Application and Comparison of Index Model and Markowitz Model in American Stock Market

Xiaoyan Zhang *

Lingnan college of Sun Yat-sen University, Guangzhou, China

* Corresponding Author Email: zhangxy837@mail2.sysu.edu.cn

Abstract. Developing a robust stock market portfolio is a critical endeavor, necessitating strategic methodologies supported by quantitative analysis. This abstract outlines a study focused on constructing an optimal portfolio comprising 12 diverse stocks and SPX, operating under five pragmatic constraints. Both the Markowitz and Index models are employed to analyze and optimize the portfolio. With powerful tools in Python, the outcome of those thirteen-asset allocations is easily revealed, and by comparison between the two models, similarities and differences are concluded in charts and figures. The major conclusion is that MM and IM can reach a similar Sharpe ratio even though the allocations of different assets are very different in the American Stock Market. Besides, compared with the Markowitz model, the Index model is easier to calculate, which may be helpful when the assets in the portfolio increase. Under five constraints, different portfolio presents different Sharpe ratio and Standard deviation, and thus the outcomes may vary as the constraint changes.

Keywords: Portfolio optimization; Markowitz model; index model; Sharpe ratio; modern portfolio theory.

1. Introduction

In the two biggest stock markets of America, NYSE and NASDAQ, millions of dollars flush into one of the most profitable investments, stock investment, and building an effective portfolio becomes a crucial step and how profitable the strategy is on the top of investors' concerns. In the passage, Markowitz model (MM) and Index model (IM) are introduced to tackle the problem. Ranging from 2003 to 2023, 12 stocks history data come from several stock sectors and SPX (S&P 500) history data are selected to build the portfolio. The aim is to find out the most profitable strategies under different constraints deriving from practices and compare two methodologies to conclude their advantages and drawbacks.

Two models being discussed have a relatively close relationship. MM introduced the concept of diversification, and Markowitz contended that compared with discounted return to be maximized, we could let the rate at which we capitalize the returns from particular securities vary with risk, which gave born to MPT (Modern Portfolio Theory) [1]. Markowitz heuristically created the Markowitz model in the 1950s that lay in the center of MPT, or mean-variance analysis, and then the model was modified by William Sharpe, who gave born to single-index model (SIM). IM largely reduced the calculation of correlation matrix by introducing a market index factor. William Sharpe suspected that preliminary evidence suggested that the relatively few parameters used by the model could lead to very nearly the same results obtained with much larger sets of relationships among securities, and the possibility of low-cost analysis, coupled with a likelihood that a relatively small amount of information need be sacrificed make the model an attractive candidate for initial practical applications of the Markowitz technique [2]. The new model assumed that every stock was influenced by market (beta) and firm specific value (alpha). Then several other models were introduced, such as Fama & French three-factor model, five-factor model, etc.

In recent years study, researchers from different regions had applied their models to different markets with different stocks and indexes. For instance, Indonesia researchers Putra and Dana applied those models to LQ45 stocks, and by using Wilcoxon-Mann-Whitney test, concluded that risk-averse investors were likely to use Index model, while the risk-seekers were probable to choose Markowitz model [3]. Another study came from Hong Kong stock market, the author recommended that investors who wanted to enter the Hong Kong stock market should use the index model to build their portfolios [4]. Setyantho and Wibowo took consideration into newly implemented OJK rules, the research showing that developing optimal portfolio through Single Index Model yielded higher expected return than that of Markowitz Model [5]. A study towards Indian market also showed that if investors constructed optimum portfolio using stocks of Nifty 50 by SIM, they could be assured of returns and safety [6]. Another example from China provided an alternative but very effective strategy other than Markowitz method for the portfolio allocation of renewable energy stocks in China, which gave us new insight that Markowitz analysis didn't hold true in every stock market [7]. An insightful study from Iran proposed a new model that combined Markowitz and the cross DEA models and implemented a case study based on information of 50 active enterprises in the Tehran Stock Exchange and solved the proposed model using the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [8]. What's more, there were several other studies to help broaden our visions and uncover the unknown grounds, like Georgalos and Paya, who explored deeply into the risky choice by using Markowitz model of utility [9]. Another study that included intricate mathematical skills to investigate the continuous-time Markowitz mean-variance problem for a multivariate class of affine and quadratic Volterra models, which was another innovative trial to modify Markowitz model [10].

