

Pricing and Optimization Model of European Options

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Abstract. Financial market forecasts are gradually gaining the attention of investors. The options market is an integral and important part. Commonly used analysis methods include traditional BS formula and CRR model. This article obtained the transaction data of SP&500 from February 2013 to February 2018, used the data of the first 4 years to train the model, and finally explored the actual price and predicted value of the European call option. To forecast the stock price, a hidden Markov model (HMM) is first applied. And on this basis, improve the parameter estimation in the traditional method BS-CRR, and compare each model with the actual value of the option. Compared with the traditional model, to some extent, the Hidden Markov Model can predict the state of the stock market, get rid of the limitations of the BS model assumptions, predict the direction of the price trend, and verify the feasibility of its application in the financial field.

Keywords: BS formula; optimized model; European call option; the hidden Markov model.

1. Introduction

Investors' interest in financial market forecasting has grown over time. Traditional BS formulas, machine learning, and its variations make up most of the typical analysis techniques. To forecast stock prices, the enhanced CRR model with Hidden Markov Model (HMM) is used. On the other hand, although e-money may be more practical as a payment method, concerns have been raised about its stability of value. Consequently, anticipating stock prices correctly can help investors avoid risks and generate enormous wealth. In terms of the nation, it can ensure that the economy is running smoothly and prevent several domino effects brought on by a stock market crash. Additionally, it can use technical tools to efficiently address the drawbacks of the current monetary system and advance economic development in a way that is safer, more effective.

However, in the complex real financial market, arbitrage emerges endlessly. It is not realistic to directly obtain the theoretical value. Monte Carlo provides us with a new way of thinking, which is to fit multiple future scenarios, use the mean to estimate the future situation, and price options based on this, which is similar to the CRR model. In the CRR model, there are two key parameters, the first is u factor and the other is p . But the acquisition of these parameters is based on the BS model. Once the BS model assumption is not established, the actual significance of these parameters remains to be investigated.

In this article, this paper will propose a model that can improve option pricing. This paper use the Hidden Markov Model to predict bear market probabilities as well as expected returns. Since the HMM model is more reliable in short-term forecasting, this paper can use this model to price short-term European options. This paper will use multiple stock data to examine BS model, CRR--HMM model, and actual option returns. Measure the accuracy of the two models and make an evaluation.

According to the Efficient Market Hypothesis (EMH), prices reflect all the information that is currently accessible and are relevant to a market [1]. Yet, numerous analyses of stock data have been done to demonstrate the market's predictability. However, Sewell show that the strict condition of the definitional "completely" implies that no real market could ever be efficient, indicating that the EMH is virtually probably untrue [2].

Zhang employed artificial intelligence techniques to anticipate the trend of companies and indices [3] using the SVM method. Xie et al. employed a neural network to predict stock price [4]. More prediction accuracy can be achieved using artificial intelligence techniques, but the models can be

confusing to understand, and a lot of training data is needed. Nevertheless, there aren't many data in the real process, and getting precise and complete data can be challenging.

Hamiton et al. made the initial suggestion for the Hidden Markov Model (HMM) [5]. This model has been improved in future uses and is a variant of the discrete Markov model. However, the traditional Hidden Markov Model has some room for improvement in the stock price prediction method [6]. In comparison to artificial intelligence techniques, HMM sets random initial parameters for customs clearance using the idea of data pattern recognition and iterates continually to find the optimal data model, which is more intelligent.

Chen showed that the significant flaws and restrictions of BS option pricing models, which are frequently shown to be inadequate when compared to actual market data since they are based on restrictive and irrational assumptions [7]. Hou et al. showed that when volatility is smaller, the Monte Carlo simulation method outperforms, while the Black-Sholes and Binomial models outperform across the board while taking moneyness into account [8].

