Comparative Study of Multi-Objective and Single-Objective Optimisation of Colour Matching for Opaque Products

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Abstract. Opaque products usually refer to all types of products with opaque packaging. In order to improve the market competitiveness of opaque products, it is crucial to determine the optimal colour scheme for the products. In this paper, we study the colour matching scheme for opaque products, taking different concentrations of red, yellow and blue dye as the research variables, taking the minimum chromatic aberration, the lowest cost, and the type of dye as the objective function, establishing the mathematical model of the optimal colour matching scheme, adopting the single-objective particle swarm algorithm and multi-objective hierarchical sequential method respectively, and using specific examples to carry out a comparative analysis to check the two methods. The results show that the single-objective method can provide the colour matching scheme with the minimum chromatic aberration from the ideal colour, and the multi-objective method can consider the cost and price of dyes as well as the types of dyes to provide a more scientific colour matching scheme. The computational results also verify the validity of the model and show that the model can provide decision-making and support for colour matching schemes for opaque products.

Keywords: Opaque products, Color matching of products, multi-objective optimization, Particle swarm algorithm, Layered sequence method.

1. Introduction

With the continuous development of computer science and technology, an increasing number of fields are embracing computer-assisted solutions for transformation and improvement. Opaque product coloration is no exception. Traditional manual color matching [1][2] is done by visually examining color samples, making approximate judgments of the dye proportions based on experience, or comparing them to color charts to find the closest match. This method relies on the skills and experience accumulated by the operators over the years, introducing subjectivity and uncertainty. With the continuous advancement of color management [3], image processing, and computer graphics technology [4], computer-based methods for color matching can ensure that the colors used meet expectations by controlling chromatic aberration. This provides users with a better color selection and matching experience. This article employs the Kubelka-Munk single constant theory [5][6] (hereinafter referred to as the K-M theory) to study the color variable relationships in the dyeing process and establish a color-matching model for opaque products. It utilizes both the single-objective particle swarm algorithm and the multi-objective hierarchical sequencing method to solve the model while comparing the advantages and disadvantages of these two approaches.

2. Description of the Problem

The colourful opaque tinted products of everyday life are dyed with colourants. The opaque colour matching problem is to use the three primary colourants as raw materials and to control the colour difference by computer methods to ensure that the colours used are as expected. The variables involved in the optimisation problem of colour matching for opaque products mainly include the concentration of the colourant C_i ($i=1,2,3$ represent red, yellow and blue colourants respectively),

$(K/S)_\lambda$ value, reflectance R_λ, colour parameters L^* a^* b^*, cost and price of the dye ρ_i. Among them: $(K/S)_\lambda$ value is the ratio of absorption coefficient and scattering coefficient of the substrate material at a specific wavelength λ, R_λ is the ratio of reflected light to incident light at the interface of the colourant, C_i is the concentration of red, yellow and blue colourant, and the parameters L^*, a^*, b^* denote the luminance, red-green and yellow-blue degrees, respectively. Changes in the type of colourant, its concentration, and its cost and price result in new colour schemes. Finding the optimal combination of the variables to minimize the difference between the dyed colour and the design value is the problem to be solved in this paper.

3. Color Matching Argumentation Model for Opaque Products

3.1. Calculated Relationships for Color Variables

The selective absorption of light by colorants is the reason why objects appear in different colors. In opaque products, the scattering coefficient of the colorant is negligible with respect to that of the substrate, and it is appropriate to adopt the K-M single constant theory.The K-M single constant theory solves the problem of reflecting one by one on the surface of the colorant particles. Under the K-M theory, the reflectance and K/S value satisfy equation (1).

$$\frac{(1-R_\infty)^2}{2R_\infty}=(\frac{K}{S})_\lambda . \tag{1}$$

The K/S values of mixed dyes satisfy the color addition and mixing laws.

$$\left(\frac{K}{S}\right)_\lambda=\left(\frac{K}{S}\right)_{\lambda 0}+\sum_{i=1}^{3}C_i\left(\frac{K}{S}\right)_{\lambda i} . \tag{2}$$

The tri-stimulus values X, Y, and Z of the CIELAB color space are obtained from equation (3)[12]。

$$\begin{cases} X=k\int_{400}^{700}S(\lambda)\bar{x}(\lambda)R(\lambda)d(\lambda) \\ Y=k\int_{400}^{700}S(\lambda)\bar{y}(\lambda)R(\lambda)d(\lambda) . \\ Z=k\int_{400}^{700}S(\lambda)\bar{z}(\lambda)R(\lambda)d(\lambda) \end{cases} \tag{3}$$

