The Relationship between SOFR and Market Yield of U.S. Treasury Securities

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Abstract. This paper introduces the characteristic of the new benchmark rate Secured Overnight Financing Rate (SOFR), explain its connection to the futures price, and show its potential to provide new insight into the argument on interest rate term structure. Based on the future price discovery theory, this study set up a hypothesis that short-term interest rates should contribute to forecasting long-term ones. The causality between interest rates in different terms of, which are separately represented by bilateral repurchase rate and the market yield of long-term U.S. government bonds, is explored in this research using the VAR model, as well as the Granger causality test. The conclusion of this research proves the hypothesis and indicates a single-direction causality between interest rate of different terms. However, the research also has drawbacks such as ignoring non-linear relations and evading the change of government policy. These problems need to be solved in the following study.

Keywords: SOFR; Interest rates; Treasury securities; Price discovery; Granger causality test.

1. Introduction

On June 30, 2023, the world witnessed that SOFR was recognized to be the benchmark interest rate referred by all financial products cleared on the dollar. The London InterBank Offered Rate (LIBOR), which was frequently used as a reference rate in the pricing of dollar loans, mortgages, and derivatives, has been replaced by the SOFR produced by the Alternative Reference Rates Committee. It's a backward-looking rate generated from the treasury bonds repurchase market, and updated according to real market transactions every day. A former study carried out by White proved that compared to its predecessor which is mired in manipulation scandals, the robustness and transparency SOFR obtained make it more reliable than LIBOR [1], and maybe, a better measurement of long-term interest rate, which can bring a new insight for a centurial debate.

Since many years ago, there has been discussion over how interest rates in different terms interact with each other. According to arguments of neoclassical economics, the term structure of interest rates shows that long-term interest rates are calculated by averaging the current and anticipated values of short-term interest rates [2]. This theory seems to suggest a one-way causality. However, since the long-term interest rates is a reflection of the expectation of short-term interest rate in multiple periods, whether the long-term rate predict the moving trend of the short-term rate?

Meanwhile, among post-keynesian economists, this argument also takes place. Kaldor and Moore insisted that the policy target interest rate determined by the monetary authorities in short term is exogenous, which makes the short-term rate a benchmark followed by all internal parties [3, 4]; Pollin believed that not only can the the monetary authorities determines short term target rate,but various market forces can also shift it. It is not stable enough and should be seen as a endogenous factor. Thus the interest rates in of long periods generated from the market transaction is useful for the prediction of short term rates [5].

Besides, the empirical researches carried out in the past years also give a practical view of this argument. For instance, Akram and Li developed a model to show how the interest rates of different terms in the America interact with one another, and the results are in accordance with the expectation theory, which holds that the short-term interest rate is a critical factor in the shifting of the market yields on long-term government bonds. [6].

2. Theoretical Analysis

Limited to the arguments between different schools and theory, there exists no solid theoretical basis for most research. This encourages researchers to more focus on pure mathematical analysis without economic insight. For example, Li and Su used the 3-month treasury bills' yield to represent the short-term measurement, meanwhile the market yield on U.S. treasury securities at 10-year constant maturity (DGS10) as the interest rate in the long period and carried the VAR model [7]. This strategy contributes to finding a statistical relationship since the two statistics are similar products. However, the widely-recognized short-term rate is the cost of capital overnight, which is represented by benchmark rates like LIBOR or SOFR, but not the yield of short-term treasury bills.

A hypothesis is whether there exists a theory that, is free from all the arguments above, moreover, it can provide a different perspective of this problem. Surprisingly, with SOFR stepping on, a possibility to solve the problem is found from its characteristics.

The SOFR is calculated from three categories of data, the first one is the transaction-level tri-party repo data collected from Bank of New York Mellon, the second one is the GCF Repo transaction data, and the last one is the data on bilateral Treasury repo transactions cleared through FICC's DVP service (REPO_DVP). The volume weight of the bilateral treasury repo (bi-repo) exceeding 60%. This data indicates that SOFR is highly related to bi-repo. Meanwhile, in the bilateral repo, lenders can dispose of treasury securities freely during the period of the contract. Then it is acceptable to suppose that the lender may sell the securities at first and buy back from the market by the end of the period. By using this strategy, the profit and loss (Y) of lenders could be expressed as:

$$Y = P_{present} - P_{future} + P_{present} \times R_{repo} \tag{1}$$

$P_{present}$ Represents the spot price of treasury securities, P_{future} represents the future price of treasury securities, $and\ R_{repo}$ is the bilateral repurchase rate.