2. Methodology

2.1. Data Selection

In the passage, a portfolio is constructed using data from 12 US stocks listed on NYSE and NASDAQ, including QCOM, AMD, AAPL, NVDA, MSFT, F, KO, WMT, WELL, XOM, CVX, and IMO. Historical closing prices of these stocks, along with S&P 500, are collected spanning from 2003 to 2023. The portfolio is developed under five distinct practical constraints. The optimization process involves two models aimed at maximizing the Sharpe ratio (a measure of excess premium over standard deviation). This analysis delves into the weight allocation for twelve of the stocks and one index and presents a comparison between the two models.

Additionally, Monte Carlo simulation is employed to provide visualizations of both models, enhancing our understanding of their differences and commonalities in practical, long-term investment scenarios. The data for the 12 US stocks is sourced from Kaggle, while the SPX index and Federal Reserve interest rates are extracted from Yahoo Finance. These stocks represent diverse sectors such as technology, manufacturing, energy, and consumer cyclical, contributing to risk diversification within the portfolio.

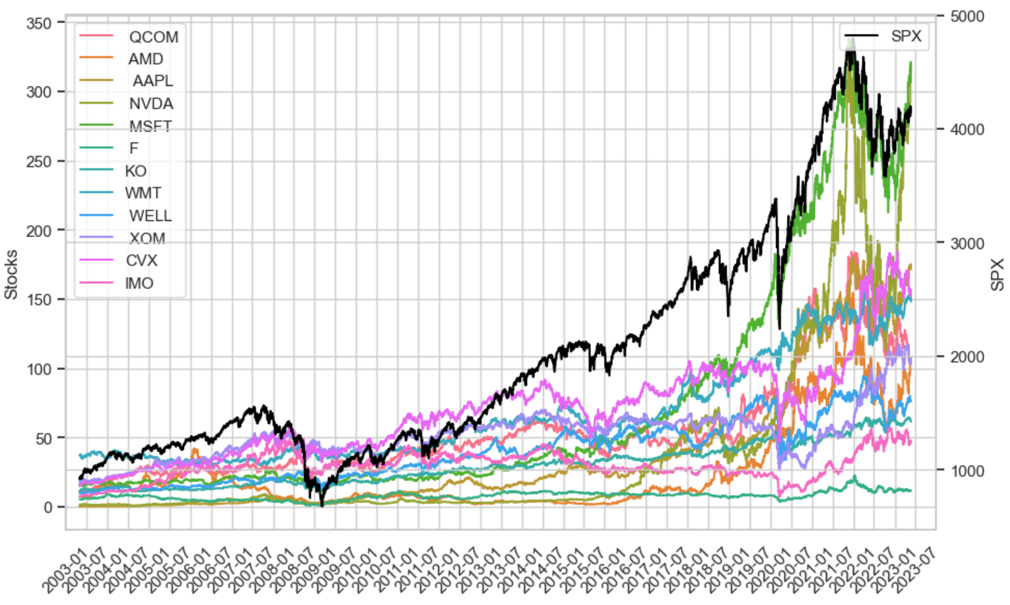

Fig 1. Stock prices over time.

Figure 1 illustrates the daily prices of 12 stocks and the SPX index, employing dual y-axes coordinate system. The graph indicates an apparent trend of daily growth, reinforcing the rationale for selecting these stocks for inclusion in the portfolio.

In practice, a refinement in data selection is executed by focusing on the last day of each month. This practice enhances model performance by aligning the data more closely with a normal distribution. As a result, biases are reduced, and statistical accuracy is improved. The assessment of skewness and kurtosis further corroborates the suitability of the selected data for the normality test.

Within the Table 1, it is evident that the skewness and kurtosis of most stocks, when presented in a monthly format, exhibit smaller deviations from zero compared to their counterparts in daily form. This observation suggests that monthly data aligns more closely with the characteristics of a normal distribution compared to the daily data.

Table 1. Monthly and Daily data comparison.

