Comparison of Portfolio Optimizations under Markowitz Model in Technology Sector and Financial Services Sector

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Abstract. In the period of Covid-19, different sectors received different levels of shocks, which gave investors a degree of caution when investing in various sectors. Therefore, portfolio optimization - using specific model to assign weights of stocks to achieve a higher return while reducing risk – becomes a popular strategy. This paper chooses the Markowitz model to find optimal sector-based portfolios, specifically in technology sector and financial services sector, as well as portfolios that contains stocks in both sectors. The study uses Python to do Monte Carlo simulation, finding two optimal portfolios with maximum Sharpe ratio and minimum volatility for each sector(s), and finally comparing performances to test if the sector-based portfolio works better than the inter-sector portfolio. According to results, the minimum volatility portfolio in combined sectors reaches the same return of 0.11 as the minimum volatility portfolio in technology sector, but with lower volatility. It means the inter-sectors portfolio is better off when seeking minimum volatility. On the other hand, the maximum Sharpe ratio portfolios in technology sector, financial services sector, and combined sectors have values of returns and volatility ordering from highest to lowest. As a result, with current information, without investors' investment preference, the optimal maximum Sharpe ratio portfolio cannot be determined and needed further exploration.

Keywords: Portfolio optimization; Markowitz model; Monte Carlo simulation; maximum Sharp ratio; minimum volatility.

1. Introduction

During the period of Covid-19 pandemic, many sectors experienced an unexpected growth in the stock market. According to Sun-Yong Choi, the pandemic boosted volatility spillovers among sectors, and the technology sector also received the shock [1]. Moreover, due to the expectations of economic recovery during the financial market rebound period, there was a tendency of declining volatility shock delivered by the financial sector during the pandemic, even though other sectors did not show any significant change at the same time [1]. Therefore, investors should have responded savvily to the dynamic and interactive trends in the stock market. It is also feasible that investors may look more at the technology and financial services sectors than before the pandemic. In the principle of investment, investment investors are generally risk averse, pursuing higher portfolio returns while avoiding higher volatilities. The study is based on this goal to look in depth to the portfolio optimization mainly in both technology sector and financial services sector in the stock market.

Portfolio optimization is one of the most significant elements in respect of investment. It means that investors would have a bunch of stocks in their portfolios, in which each stock would have a weight out of 1 so that the expected returns of all stocks would reach a criterion such as the minimum variance or maximum Sharpe ratio. There are two classical models for portfolio optimization. One classical model is the Markowitz model, which is set up by Harry Markowitz in 1952. The model is the foundation to modern portfolio theory (MPT). It aims to utilize the idea of diversification, holding different assets in the portfolio to minimize the single risk from one asset, to find the maximize the expected return for a certain level of portfolio risk. The Markowitz model is also called the mean-variance model because it collects historical data of stocks in the portfolio to calculate the not only expected returns but also standard deviation of the portfolio. By obtaining these parameters, the model helps investors to withstand relatively low risks with efficient returns.

The Markowitz model is applicable in real world finance. According to V.N. & Mathew, the Markowitz model is applied for the selection of the portfolio in terms of the risk and return [2]. Given the correlations of selected stocks, investors can know which stocks to put into the portfolio to diversify the portfolio risk, thereby reducing the portfolio standard deviation. It is also proven to be effective in conducting portfolio optimization on Indonesia Stock Exchange during the Covid 19 in LQ45 Index Company, due to the results generated by authors that the portfolio return is higher via the use of the Markowitz model than the single-index model [3].

Even though the Markowitz model is widely used for the purpose of portfolio optimization and can be applied to a wide range of portfolio situations, as claimed by Qi of Review of Research on Markowitz Model in Portfolios [4], it has advantages and drawbacks. In the paper Explore to Find the Optimal Portfolio in The Financial Market, authors claim that returns in the Markowitz model are always higher than the single-index model under all 5 constraints mentioned in the paper [5]. Since the requirement of finding the covariance of each asset in the portfolio, the Markowitz model can produce a more nuanced model, which specifically targets to stocks chosen into the portfolio. But this can also cause some problems. The increasing number of stocks in the portfolio could complicate the covariance, which would further complicate the whole process of optimization. The complexity of the covariance would also produce more errors, making the portfolio less effective. Furthermore, the model requires normally distributed data, analysis of non-normally distributed data would be inaccurate [5].