Finally, the corresponding color parameter L^*, a^*, b^* is derived as shown in equation (4).

$$\begin{cases} L^*=116(\frac{Y}{Y_0})^{1/3}-16 \\ a^*=500[(\frac{X}{X_0})^{1/3}-(\frac{Y}{Y_0})^{1/3}] . \\ b^*=200[(\frac{Y}{Y_0})^{1/3}-(\frac{Z}{Z_0})^{1/3}] \end{cases} \tag{4}$$

3.2. Objective Function

In order to measure the advantages and disadvantages of the color matching scheme for opaque colored products, three main objectives are considered, namely, the total chromatic aberration ΔE, the total cost of the dyes ρ, and the type of dyes N, as shown in equation(5).

$$\begin{aligned} &\min \Delta E = \sqrt{\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2}} \\ &\min \rho = \sum C_i \rho_i \\ &\min N, N = 1,2,3 \end{aligned} \quad . \tag{5}$$

Where L^* is the luminance, a^* is the red-green degree, b^* is the yellow-blue degree, ρ_i $(i = 1,2,3)$ is the unit price of the red, yellow, and blue color colorants, respectively, and N is the number of colorant species used for the matching.

3.3. Restrictive Condition

In order to be compatible with the colorant K/S value base database, the concentration range should satisfy equations(7).

$$0 < C_i < 5 \, , i = 1,2,3 \, , \tag{6}$$

$$\frac{K}{S} \in f(C_i) \, . \tag{7}$$

Reflectance R refers to the proportion of light or other waves that are reflected when they intersect a surface. Its value usually ranges from 0 to 1 and is expressed in equation (8).

$$0 < R < 1 \, . \tag{8}$$

According to the requirements of the actual situation, the optimization process should reduce the model complexity without affecting the results by stipulating that the total chromatic aberration is less than 1, namely, equation (9).

$$\Delta E < 1 \, . \tag{9}$$

4. Algorithm Design

The color-matching model for opaque products involves 5 variables and 3 objective functions, rendering it a multi-objective optimization problem. Historically, intelligent optimization techniques have been widely employed to solve similar problems, primarily within the domains of electricity and engineering[10][11]. However, in the context of dye matching, it is crucial to select distinct optimization algorithms tailored to the unique attributes of each target property. This article outlines two distinct algorithmic approaches, both of which are detailed in the algorithm flow chart presented in Figure 1.

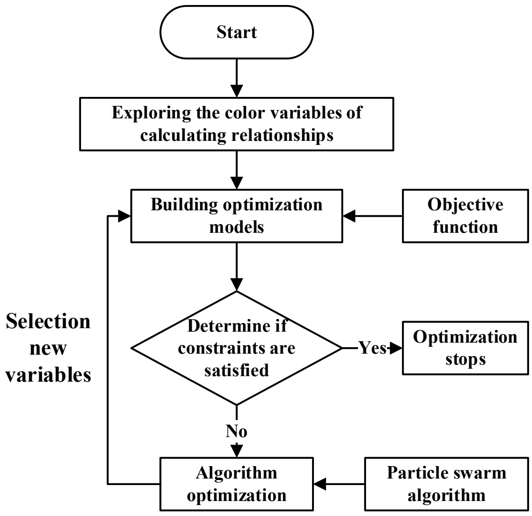


Figure 1. Algorithmic process

4.1. Single-objective Particle Swarm Algorithm

The single-objective approach focuses on studying the most important chromatic aberration objective among the three and utilizes the particle swarm algorithm for solving. The particle swarm algorithm[10], inspired by the behavior of a group of individuals, is employed for optimization problems. Its basic idea is to simulate cooperation and competition among individual particles to find the optimal solution to a problem.

Step1: Determine the feasible domain of the model T.

Step2: Within the feasible domain, establish a group of particles randomly distributed and calculate their fitness.

Step3: Based on fitness, the algorithm determines both the individual best solution for each particle and the overall best solution.

Step4: Update the position of each particle based on the overall best solution and individual best solutions, then recalculate fitness.

Step5: Check if the maximum iteration count is reached; if so, end the process, otherwise, go back to steps 3-4.