Name	Monthly Skewness	Monthly Kurtosis	Daily Skewness	Daily Kurtosis
QCOM	0.875	3.395	0.350	9.308
AMD	0.388	0.138	0.666	12.597
AAPL	-0.211	1.036	0.138	5.077
NVDA	0.656	3.148	0.045	12.000
MSFT	0.124	0.702	0.253	9.996
F	3.026	28.941	0.683	16.499
KO	-0.479	1.449	0.039	12.570
WMT	-0.156	0.722	0.234	12.229
WELL	-0.848	3.958	0.039	20.185
XOM	0.438	3.286	0.221	10.631
CVX	0.392	2.137	0.119	21.410
IMO	-0.129	4.697	-0.137	10.177
FEDL01	1.253	0.379	1.214	0.209
SPX	-0.640	1.508	-0.255	12.797

2.2. Theory

In the analysis, two important models are included. The Markowitz model employs mean-variance analysis as a heuristic approach. Constructing the model involves a series of steps, beginning with the computation of both the expected average rate of return (RoR) and the average standard deviation (std). This process is outlined as follows:

$$E(\bar{R}) = \sum_{i=1}^{13} \omega_i \cdot E(\bar{r}_i) \tag{1}$$

$$\sigma_R = (\vec{\omega} \cdot \vec{\sigma}^T) \begin{pmatrix} corr(\bar{r}_1, \bar{r}_1) & \cdots & corr(\bar{r}_1, \bar{r}_{13}) \\ \vdots & \ddots & \vdots \\ corr(\bar{r}_{13}, \bar{r}_1) & \cdots & corr(\bar{r}_{13}, \bar{r}_{13}) \end{pmatrix} (\vec{\omega} \cdot \vec{\sigma}^T)^T \tag{2}$$

To highlight an interesting point, calculating the correlation matrix among various stocks requires complex computations. As a result, incorporating numerous stocks into your portfolio becomes challenging. This limitation necessitates the modification of the model to reduce computational demands. This is precisely why the index model has been introduced to provide a solution that requires fewer calculations.

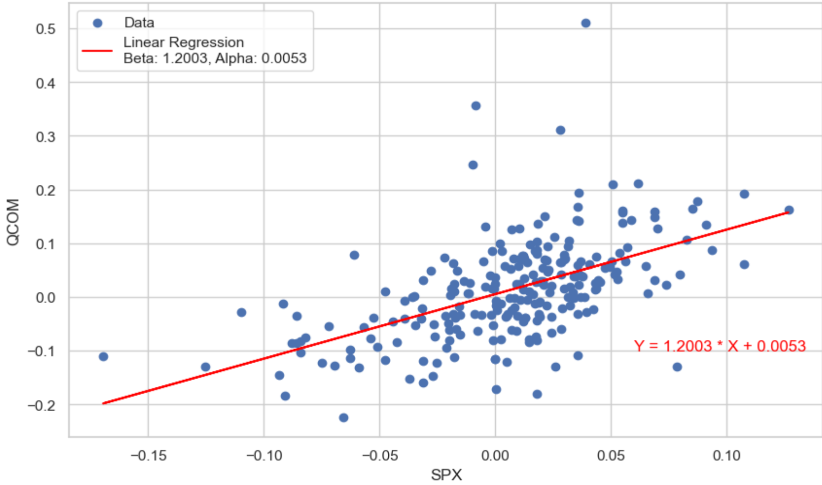

Fig 2. SPX and QCOM Linear Regression.

Index model, based on the previous model, is modified by adding in a new factor, market index factor. Loosely speaking, the model calculates the linear relationship between market index and stocks prices, and construct the model as follows:

$$r_i = r_f + \alpha + \beta_i(R_M - r_f) \tag{3}$$

In the equation, every stock's excess premium fluctuates near the market excess premium, and the rate of change is beta, but every stock may exceed or below market performance, which is named as alpha. In Figure 2, it is depicted that the linear regression of SPX and one of the stocks, QCOM has a positive beta (around 1.2) and a small positive alpha (around 0.005). In the following (4) and (5), RoR and std are revealed by two equations:

$$E(\bar{R}) = \sum_{i=1}^{13} \omega_i \cdot E(\bar{r}_i) \tag{4}$$

$$\sigma_R = \sqrt[2]{\sigma_M{}^2\left(\vec{\omega} \cdot \vec{\beta}^T\right)^2 + \sum_{i=1}^{11} \omega_i{}^2 \sigma_{residual_i}{}^2} \tag{5}$$

The σ_M represents market risk and $\sigma_{residual_i}$ represents the std of residual return (excess premium minus market return). As we can see here, the portfolio std, or σ_R, is requires less terms compared to the previous one, the complexity of the equation declines from the exponential growth to linear growth, which saves lots of time and energy.