With those concerns, it is hard to determine if the Markowitz model can be used in the optimization under specific conditions. Thus, the study can expand previous studies by exploring if model is better used in some circumstances. Many studies, like those mentioned above, study portfolios with stocks from many different industries, not quite a few scholars choose to explore the applicability in industry-leveled portfolios. Senthilkumar, Namboothiri, and Rajeev point out the paucity of evidence on inter- and intra-sector relationships [6], which also indicates the need for research in assessing the model at industry level. According to their paper does portfolio optimization favor sector or broad market investments, they found that in the Indian context, Sharpe's model performs better in intra-sector technology portfolios [6, 7]. The study can take this previous research as an anchor to further analyze whether technology sector-based portfolio or financial services sector-based portfolio performs better under the Markowitz model. Or does the combined portfolio with stocks in both technology sector and financial services sector have a better performance than either of the single sector portfolio.

2. Methods

2.1. Data Source

The study chooses 20 stocks in Yahoo finance during the period from 2001/5/11 to 2021/5/12. 10 out of 20 are stocks from technology sector. The rest 10 out of 20 stocks are from financial services sector. Choosing 20 years data ensures to investigate the long-term trends of stocks in two sectors [8].

2.2. Index Choice

In this study, S&P500, short in SPX in the following paper, is chosen as the equity index, 1-month Fed Funds rate as a proxy for risk-free rate. All stocks' information, including ticker, full name, and sector, is listed in Table 1. The raw data of 20 stocks is their daily prices, while the monthly prices are the last day prices in each month [9, 10].

Table 1. Stocks information.

Ticker	Full Name	Sector
AAPL	Apple Inc.	Technology
ADBE	Adobe Inc.	Technology
CSCO	Cisco Systems, Inc.	Technology
IBM	International Business Machines Corporation	Technology
INTC	Intel Corporation	Technology
MSFT	Microsoft Corporation	Technology
NVDA	NVIDIA Corporation	Technology
ORCL	Oracle Corporation	Technology
QCOM	QUALCOMM Incorporated	Technology
SAP	SAP SE	Technology
BAC	Bank of America Corporation	Financial Services
C	Citigroup Inc.	Financial Services
WFC	Wells Fargo & Company	Financial Services
TRV	The Travelers Companies, Inc.	Financial Services
JPM	JPMorgan Chase & Co.	Financial Services
BRK/A	Berkshire Hathaway Inc.	Financial Services
GS	The Goldman Sachs Group, Inc.	Financial Services
USB	U.S. Bancorp	Financial Services
TD CN	The Toronto-Dominion Bank	Financial Services
ALL	The Allstate Corporation	Financial Services

2.3. Steps and Processes

Since the Markowitz model assumes the data set is normally distributed/Gaussian, so we should check if the rate of returns of, which is the price of that day divided by the previous price and minus 1, 20 stocks as well as the S&P500 is normally distributed. Therefore, before process the Monte Carlo Simulation to generate the optimized portfolios as well as the exact weights allocation of portfolio, Quantile-Quantile plot should be drawn to check the normality of the data set of both the daily data and monthly data.

According to the Markowitz model, it aims to balance returns and risks, so we need to calculate the portfolio returns and its variance to denote the models' returns and risks, respectively. The formula of portfolio return is shown below:

$$E(r_p) = \sum \omega_i E(r_i). \tag{1}$$

Where $E(r_p)$ refers to the expected portfolio return, ω_i refers to the weight of each stock, and $E(r_i)$ refers to the expected rate of return of each stock in the portfolio. The variance of portfolio (with two stocks) is calculated as follows,

$$\sigma_p^2 = \omega_i^2 \sigma_i^2 + \omega_j^2 \sigma_j^2 + 2\omega_i \omega_j Cov(r_i, r_j) \tag{2}$$

Where σ_p^2 refers to the variance of the portfolio, w_i and w_j refer to the weights of stocks, σ_i^2 and σ_j^2 refer to the variance of stocks, and $Cov(r_i, r_j)$ refers to the covariance of two stock returns. To find the covariance, the correlation matrix for stocks in the portfolio needs to be calculated.