Then with arbitrage and one price law, in the long term run, there would not be any profit or loss through this strategy. It means Y equals 0. Thus:

$$P_{present} - P_{future} + P_{present} \times R_{repo} = 0 \tag{2}$$

$$P_{future} = P_{present} \times (1 + R_{repo}) \tag{3}$$

Notice that theR_{repo}, which is the basis of SOFR, represents the future price of government bonds. Meanwhile, the price of treasury securities is also representative of the interest rates in long period. Then this equation inspires an idea that the repo rate is strongly related to the future price of treasury securities. This paper can utilize some theories of futures prices to explain the relationship between interest rates in different terms.

Chan discovered a leading lag relationship in the index yield of cash and the futures of the main index market [8]. Poskitt also had a similar conclusion in his research of the spot and futures interest rate market [9]. It is found that the futures market plays the function of price discovery in the spot market. Thus, it is sensible to infer this discovery also works in the transactions of treasury securities, for some characteristics shared by the stock market and bond market, such as large volume, high liquidity, and frequency.

Based on this idea, a hypothesis can come up:

The bilateral repurchase rate should contribute to the forecast of yield of long-term U.S. government bonds.

Using data from authoritative sources and statistical techniques like the VAR model as well as Granger causality test, this study aims to clarify the causality between interest rates in different terms. It will be an investigation into how interest rates of different terms interact with each other and can be helpful in finding a solution to this issue.

3. Methodology

This article constructs the VAR model and implies the Granger causality test to examine whether one stationary time series variable *X* can be used to predict another stationary time series variable *Y* [10].

Considering this research only has two variables, the particular, vector autoregression model on this premise can be defined as:

$$Y_t = \sum_{i=1}^{k} \alpha_k Y_{t-k} + \sum_{i=1}^{k} \beta_k X_{t-k} + \mu_\tau \quad (4)$$

In the equation above, *X* is the independent variable in the situation where *Y* is the dependent variable, α and β are the variable's parameters, k stands for lagged order, and μ_τ is the random error.

With this model constructed, a Granger causality test can be carried out, in which the null hypothesis is:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \cdots = \beta_k = 0 \quad (5)$$

Meanwhile, the alternative hypothesis is:

For each β_i ($i \in (1,k)$):

$$H_1: \beta_i \neq 0 \quad (6)$$

If the null hypothesis is not accepted, then it proves time series *X* is useful for the forecast of time series *Y*.

4. Data

4.1. Data Description

This paper chose the time series data of REPO_DVP (tenor less or equal to 30 days) and DGS10 from 8 May 2023 to 7 Jul 2023, the weekend gaps were filled with Fridays. This paper chose this time quantum for it's a relatively long period where the interest rate target of the Federal Reserve was relatively stable, which can reduce the possible externality caused by the change of monetary policy.

REPO_DVP is one of the most important components of SOFR, which is sourced from the Office of Financial Research (OFR). The weight of REPO_DVP in SOFR calculation is above 60%. It is reasonable to assume that these results can somewhat support the research's hypothesis.

DGS10 is a well-known benchmark for long-term Market Yield of U.S. Treasury Bonds, which was released by the Board of Governors of the Federal Reserve System. It is the long-term interest rate indicator used in this paper.

The summary of two series is concluded in Table 1:

Table 1. Descriptive statistics.

	N	mean	sd	min	max	Variance	skewness	kurtosis
REPO_DVP	61	5.103	0.080	4.560	5.180	0.006	-5.188	36.34
DGS10	61	3.720	0.133	3.390	4.060	0.018	-0.315	3.812

4.2. Unit Root Test

Before building a VAR model, this paper should initially ensure that the two series are stationary. To reach that goal, the research chooses to process data with logarithms, then introduce the first difference and discuss the stationary of the changing rate of two variables.

Define the changing rate of REPO_DVP as R_{repo}, changing rate of DGS10 as R_{dgs}. Then this paper conducted the ADF test to examine stationary, the test can be seen in Table 2:

Table 2. ADF unit root test.

variables	test statistics	Dickey-Fuller critical value			p-value
		1%	5%	10%	
R_{repo}	-11.538	-3.566	-2.922	-2.596	0.0000
R_{dgs}	-8.673	-3.566	-2.922	-2.596	0.0000

From Table 2, the changing rate series of two variables can concluded as two stationary time series.