2.3. Explanation and Selection of Indicators

In the analysis, the annual RoR and annual std indicate whether the portfolio is profitable and risky. By choosing different model and different weights, $E(\bar{R})$ and σ_R will change, the aim is to find the maximum of Sharpe ratio and the Sharpe ratio, showing as follows:

$$Sharpe\ ratio = \frac{E(\bar{R}) - r_f}{\sigma_R} \tag{6}$$

3. Results and Discussion

3.1. Preparation for the Calculation

In practical application, data analysis primarily takes place using Excel, while Python is employed for generating graphs and tables. Prior to the analysis, the raw data must be organized and aggregated into monthly intervals using the VLOOKUP function. For the MM, the construction of a correlation matrix is necessary. In Figure 3, a heatmap illustrating the correlation matrix displays relatively lower correlations among the stocks. IM model needs to calculate beta and residual std, and the results shows in Table 2.

Table 2. Statistic characteristics of 13 assets.

Name	Annual avg	Annual std	beta	alpha	residual std
QCOM	0.156	0.325	1.200	0.066	0.274
AMD	0.287	0.604	2.243	0.119	0.538
AAPL	0.377	0.331	1.278	0.282	0.275
NVDA	0.391	0.514	1.816	0.255	0.456
MSFT	0.159	0.224	0.964	0.086	0.173
F	0.128	0.482	1.787	-0.006	0.420
KO	0.086	0.162	0.577	0.043	0.151
WMT	0.070	0.171	0.393	0.041	0.184
WELL	0.122	0.243	0.874	0.056	0.207
XOM	0.104	0.228	0.820	0.043	0.195
CVX	0.135	0.237	0.998	0.060	0.187
IMO	0.135	0.323	1.215	0.044	0.271
SPX	0.075	0.147	1.000	0.000	0.000

In Table 2, annual RoR and annual std are presented. They represent the assets expected rate of return and risk, which will further used in (2), (3) to construct MM. In the other hand, beta and residual std help to construct IM, by using formula (5), (6). Then the data is fully prepared for the optimization process.

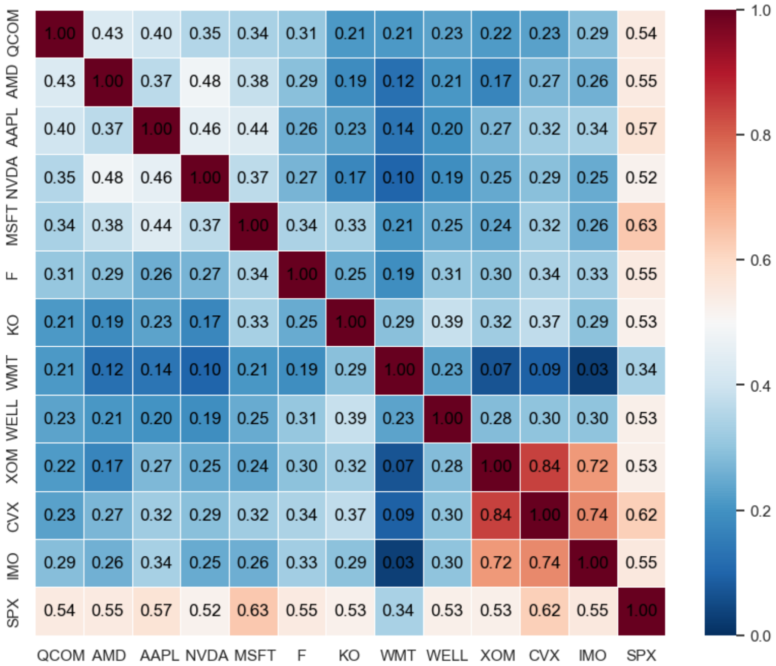

Fig 3. Correlation Heatmap.

3.2. Five Different Constraints

The last step is to calculate the maximized Sharpe ratio points and minimum standard deviation points by using Slover, one of power tool packs in Excel. Before jumping into the conclusion, five constraints are needed to introduce to simulate a more realistic scenario.