The Monte Carlo Simulation simulates the portfolio results by generating random values of stock weights in the portfolio. In this study, the simulation runs repeatedly 100 thousand times and results are plotted in dots. The outline of dots is the efficient/inefficient frontier of the portfolio, where the curve below the minimum volatility point is inefficient frontier while the curve above is efficient frontier. The point with the highest return to volatility ratio (which is called Sharpe ratio) indicates the maximum Sharpe ratio portfolio allocation. On the other hand, the point with the lowest volatility indicates the minimum volatility portfolio allocation. In python, two points and their according weights of each stock in the portfolio needs to be found.

3. Results and Discussion

3.1. Data Preprocessing

To determine which data set to use for later portfolio optimization, Quantile-Quantile plots of the daily and monthly rate of return are both drawn via Python? Quantile-Quantile plot, short in Q-Q plot, is a graphical device assessing the distributional relationship between the quantiles of two data sets. The more straight the scatter plot line is, the more same the distributions of two data set are. In this study, the distribution of the rate of return of each stock is going to compare with the normal distribution. The x-axis represents the theoretical quantiles, which shows the normal distribution, while the y-axis represents the sample quantiles, which shows the distribution of the stock's rate of return. If the blue dots match more closely to the red dashed line, in general forming a straighter line, the distribution of the stock's rate of return is more normal or Gaussian. Figure 1 shows Q-Q plots of all stocks' daily rate of return, as well as SPX's daily rate of return. Between theoretical quantiles range from -2 and 2, the blue dots match closely to red dashed lines, forming relatively straight lines for each scatter plot. On the other hand, however, dots outside the -2 to 2 theoretical quantiles becomes more like outliers, deviating from the dashed red line more as they are way farther from the edges of the range that shows partial normal distribution.