4.3. Granger Causality Test

According to Table 3, the optimal lag of the VAR model could be sorted out as 1. Thus this research selected a lag order equal to 1 and constructed a VAR model.

Table 3. Lag-order selection criteria.

LAG ORDER	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	299.182				8.40E-08	-10.6137	-5.188	-10.5413*
1	306.726	15.088*	4	0.005	7.4e-08*	-10.7402*	-10.6561*	-10.5232
2	309.339	5.2248	4	0.265	7.80E-08	-10.6907	-10.5504	-10.329
3	310.629	2.5807	4	0.63	8.60E-08	-10.5939	-10.3976	-10.0876
4	312.734	4.2094	4	0.378	9.20E-08	-10.5262	-10.2738	-9.87519
5	317.293	9.1195	4	0.058	9.10E-08	-10.5462	-10.2377	-9.75052

After the construction of the VAR, calculate the AR Characteristic Polynomial. It could be found that all inverse roots lie in the unit circle, as it can be seen in Figure 1.

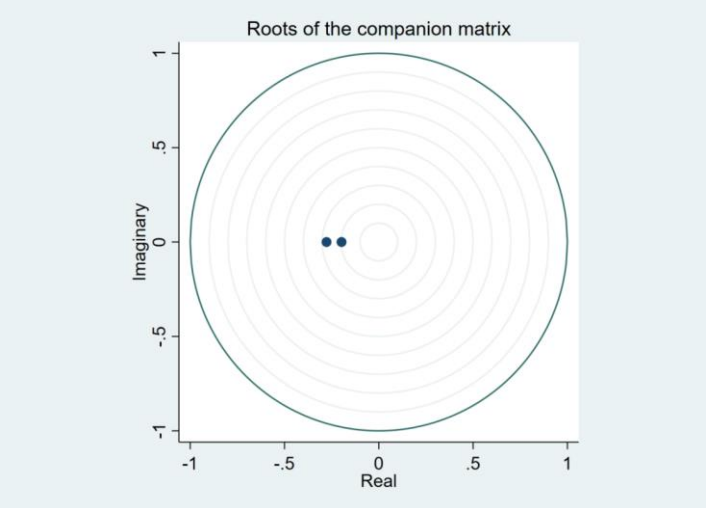

Figure 1. VAR eigenvalue examination.

It indicates that this VAR (1) satisfies the stability condition. Based on the VAR, a Granger causality test could be carried out (see Table 4).

Table 4. Granger causality Wald tests.

Equation	Excluded	chi2	df	Prob > chi2
R_{repo}	R_{dgs}	0.67994	1	0.410
R_{repo}	ALL	0.67994	1	0.410
R_{dgs}	R_{repo}	5.3085	1	0.021
R_{dgs}	ALL	5.3085	1	0.021

The chart shows that R_{repo} is a Granger cause ofR_{dgs}. Meanwhile, R_{dgs} is not a Granger cause ofR_{repo}, that is, there exists no bidirectional causality.

5. Conclusion

This article illustrates the continuous study of the causality exists in interest rates in different terms, points out the deficiencies of former research, and attempts to explore another insight into this problem based on the possible connection between SOFR and future price discovery theory. To prove this thought, the following statistical analysis via constructing the VAR model and Granger causality test is implemented. Empirical analysis indicates a single-direction causal relationship from R_{repo} toR_{dgs}, which meets the hypothesis, that is, the bilateral repurchase rate should contribute to the forecast of yield of U.S. treasury bonds. The absence of bidirectional causality also matches the theory of price discovery which the study is based. The result of the article might have a positive influence on the perfection of interest rate term structure, provide a new possible way to understand this problem, moreover, work as a reference for decision-making on investment or policy.

This research also along with several drawbacks. Initially, this research only focused on the linear Granger causality. However, most economic relationships are non-linear and have convexity. This may lead to a relatively large offset while the variability is high. To improve this problem, a non-linear Granger causality test based on the VAR model in further research is necessary.

Meanwhile, the selection of data evades the change of the target interest rate, which is impractical under most circumstances. The result this paper conducted only reflects the force of the market but evades intervene from the government. In the following research, this paper will expand the time series to select a period including multiple rate-setting meetings.

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