Regulation T Constraint is the first constraint is designed to simulate the Regulation T by FINRA, which allows the broker-dealer gives their customers borrowed stocks. It allows the customer to have short position, with at least 50% equity, so the total weight of 13 assets is less or no more than 2:

$$\sum_{i=1}^{13} |w_i| \leq 2 \tag{7}$$

Arbitrage box constraint is the second constraint is designed to simulate some arbitrary "box" constraints on weights:

$$|w_i| \leq 1, for\ \forall i \tag{8}$$

Free problem is the third constraint is a free problem, so there isn’t any condition except for all the weights added up need to equal one. Without any short positions is the fourth constraint simulates the typical limitations that US funds can’t have short positions.

$$w_i \geq 0, for\ \forall i \tag{9}$$

Exclusion of broad index is the fifth constraint exclude the broad index factor to make a comparison with the rest of others:

$$w_1 = 0 \tag{10}$$

3.3. Presentation and Explanation of Results

By utilizing the Solver tool, the optimal weights for the portfolio have been successfully determined. These weights reflect a careful consideration of various factors and constraints. Armed with a comprehensive set of five constraints, we are now presented with diverse portfolio outcomes that are detailed in Figure 4. This graphical representation sheds light on the variations in portfolio composition under distinct conditions and constraints.

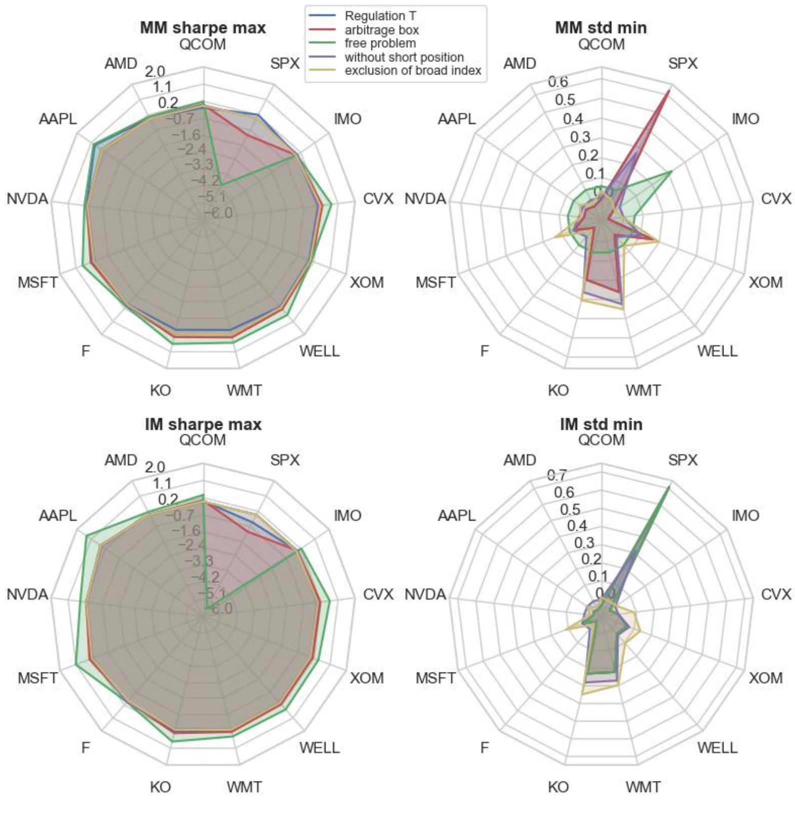

Fig 4. Radar charts for different conditions.

Figure 4 vividly depicts the dynamic nature of portfolio optimization. The graph showcases how different combinations of constraints and conditions lead to varying allocations among the stocks. Each point on the graph signifies a unique set of constraints and conditions and the resulting allocation percentages for each stock. Analyzing these data points can provide valuable insights into how the portfolio's composition changes in response to different optimization criteria.

Complementing the insights from Figure 4, Table 3 provides a comprehensive overview of the weights assigned to stock QCOM, AMD, AAPL, NVDA, MSFT, and F for different conditions and constraints. This tabular presentation offers a more detailed breakdown of the portfolio allocations, enabling a closer examination of the interplay between constraints, conditions, and stock weights. The values in the table allow for a quantitative assessment of how specific constraints influence the weight distribution across the portfolio.

Table 3. Different proportions of weights of portfolio.