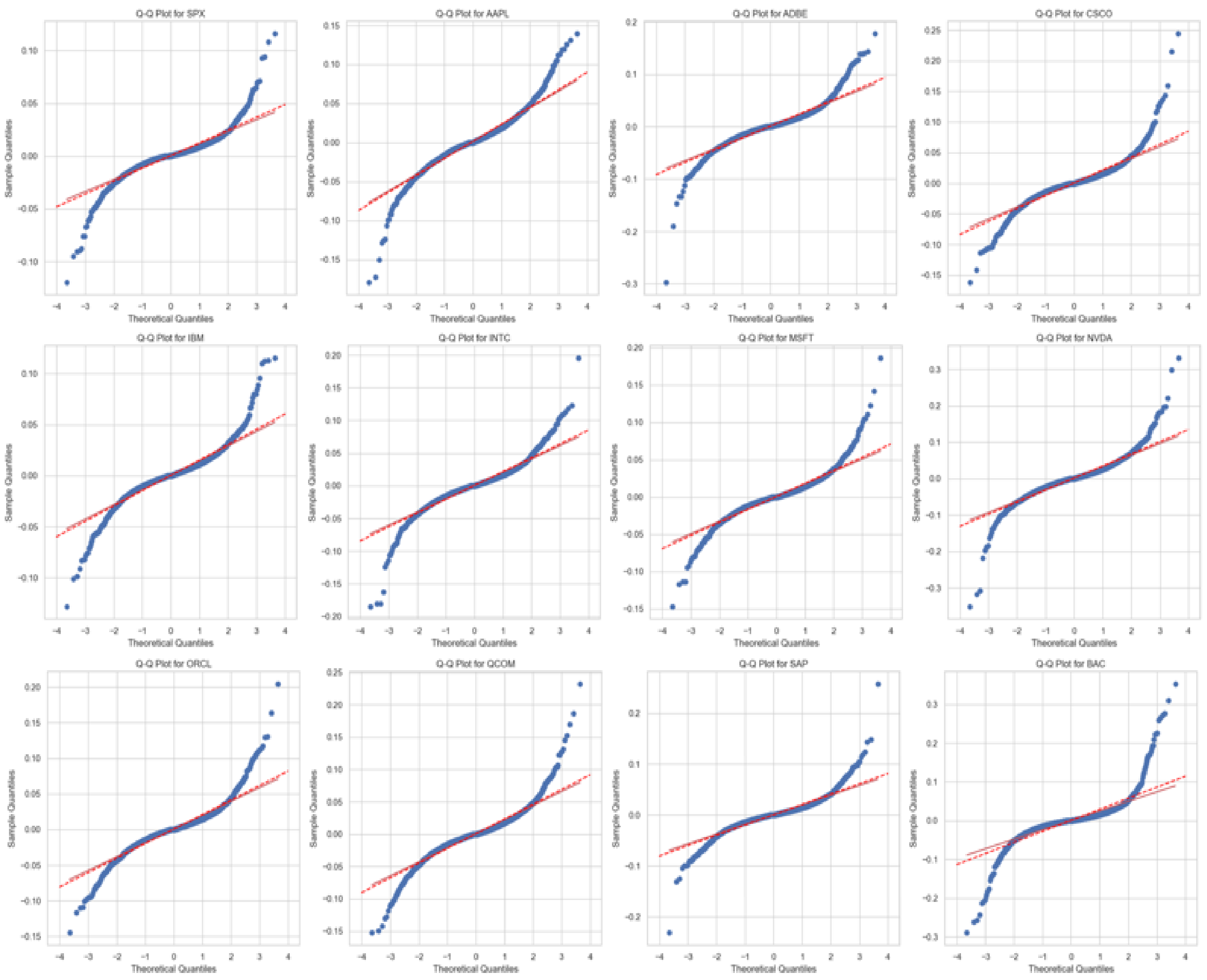

Fig. 1 Part of Q-Q plots for daily rate of returns.

Figure 2 shows Q-Q plots of SPX's and all stocks' monthly rate of return. Compared to Q-Q plots of daily rate of return, the lines in Figure 2 are straighter overall. Less dots are outside of the range from -2 to 2. There are few dots behave like outlier, lying outside of the red dashed line. Overall, the total number of blue dots that lie outside of the range from -2 to 2 in each Q-Q plot in Figure 2 is approximately 10, whereas the number in Figure 1 is apparently larger than this number. Nevertheless, it is still true that the father dots lying away from -2 or 2, the more extent the dots are far away from the red dashed line. Therefore, as the comparison of Q-Q plots of daily return rate and Q-Q plots of monthly return rate, the monthly rate of return is more Gaussian/normally distributed than the daily rate of return, so it is appropriate to use the monthly rate of return as data set for portfolio optimization under the Markowitz model.