2. Method

2.1. Data Source

Representative S&P 500 stocks are chosen for study and prediction to thoroughly assess the prediction performance of the HMM in light of the current policy direction. All of the information is taken from <https://www.kaggle.com>. The Bitcoin price data used in this study ranges from 2013/2/8 to 2017/2/7. The open price and trading volume are used for the daily price data for stocks. Following that, this study will calculate the model's parameters using training data, forecast future stock closing prices using the CRR model's optimized parameters, and assess the model's predictive power using test data.

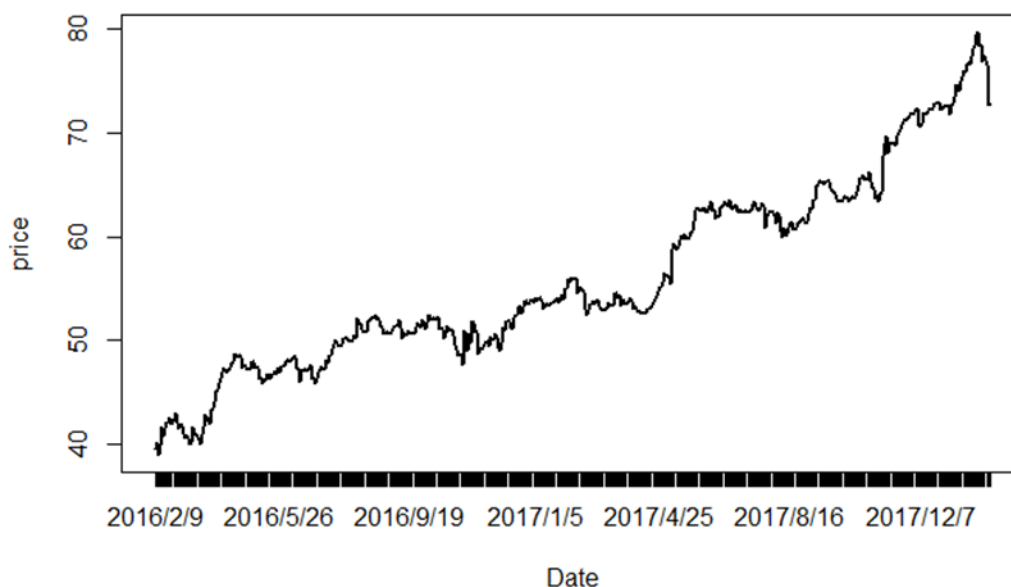


Fig. 1 AAL price chart

Figure1 shows the AAL price chart. It is not difficult to see that in the selected time frame, the price of AAL stock continued to rise steadily. This paper will first use the HMM model to predict its price. After analyzing the results, use BS, CRR, CRR-HMM models to price the European call option.

2.2. Hidden Markov Model

A probabilistic model of time series is the Hidden Markov model. It explains how to randomly create a hidden Markov chain's unobservable state (D6 in Figure 2), then randomly create an observation (1 in Figure 2) from each state to create a random sequence of observable states. An HMM is a finite model that explains a probability distribution over an infinite number of potential sequences, and this is the main concept [9].