Constraints	Different conditions	QCOM	AMD	AAPL	NVDA	MSFT	F
1	MM max	-12.85%	0.00%	87.90%	12.72%	16.42%	-8.81%
1	MM min	-0.81%	-3.67%	-2.01%	-2.46%	3.50%	-6.04%
2	MM max	4.03%	0.72%	45.86%	10.18%	27.30%	-2.92%
2	MM min	-0.81%	-3.67%	-2.01%	-2.46%	3.50%	-6.04%
3	MM max	18.64%	9.05%	94.69%	24.47%	73.13%	5.53%
3	MM min	5.67%	5.67%	5.67%	5.67%	5.67%	5.67%
4	MM max	0.00%	0.00%	42.07%	6.93%	7.38%	0.00%
4	MM min	0.00%	0.00%	0.00%	0.00%	3.80%	0.00%
5	MM max	-3.00%	-2.21%	42.91%	8.43%	10.86%	-6.76%
5	MM min	3.28%	-1.85%	1.78%	-0.92%	13.63%	-3.78%
1	IM max	3.08%	0.18%	46.01%	12.29%	25.65%	-4.59%
1	IM min	-3.33%	-5.35%	-4.58%	-4.89%	1.48%	-5.54%
2	IM max	6.85%	1.71%	51.50%	14.54%	34.56%	-6.41%
2	IM min	-3.33%	-5.35%	-4.58%	-4.89%	1.48%	-8.81%
3	IM max	34.11%	15.68%	140.16%	46.26%	110.86%	-6.04%
3	IM min	-3.32%	-5.34%	-4.58%	-4.89%	1.46%	-2.92%
4	IM max	0.00%	0.00%	49.09%	12.67%	17.83%	-6.04%
4	IM min	0.00%	0.00%	0.00%	0.00%	1.78%	5.53%
5	IM max	0.78%	-0.02%	49.11%	13.22%	21.05%	5.67%
5	IM min	0.97%	-3.91%	-0.22%	-3.08%	11.56%	0.00%

Table 4 showcases the performance of individual stocks (KO, WMT, WELL, XOM, IMO, SPX) under various constraints and conditions, providing a more comprehensive view than what was presented in Table 3. It's important to note that certain figures in the table appear to be unrealistic. For instance, in Constraint 3 and Condition IM max, the weights assigned to SPX are indicated to be below -500%. Such extreme values seem inappropriate and raise concerns about the validity of these results.

Additionally, when comparing the outcomes of the two different models, it's evident that the assigned weights can be quite similar under specific conditions, but can diverge significantly in other instances. This variability highlights the sensitivity of the models to different constraints and conditions, reinforcing the need for careful interpretation of the results. It's recommended to scrutinize and validate the data to ensure that such unrealistic figures are corrected and that the findings drawn from the table are accurate and meaningful.

In conclusion, the amalgamation of insights gleaned from both Figure 4 and the comprehensive data laid out in Table 3 yields a panoramic and intricate panorama of the portfolio optimization process. This synthesis illuminates the delicate dance between the imposed constraints, the contextual conditions, and the resulting distribution of stocks within the portfolio. Consequently, it emerges as an invaluable compass for decision-makers who are vested with the responsibility of sculpting their investment strategies to align precisely with their distinct and defined objectives.

Table 4. Different proportions of weights of portfolio.

Constraints	Different conditions	KO	WMT	WELL	XOM	CVX	IMO	SPX
1	MM max	-8.08%	-7.29%	7.32%	-6.50%	5.40%	-6.47%	20.24%
1	MM min	19.83%	26.23%	-1.22%	16.47%	-7.81%	-4.45%	62.43%
2	MM max	30.91%	32.50%	24.15%	5.62%	27.90%	-6.25%	-100.00%
2	MM min	19.83%	26.23%	-1.22%	16.47%	-7.81%	-4.45%	62.43%
3	MM max	67.52%	61.06%	63.96%	5.30%	76.07%	-4.09%	-395.33%
3	MM min	5.67%	5.67%	5.67%	5.67%	5.67%	31.98%	5.67%
4	MM max	12.59%	16.22%	9.41%	0.00%	5.39%	0.00%	0.00%
4	MM min	26.31%	32.53%	0.37%	10.19%	0.00%	0.00%	26.80%
5	MM max	12.58%	16.78%	12.94%	-1.16%	17.51%	-8.89%	0.00%