Step6: Output the overall best solution and the corresponding decision variable set.

4.2. Multi-objective Hierarchical Sequence Method

The hierarchical sequencing method is an approach for solving multi-objective optimization problems [11]. It divides multiple objectives into different priority levels and iteratively seeks the optimal solution for the next objective within the optimal solution set of the previous objective. This process continues until a common optimal solution is obtained. In the context of minimizing chromatic aberration and minimizing cost, the hierarchical sequencing method is used to progressively optimize these two objectives.

The objective of minimizing the number of dye types is binary in nature, where for a specific dye, there are only two possibilities: either selecting it or not selecting it. Therefore, in the previous optimization step (where the chromatic aberration was $\Delta E'$), an "incentive mechanism" is added to the fitness function. If the particle's position falls within the allowable range of total chromatic aberration, which is $|\Delta E - \Delta E'| < 0.25$, and one of the three position coordinates (representing the concentration of the three colorants) is set to 0, then the fitness is increased by 100. This is done to encourage particles to achieve the goal of using as few dye types as possible. The specific fitness function I is defined as per equations (11).

$$I = \begin{cases} -\rho + a \ , & |\Delta E - \Delta E'| < 0.25 \\ -1000\Delta E + a \ , & else \end{cases} , \tag{10}$$

$$a = \begin{cases} 0 & , N = 3 \\ 100 & , N = 2 \\ 200 & , N = 1 \end{cases} . \tag{11}$$

In the equations, to restrict the chromatic aberration within a certain range, there is a "penalty mechanism" in the fitness function when the chromatic aberration does not meet the requirements, denoted as $I = -1000\Delta E + a$.

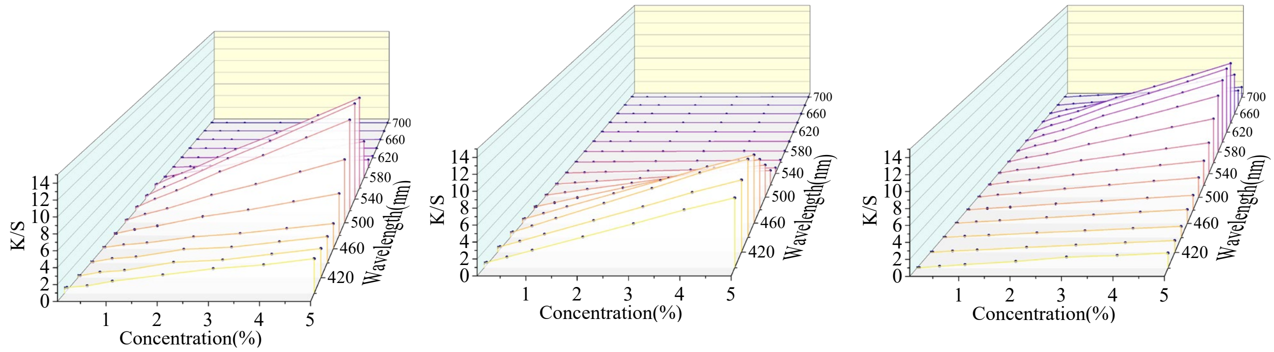
5. Example Analysis

5.1. Data Preparation

In order to check the accuracy of the model, this paper uses five sample colors of a dyeing factory for color matching example verification. The reflectance values of the target samples are shown in Table.1, and the unit prices of red, yellow and blue colorants are 60 yuan/kg, 65 yuan/kg and 63 yuan/kg, respectively. Now, according to the basic database of colorant values (Figure 2), we need to match the color of 2 kg of base material and find out the closest formula to the target color for each sample.

Table.1. Reflectance of 5 target samples at different wavelengths

Samples	400nm	440nm	480nm	520nm	560nm	600nm	640nm	660nm	700nm
1	0.51	0.55	0.49	0.37	0.37	0.72	0.89	0.90	0.91
2	0.68	0.75	0.73	0.66	0.64	0.72	0.73	0.77	0.86
3	0.63	0.69	0.65	0.55	0.53	0.68	0.71	0.75	0.85
4	0.60	0.65	0.64	0.59	0.50	0.44	0.44	0.50	0.69
5	0.64	0.70	0.69	0.65	0.57	0.51	0.51	0.57	0.74