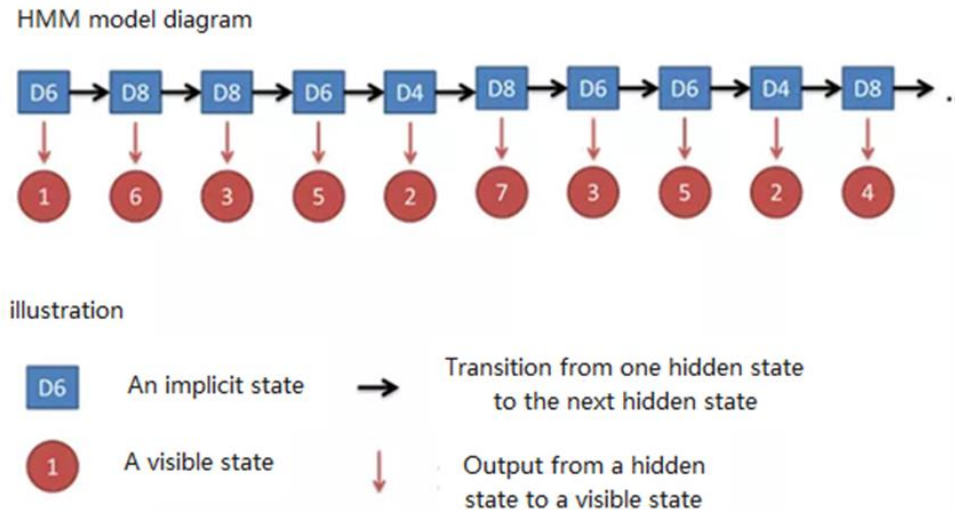


Fig. 2 HMM model diagram

The Hidden Markov model has two basic problems: First, the probability calculation problem. Given the model and the observation sequence, calculate the probability $P(O|\lambda)$. Secondly, the problem of learning. Given the observation sequence, estimate the model parameters so that the observation sequence probability $P(O|\lambda)$ maximum.

To determine a set of historical data whose trend is like the current trend and anticipate the value of the stock price, HMM employs the probability value estimation method. Start by gathering raw data. Next, ascertain how many hidden states there are. To ascertain the quantity of stable hidden states, train HMM models using different state counts using the AIC and BIC criteria. The calculation is stopped after the allotted number of iterations has been reached. Lastly, find patterns that are comparable and get anticipated values [10].

3. Results and Discussion

3.1. Model

3.1.1 Black-Scholes model

A key idea in contemporary financial theory is the Black-Scholes model, commonly referred to as the Black-Scholes-Merton (BSM) model. Considering the effects of time and other risk factors, this mathematical equation calculates the potential value of derivatives based on other investment instruments. In notation for mathematics:

$$C = SN(d_1) - Ke^{-rt}N(d_2) \quad (1)$$

Where:

$$d_1 = \frac{\ln\frac{S}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} \quad (2)$$

$$d_2 = d_1 - \sigma\sqrt{t} \quad (3)$$

Where C represents call option price, S represents current stock, K represents strike price, r represents Risk free interest rate, t represents Time to maturity, N represents normal distribution.

3.2. HMM Results

We first used the HMM model to fit the AAL stock pricing. Figures 4, 5 and 6 demonstrate that HMM model performs poorly. The HMM model will provide very substantial errors when employed for medium- and long-term forecasts, making it challenging to accurately forecast the stock price in the face of significant market volatility. The accuracy of projections drastically declines with time.

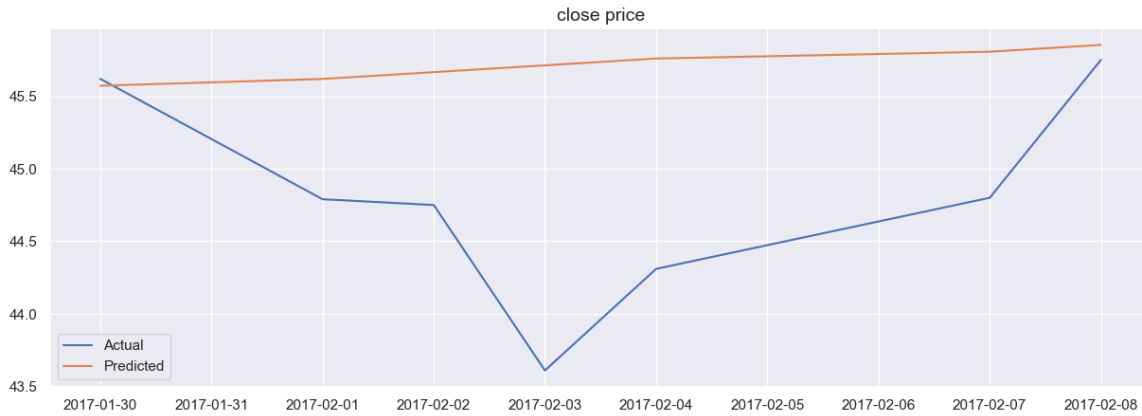


Fig. 4 Short-term HMM model



Fig. 5 Mid-term HMM model



Fig. 6 Long-term HMM model

However, some parameters in the HMM model give us the inspiration. One of the flaws of the BS and CRR model is that it is difficult to satisfy market assumptions. Therefore, the parameters in the model are not accurate enough compared with reality. We can use the HMM model to fit the data and count the proportions of different states to estimate p in the CRR model. At the same time, according to the mean of increment of different states, u in the CRR model is estimated.

