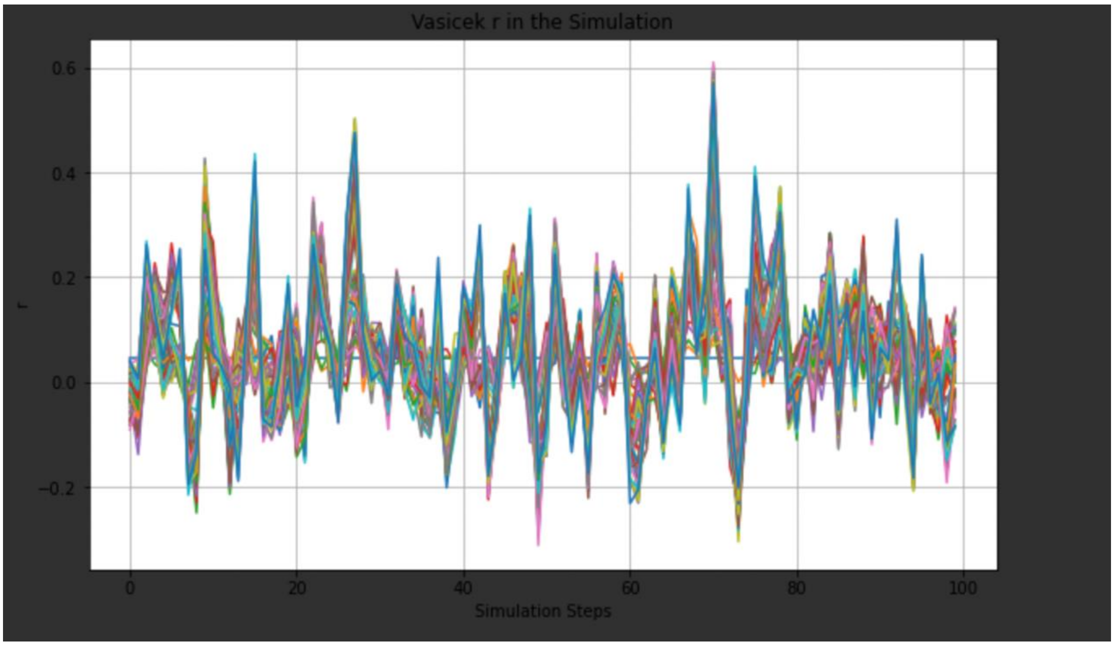

Figure 2. V-Model's graph of interest rate over tim**e**

4. Discussion

This research estimated the Call Option Prices of the BS-Model, the V-Model and the G-Model by using MCS to simulate the stock paths. By using MSE in calculating the accuracy of the three models in the estimation, the results revealed that the G-Model is the most accurate model and that the V-Model barely improves the accuracy of the BS-Model as the MSE value of the V-Model is greater than that of the BS-Model.

Aduda partly investigated whether or not the N-G-Model, a type of the G-Model, improves the BS-Model in estimating foreign currency pricing options. The results corroborated with the results provided by this research. However, as K. C. Hsieh and P. Ritchken claimed in their research, the N-G-Model turned out to perform better than the traditional G-Model since the N-G-Model won't take biases from pricing residuals into consideration and is able to withstand the modifications in modelling variables. As a consequence, it can be concluded that Aduda's research gave a more in-depth analysis and gave a more thorough conclusion about a particular type of the G-Model which had improved the BS-Model in estimating Call Option Prices.

Chaudhury and Jason used the combination of Empirical Martingale Simulation and MCS to simulate stock paths. This was then used in estimating Option Prices for the P-G-Model (a type of the G-Model), the BS-Model and the Modified BS-Model. Their results corroborated with the results provided by this research. However, the approach they had chosen to simulate the stock paths was better than the approach used in this research. This is because Empirical Martingale Simulation improves MCS, as the sampling error is removed, thereby improving the accuracy for simulating stock paths.

The research carried out by Steven and Saikat was partly similar to this research, where the performances of the BS-Model and the G-Model were being compared. Their results had corroborated with the results presented by this research. However, the comparison method used in their research is different from the method used in this research. Instead of calculating the MSE value between estimated Call Option Prices and market Call Option Prices, the method used in this research, they had compared the out-of-sample valuation errors of the BS-Model and the G-Model.

The research carried out by Zehra partly corroborated with the results of this research, however, in an indirect way. The result, that the V-Model doesn't improve the accuracy of the BS-Model in estimating call option prices, can be shown by carrying out the MSE test as in this research. In Zehra's research, the sensitivity of the Call Option Prices in relation to other variables, for the BS-Model and the V-Model, had been compared. One of the results had revealed that, after a long time, the interest rate will reach an equilibrium position. Therefore, an indirect conclusion can be made: since interest rate will reach an equilibrium value, which can be roughly viewed as a constant, adding the V-Model to the BS-Model won't necessarily improve the accuracy of the BS-Model.

5. Conclusion

In this research, the Call Option Prices of the BS-Model, the V-Model and the G-Model are estimated, by simulating the stock paths using MCS. The accuracies for the three models are compared by applying the MSE test on the estimated Call Option Prices of the three models, by varying K value for 20 days, with the market Call Option Prices of these 20 days. The value for G-Model is the least, suggesting that it has the greatest accuracy in estimating Call Option Prices compared to the other two. The value of the V-Model is greater than that of the BS-Model, indicating that the BS-Model is not improved by the V-Model in this research. The significance of this research is to provide a clear picture of considering the variation in which parameters in a Call Option Pricing Model can improve the accuracy of the original one. The results of this research revealed that improving the interest rate component won't improve the accuracy, whereas improving the volatility component will improve the accuracy.

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