5	MM min	30.42%	35.17%	6.11%	19.71%	-0.46%	-3.09%	0.00%
1	IM max	20.52%	17.84%	8.96%	6.04%	9.44%	0.00%	-45.41%
1	IM min	23.15%	22.22%	3.63%	5.89%	0.04%	-3.67%	70.94%
2	IM max	29.07%	22.86%	14.52%	11.95%	16.79%	2.08%	-100.00%
2	IM min	23.15%	22.22%	3.63%	5.89%	0.04%	-3.67%	70.94%
3	IM max	74.48%	47.25%	50.81%	44.30%	66.98%	23.85%	-553.93%
3	IM min	23.14%	22.22%	3.64%	5.89%	0.05%	-3.66%	70.94%
4	IM max	7.83%	10.01%	1.98%	0.00%	0.58%	0.00%	0.00%
4	IM min	28.12%	27.01%	4.41%	7.16%	0.05%	0.00%	31.47%
5	IM max	10.62%	11.02%	4.40%	0.19%	4.08%	-4.57%	0.00%
5	IM min	34.91%	29.62%	10.47%	13.52%	8.81%	0.77%	0.00%

In Table 5, the data presents the RoR (Rate of Return), standard deviation, and Sharpe ratio under various constraints and conditions. Under the third constraint, the Sharpe ratios of both models reach their highest values (MM max 1.547 and IM max 1.536). This suggests that MM and IM perform similarly when no extra constraints are applied. Moreover, the theoretically highest Sharpe ratio and lowest standard deviation should be attainable in a constraint-free scenario.

Interestingly, the MM model's lowest standard deviation arises under the first constraint. This might imply that the numerical solutions are not entirely accurate, and as a result, there could be multiple local minima, but only one of them represents the global minimum. Inaccurate solutions could stem from the initial weights provided. If the initial point is significantly distant from the optimal point, the outcomes might appear unsuitable and unrealistic.

The suboptimal outcome indicates the potential for improved methods to achieve more accurate optimal points. This insight underscores that the optimal point could vary depending on the initial starting point, leading to slight differences each time we solve the problem.

Table 5. RoR, std and Sharpe ratio under different constraints.

Constraints	Different conditions	RoR	std	Sharpe ratio
1	MM max	0.380	0.330	1.153
1	MM min	0.050	0.114	0.441
2	MM max	0.300	0.206	1.455
2	MM min	0.050	0.114	0.441
3	MM max	0.617	0.399	1.547
3	MM min	0.162	0.205	0.788
4	MM max	0.239	0.194	1.228
4	MM min	0.083	0.121	0.685
5	MM max	0.241	0.192	1.258
5	MM min	0.094	0.120	0.778
1	IM max	0.288	0.212	1.358
1	IM min	0.033	0.112	0.292
2	IM max	0.335	0.233	1.437
2	IM min	0.033	0.112	0.292
3	IM max	0.896	0.584	1.536
3	IM min	0.033	0.112	0.292
4	IM max	0.280	0.226	1.239
4	IM min	0.082	0.123	0.670
5	IM max	0.281	0.221	1.267
5	IM min	0.082	0.121	0.679

3.4. Monte Carlo Simulation

A Monte Carlo simulation is a model used to predict the probability of a variety of outcomes when the potential for random variables is present. In the previous analysis, by using solver, which is numerical Monte Carlo way to find out the optimal portfolio. However, the visualization of the models still lacks. By using Python, the program randomly throws out 300000 points on the charts, which draw out the efficient frontier of both MM and IM. In the Table 6, the detailed Algorithm 1 are illustrated.

Table 6. IM and MM visualization.

Algorithm 1 Monte Carolo simulation of IM and MM under free-problem constraint
Step 1. (Random weights generator) Create a NumPy series with 13 numbers, to represent 13 assets different weights. Use the random lib and then do the normalization to control 13 number range in -1 to 1. The outcome should be like: $\{\omega_1, \omega_2, \dots \omega_{13}\}$
Step 2. (300000 loops of Monte Carlo simulation) Using the asset RoR and std to create portfolio RoR and std and repeat the process 300000 times. For the MM, the equation (1) and (2) should be formatted in Python codes. For the IM, the equation (4) and (5) need to transform to the Python codes either. Lastly, repeat the above process in a 300000 times loop and you should get a table with 300000 points, and each point has two values RoR and std, which represents a random portfolio weights allocation.
Step 3. (Visualization representing different Sharpe ratio) Using the matplotlib in python lib and using the cool-warm to visualize different Shapre ratio, and the points should be restricted in a hyperbola-like area.