Next, we will use different models to price the call option of the four stocks and compare them with their true values. In detail, we fit the model using stock prices from February 8, 2013, to February 7, 2017. And choose the strike price according to the last day's price. Finally, compare the one-year call option pricing of different models and the actual value of the option based on the stock price on February 7, 2018.

3.3. Comparison of Different Pricing Models

3.3.1 AAL

To begin with, we first analyse AAL stock. The last price is 45.75 and maturity price is 50.91. Strike price ranges from 50 to 70. It is clear that the true value of call option is almost 0 whereas the CRR and BS methods show that this option is valuable and worth the investment. Using improved parameters from HMM model, the CRR-HMM model price matches the actual. In this case, the improved model performs the best through all models, as figure 7 shows.

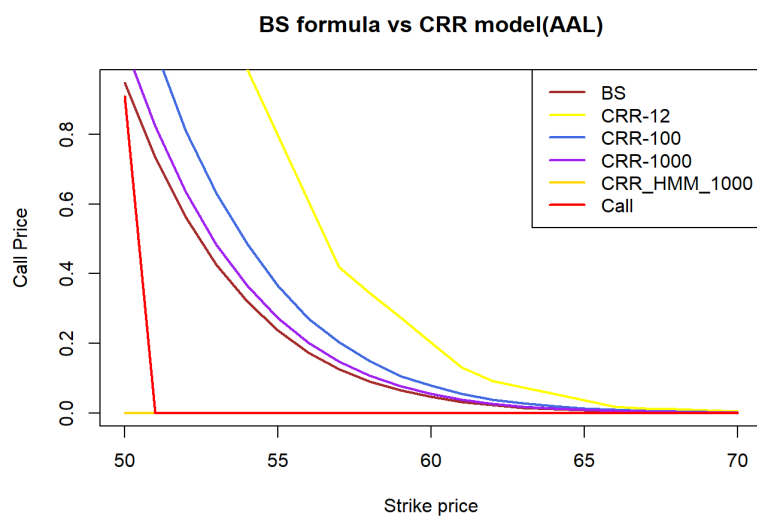


Fig. 7 Call option pricing model (AAL)

3.3.2 AAPL

Now, let's turn to AAPL stock. The last price is 130.54 and maturity price is 163.085. Strike price ranges from 130 to 150. In this case, CRR model with higher depth performs better. The theoretical BS model matches the true value while CRR-HMM model performs inferior. Now let's check the trend of the price, as Figure 8 shows.

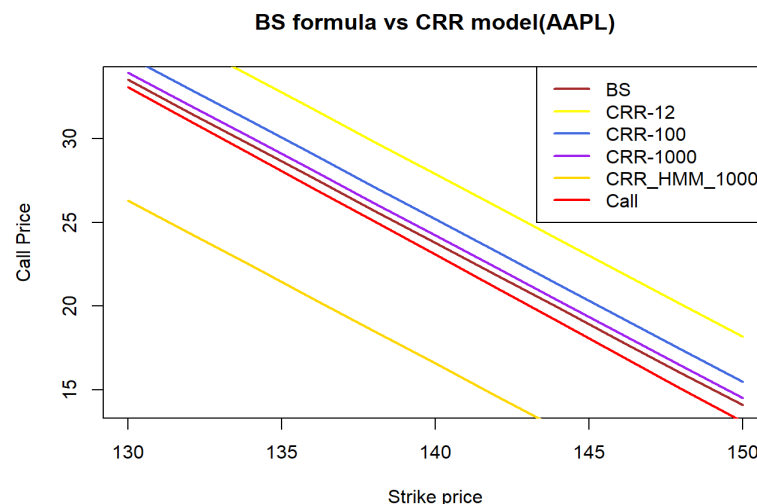


Fig.8 Call option pricing model (AAPL)

Figure 9 shows periodicity to some degree and a strong increasing trend with volatility which strong supports the BS model assumptions. When it comes to ideal situation, BS model performs best as the model deviation using HMM is inevitable.