Red Yellow Blue

Figure 2. K/S of colorants at different wavelengths value versus concentration

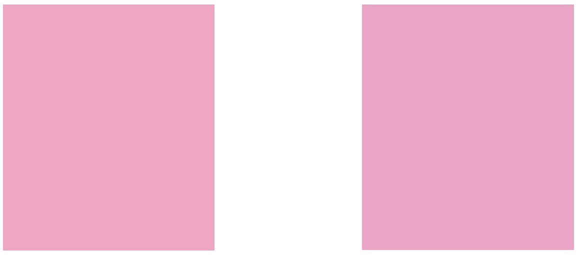
5.2. Results and analysis

Single-objective Particle Swarm Algorithm was used to optimize the calculation of the optimal color matching model for opaque products, the number of particles was set to 100, and after 50 iterations, the particles were concentrated near the point where the total chromatic aberration ΔE was the smallest. The formulation closest to the target-like color was produced in the process of approximating the optimum value, and the results are shown in Table.2.

Table.2. Values of the parameters of the single-objective method

Target number	Concentration (red)%	Concentration (yellow)%	Concentration (blue)%	ΔE	ρ (Yuan)
1	0.4049	0.001	0.0445	0.5951	54.325
2	0.1087	0	0.1574	0.9874	32.8764
3	0.197	0	0.1825	0.9235	46.635
4	0	0.041	0.4068	0.7767	56.5868
5	0	0.0154	0.3386	0.3794	44.6656

For target sample 1, the colour scheme with the minimum chromatic aberration from the target sample is [0.4047%, 0%, 0.0442%], which represents a dye made by mixing two colourants with a concentration of 0.4047% for the red colourant and 0.0442% for the blue colourant, with chromatic aberration of 0.5677. CIELAB colour parameters in CIELAB colour space were calculated for the target sample 1 and for the best colour scheme obtained, respectively. The colour parameters in CIELAB colour space were calculated for the target sample 1 and the best colour scheme obtained respectively, and the colours were plotted in Figure 3.



(target) (solution)

Figure 3. Comparison of target sample 1 color(left) with the requested color scheme(right)

The single-objective particle swarm algorithm only considers the single objective of minimizing the chromatic aberration. In the actual production, the cost of color matching should also be as little as possible, at the same time, in order to improve the efficiency of color matching, the fewer the types of colorants N required for color matching, the better, but these two reductions also lead to an increase in the chromatic aberration. The hierarchical sequence method can consider the three objectives sequentially and solve them comprehensively, and the results are shown in Table.3.

Table.3. Values of each parameter of the hierarchical sequence method

Target number	Concentration (red)%	Concentration (yellow)%	Concentration (blue)%	ΔE	ρ (Yuan)
1	0.3987	0	0	0.9795	47.849
2	0.1078	0	0.1551	0.9992	32.4801
3	0.1979	0	0.1811	0.9987	46.5638
4	0	0.0237	0.4019	0.9964	53.7128
5	0	0.0045	0.3271	0.9986	41.797

For the target sample 1, the chromatic aberration of multi-objective hierarchical sequence method increases by 0.3844 compared with the single objective particle swarm algorithm, the cost decreases by 6.476 yuan, and the corresponding type of colourant decreases by one, saving a large amount of cost. The multi-objective hierarchical sequence method provides an optimization scheme for the actual production, which can achieve cost savings with little change in colour difference and can meet the needs of decision makers to a greater extent.

6. Conclusions

For the optimal color matching design problem for opaque products, this article has established a colorant mixing color matching model. This model consists of three decision variables and three objective functions. Two approaches have been used to solve the problem. The first approach uses the particle swarm algorithm to find the color matching scheme with the minimum chromatic aberration, and the second approach uses the multi-objective hierarchical sequencing method to find a color matching scheme with acceptable chromatic aberration and the lowest cost. Based on case studies with multiple target samples, the following conclusions have been drawn:

(1) The study of the K-M theory and the functional relationship between the K/S values and concentrations of three coloring agents at different wavelengths enables a more precise selection of coloring agent proportions, yielding more feasible results.

(2) Both the particle swarm algorithm and the hierarchical sequencing method can solve the optimization problem for this model. These two methods respectively consider the minimum chromatic aberration color matching scheme and the lowest cost color matching scheme. They provide a wide coverage of options for color matching, offering more flexibility in decision-making.

(3) The computational results demonstrate that the proposed optimal color matching scheme for opaque products is effective and can provide decision support for color matching applications in opaque product manufacturing.

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