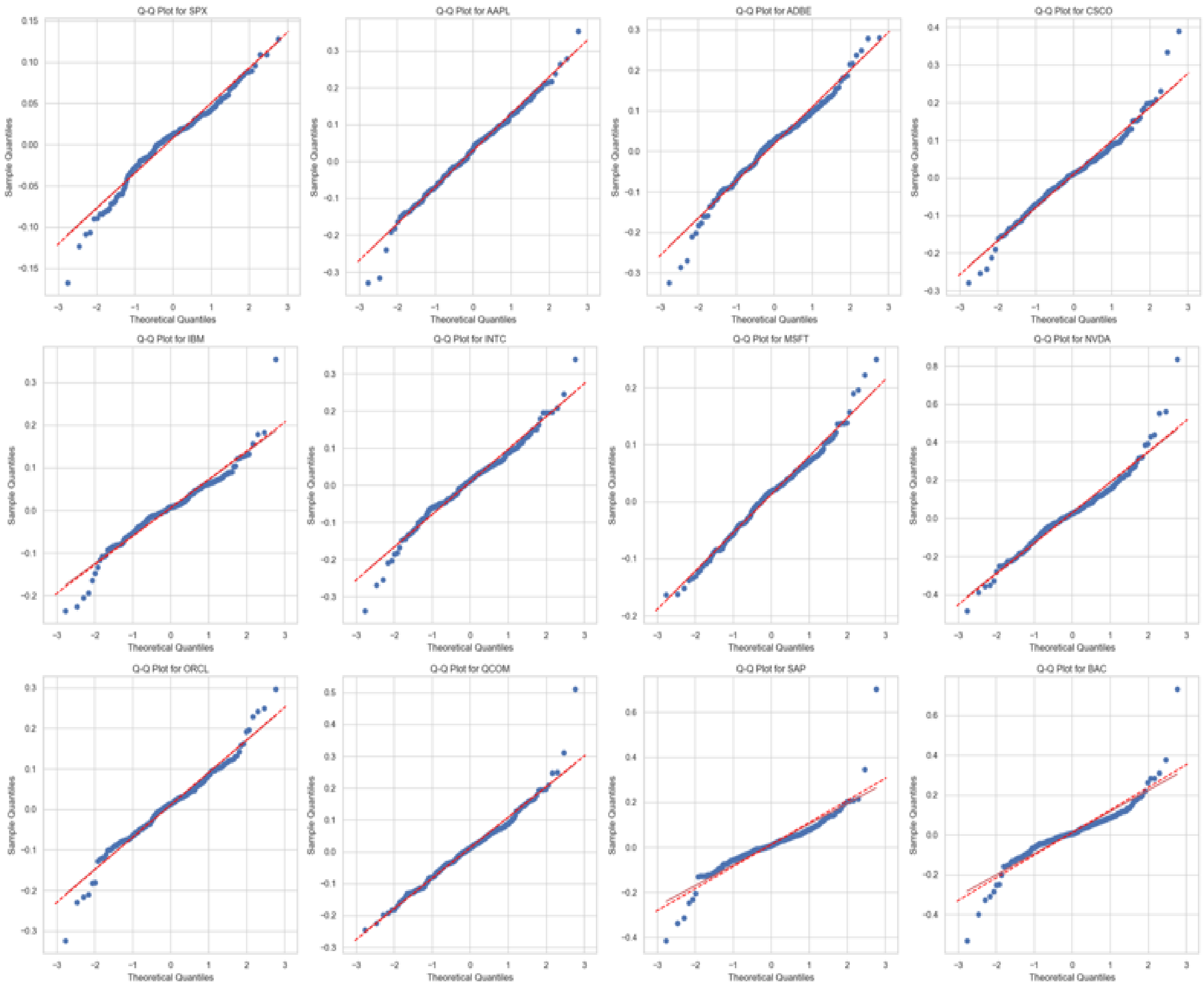

Fig. 2 Part of Q-Q plots for monthly rate of returns.

3.2. Correlation Analysis

The following graphs are heatmaps/correlation plots that demonstrate correlation between stocks within certain sectors. The correlation coefficient varies between -1 and 1, the higher the absolute value, the higher the correlation is. The positive value indicates that two stocks are positively correlated, while the negative value indicates that two stocks are negatively correlated. The x-axis and y-axis list SPX and stocks in the single sector. The bar on the right demonstrates the correlational coefficient of different colors. The warmer color means more positive correlation and the colder color means more negative correlation between two stocks. Figure 3 is the correlation plot of technology sector. All stocks are positively correlated. Except for correlation coefficients on the diagonal (the coefficients are always 1 because the correlation between stock and itself is always 1), correlational coefficients range from 0.28 to 0.66, which indicates only weak (less than 0.5) or moderate (between 0.5 and 0.75) correlation.

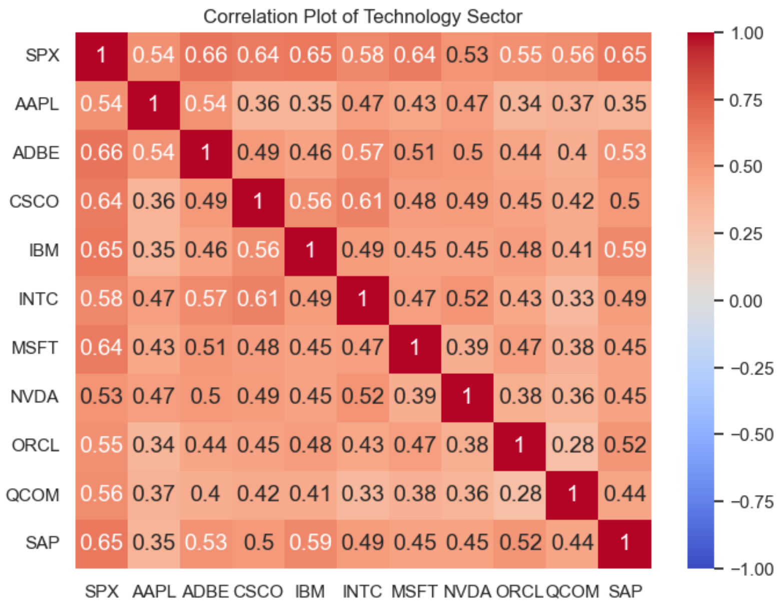

Fig. 3 Correlation plot of technology sector.