Fig. 9 AAPL price chart

3.3.3 ABBV

Now, let’s consider another stock ABBV. The last price is 60.65 and maturity price is 111.52. Strike price ranges from 60 to 80. It is not difficult to find that all the models have difficulty predicting the actual price. This is likely due to the surge in prices. Now let’s check the trend of price, as Figure 10 shows.

BS formula vs CRR model(ABBV)

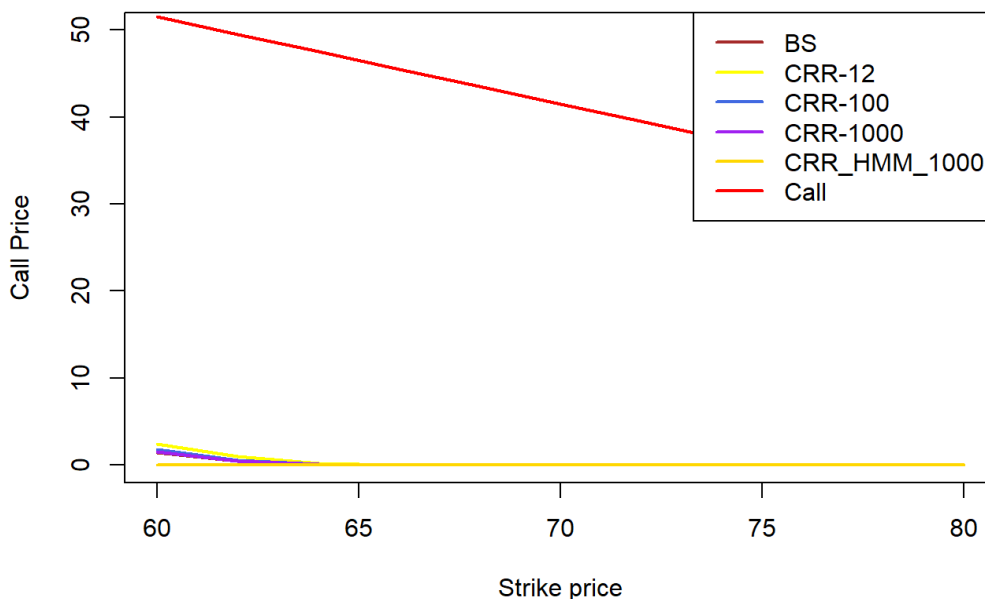


Fig. 10 Call option pricing model (ABBV)

Figure 11 show that the strike price keeps stable in our training set whereas the maturity price is nearly twice as the last price. Neither BS nor HMM model expect this. Therefore, in such extreme situation, no models are reliable.