The results successfully testify the viability of both models, showing out that the minimum point and maximum point lie on the simulation areas. And so on, the rest four constraints should have similar results, as figure 5 shows.

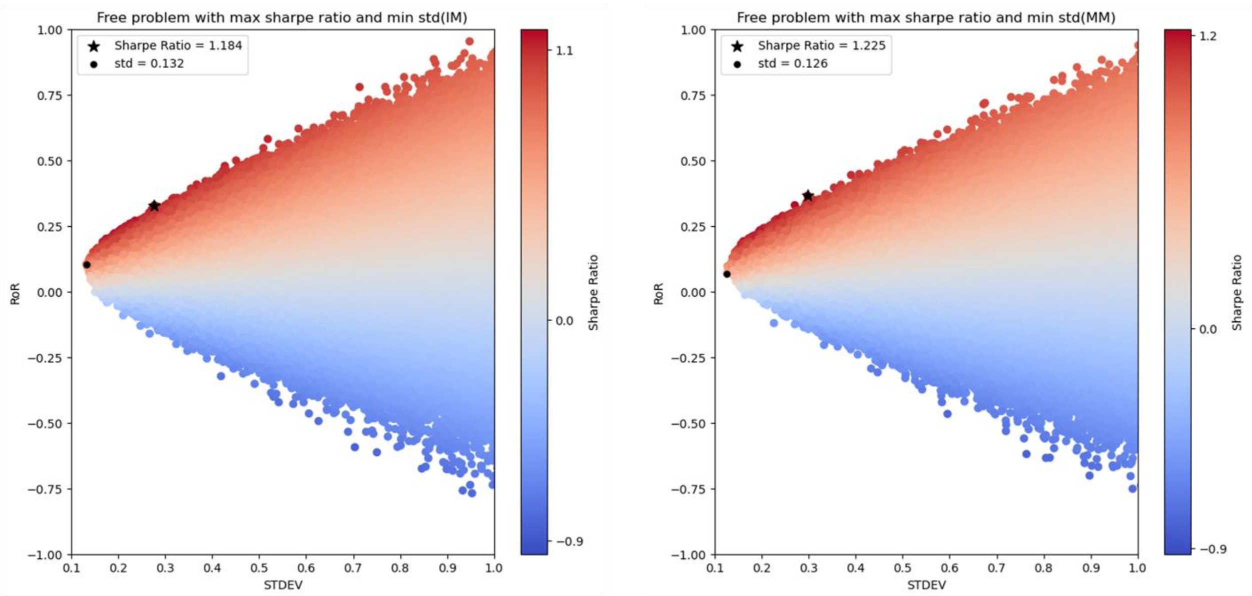

Fig 5. Comparison between MM and IM.

4. Conclusion

In summary, the portfolios are succeeded built by using the Markowitz Model (MM) and Index Model (IM). Through a comparative analysis, monthly data are finally used, as opposed to daily data, to determine the optimal weights for maximizing the Sharpe ratio and minimizing the standard deviation for both models. Subsequently, the feasibility of these selected points is tested, ensuring their placement along the efficient frontier.

The analysis yields the following conclusions: MM and IM models exhibit largely similar behavior across various scenarios. While the weightings for individual stocks may occasionally diverge significantly, the optimal points tend to align more closely. Notably, IM manages to achieve comparable results to MM in the American stock markets, simplifying the computational process.

Acknowledging the study's limitations, the accuracy of optimal points derived from Monte Carlo simulations introduces an element of uncertainty. To enhance robustness, future iterations could expand the scope by incorporating more stocks and indices, surpassing the current 13 stocks and 1 index.

It's important to recognize that the study's focus on the United States might limit generalizability to other markets like India and China. Moreover, the significant variations in weightings between the MM and IM models require further investigation for a comprehensive understanding. To augment the index model's comparability, introducing additional factors warrants consideration. This would provide a better-rounded basis for model evaluation.

In summary, this research underscores the need for future scholars to approach this study with novel perspectives. The potential application and validation of both the Markowitz and Index models across diverse scenarios and constraints hold promise. The hope is for these insights to stimulate ongoing exploration, unveiling new avenues in this field.

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