On the other hand, Figure 4 demonstrates the correlation of stocks within financial services sector. Similarly, the correlation coefficients are all positive. Other than the correlation coefficient of 1 lying on the diagonal of the heatmap, the correlation coefficients range from 0.34 to 0.83. It is worth noticing that there are few correlation coefficients exceed 0.75, including C & BAC (with coefficient of 0.83), WFC & BAC (with coefficient of 0.76), USB & WFC (with coefficient of 0.76). These pairs have strong correlation, so it is reasonable to predict that the change rate of weights if either paired stock in portfolio allocation would be highly similar.

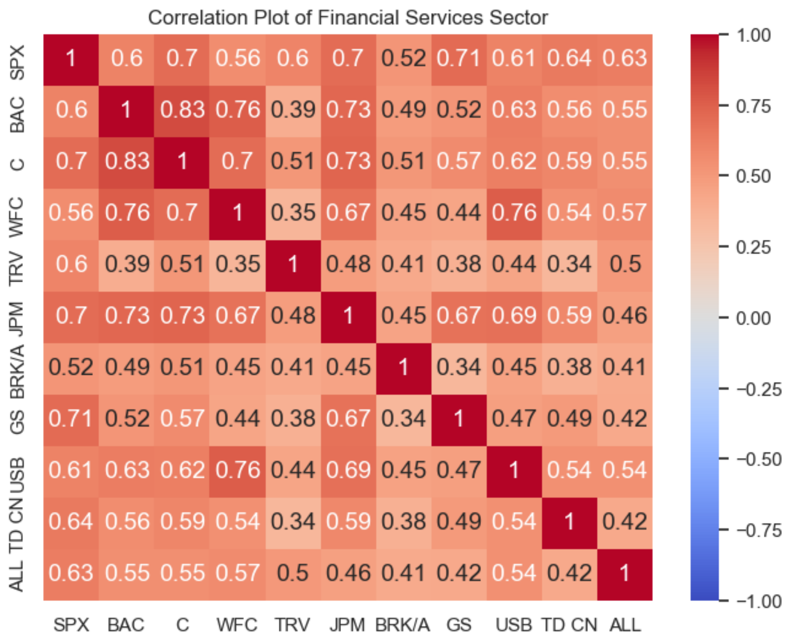

Fig. 4 Correlation plot of financial services sector

3.3. Portfolio Optimization and Monte Carlo Simulation

Then the study proceeds Monte Carlo simulation to obtain the following results of optimized portfolios in technology sector, financial services service, and two sectors combined. The bar on the right-hand side of the graph denotes the level of the Sharpe ratio: higher ratio indicates more returns per unit of volatility, and it also means larger slope of the graph. The red star shown in the graph demonstrate the point in which the portfolio with weight allocation has a maximum Sharpe ratio, the highest slope, and the max returns per unit of volatility. The green dot represents the most left point in the graph, where the portfolio volatility with weights allocated in each stock is minimal. Figure 5 demonstrates the Monte Carlo simulation of portfolio in technology sector. Its optimized portfolio with maximum Sharpe ratio has annualized return of 0.2 and annualized volatility of 0.23. The portfolio allocation comes to SPX (2.04%), AAPL (33.25%), ADBE (8.35%), CSCO (4.26%), IBM (2.4%), INTC (2.81%), MSFT (22.52%), NVDA (0.83%), ORCL (6.87%), QCOM (4.83%), SAP (11.85%). Its optimized portfolio with minimum volatility has annualized return of 0.11. And annualized volatility of 0.18. The portfolio allocation becomes SPX (26.54%), AAPL (8.65%), ADBE (4.55%), CSCO (3.14%), IBM (29.6%), INTC (1.19%), MSFT (11.18%), NVDA (0.72%), ORCL (6.72%), QCOM (4.6%), and SAP (3.12%).