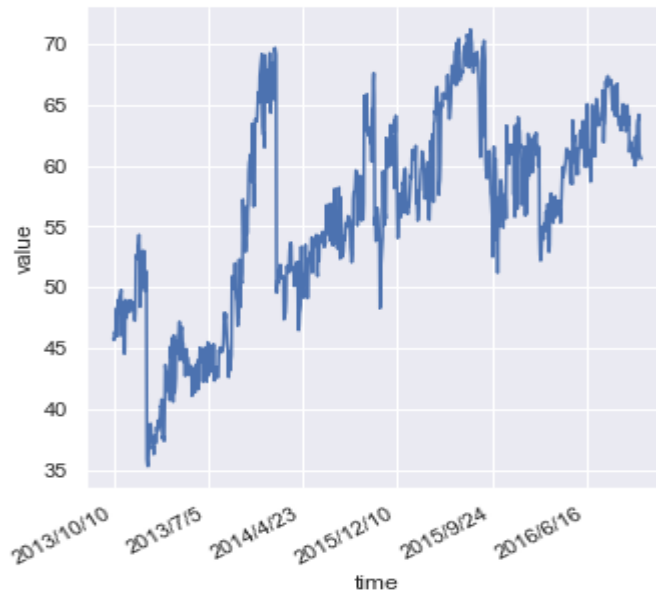


Fig. 11 AAL price chart

3.3.4 AAP

Finally, let’s turn to AAP stock. The last price is 161.21 and maturity price is 112.09. Strike price ranges from 160 to 180. BS and CRR model show that this option is valuable and worth the investment. On the other hand, CRR-HMM model gives the opposite decision which matches the reality, as Figure 12 shows.

BS formula vs CRR model(AAP)

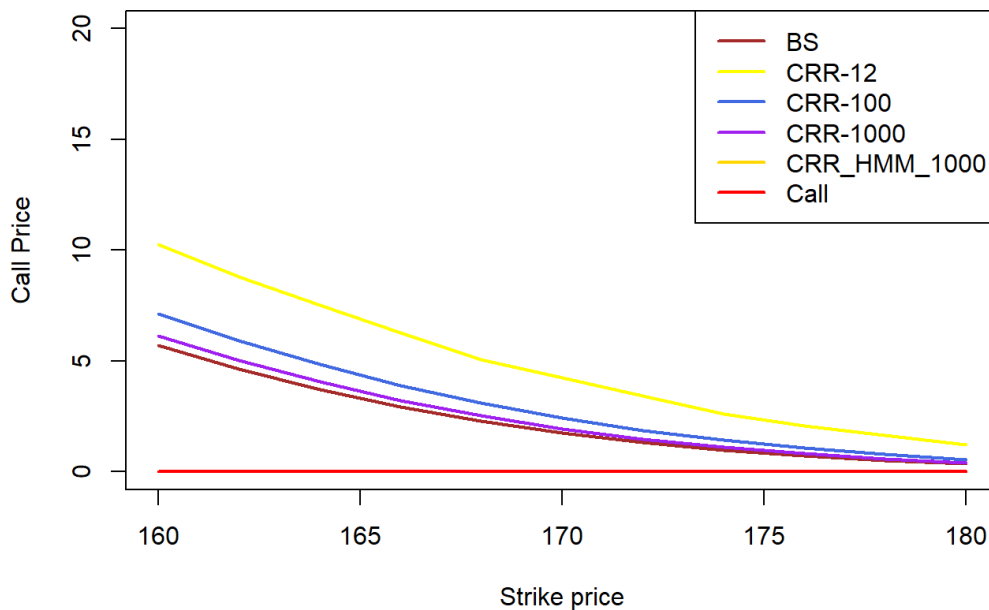


Fig. 12 Call option pricing model (AAP)

4. Conclusion

This article uses the S&P 500 transaction data from June 2013 to September 2017, using HMM model to test the accuracy model accuracy in the field of stock prices, and on this basis improves the traditional European call option pricing model BS-CRR Model. The results show that the HMM model performs poorly for long-term stock price forecasting. Although this paper selected the actual situation parameters (hidden state=3), this will lead to model underfitting. However, the HMM model provides another way to estimate the parameters of the CRR model, based on which the paper improves the parameters of the traditional BS-CRR model, using actual values instead of theoretical values. Therefore, the CRR-HMM model is interpretable and not limited by traditional BS assumptions. By comparison, the improved model is superior to the traditional model in many cases and has certain reference value for option price prediction. This paper does not have a good way of incorporating known information in parametric design. Bayesian methods can be used in the future to improve parameter estimates. At the same time, this paper only selects a small range when selecting the strike price. How to design the option maturity strike price based on the previous price is also worth exploring.

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