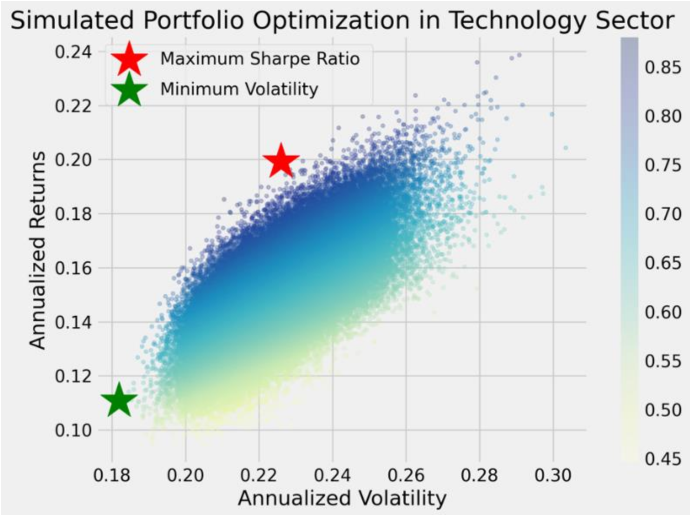

Fig. 5 Monte Carlo simulation of portfolio in technology sector.

In figure 6**,** the optimized portfolio with maximum Sharpe ratio in financial services sector generates annualized return of 0.1 and annualized volatility 0.16. Each stock is allocated as: SPX (1.31%), BAC (4.67%), C (0.53%), WFC (1.3%), TRV (25.01%), JPM (1.5%), BRK/A (15.73%), GS (4.47%), USB (3.85%), TD CN (27.6%), ALL (14.01%). In terms of the minimum volatility portfolio, the performance is not extensively different from the performance of the maximum Sharpe ratio portfolio. The annualized return is 0.09 and annualized volatility is 0.15, with portfolio allocation SPX (20.54%), BAC (1.5%), C (2.81%), WFC (1.81%), TRV (16.97%), JPM (1.13%), BRK/A (21.86%), GS (1.39%), USB (3.24%), TD CN (13.55%), ALL (15.18%).

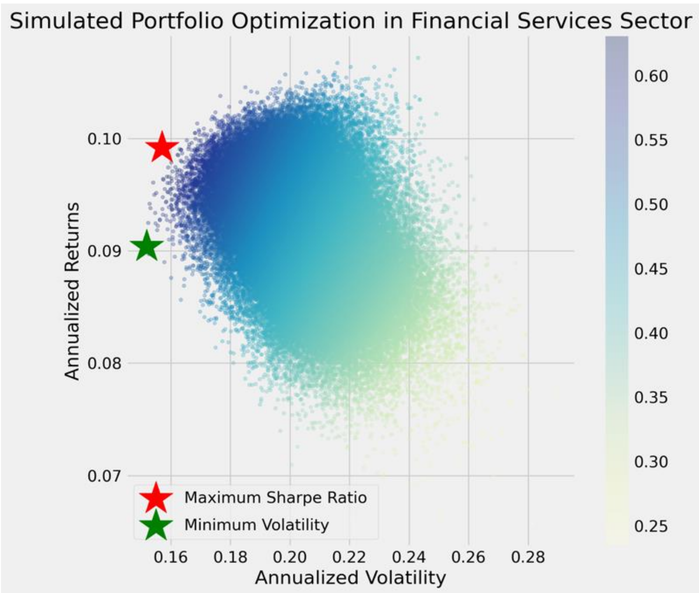

Fig. 6 Monte Carlo simulation of portfolio in financial services sector.

Comparing two sectors, the return, either of the maximum Sharpe ratio portfolio and the minimum volatility portfolio, in technology sector is higher than financial services sector, but the volatility is also higher. To check if inter-sectors portfolio can generate better results, specifically higher return with lower volatility, optimization of combined portfolios with stocks that come from two sectors is also simulated and presented below. Figure 7 is the scatter plot of portfolios of stocks in two sectors. The annualized return is 0.16 in the maximum Sharpe ratio portfolio and is 0.11 in the minimum volatility portfolio. Among the minimum volatility portfolios in three situations (technology, financial services, and combined), the return of the combined portfolio reaches the same level as the one solely in technology sector, but with lower volatility. The annualized return of maximum Sharpe ratio portfolio also outperforms the portfolio of financial services sector. Even though its return is lower than that of the portfolio of technology sector, the volatility of the combined sector becomes lower in this case. To sum up, to investors who want to invest the minimum volatility portfolio, it is better to choose portfolio in combined sectors if they prefer to maximize returns. There is no need to consider the one in technology sector as the return is the same while it is riskier. On the contrary, in terms of maximum Sharpe ratio portfolios, both return and volatility do show high, moderate, and low level in technology, combined, and financial services sectors respectively, so the optimal choice depends on the risk aversion level of investors. As Yao points out, Markowitz model does not take investors' investment preferences into account [7]. Therefore, with the model relied on the study, it is not possible to exclude any portfolio when considering the maximum Sharpe ratio point based on current information.

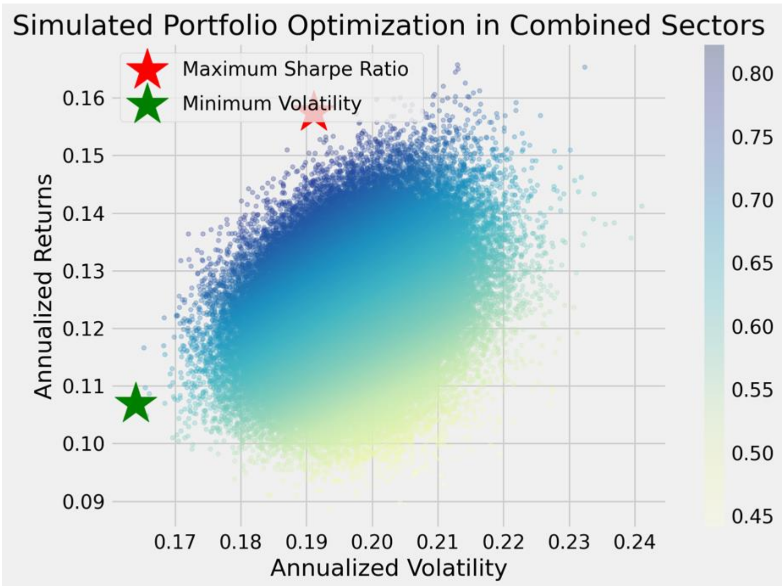

Fig. 7 Monte Carlo simulation of portfolio in two sectors combined

Morever, although putting more stocks into the portfolio can reduce the likelihood of unrepresentativeness, it also generate more errors when calculating the correlation between stocks, which complicates the analysis and reduces accuracy. Based on Nandan and Srivastava, the Single Index model, another model for conducting optimal portfolios, requires less inputs and easier calculations, compensating Markowitz model's increasing complexity [8]. However, according to Varghese and Joseph, the Single Index model also has limitation in that it does not take uncertainty that varies by time into account [9]. Similarly, based on Mistry and Khatwani, "the portfolio selection by this model is a static process whereas no other market is as dynamic as the stock market" [10].

4. Conclusion

This paper does the Monte Carlo simulation based on the monthly rate of return data to find the optimized portfolios – the maximum Sharpe ratio portfolio and minimum volatility portfolio – under the Markowitz model. To test whether the portfolio can perform better in only technology sector, financial services sector, or two sectors combined, the annualized return and volatility of two special points (max Sharpe and min volatility) are calculated under three circumstances. After comparison, the difference between the maximum Sharpe ratio point and the minimum volatility point is not huge, with only 0.01 in return and 0.01 in volatility as well. If focusing on minimum volatility portfolios, the return of the combined sectors portfolio reaches the same level as the riskier technology sector portfolio. Thus, the combined sectors portfolio is better off in this case. However, when looking at the maximum Sharpe ratio portfolios, the return in single sector, namely technology sector, can have a better result comparing to portfolio chosen in broader market. In this case, the optimum choice is not obvious. Instead, it depends on risk aversion levels of investors their personal understandings in investment, which cannot be derived from the Markowitz model.

Admittedly, the study only chooses 10 stocks in each sector, so it is likely that the results exist certain level of randomicity. In other words, the conclusion might change if we choose in a broader range in the stock market. Nevertheless, error and complexity increase as the number of stocks increases. Therefore, future studies can focus on investigation of finding a more appropriate model that allows to hold more stocks in the portfolio without extensively increasing errors, takes investors' investment preferences into account, and generates more representative outcomes to test the efficacy in portfolio optimization in single sector.

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