Research on portfolio in the US stock

Chenye jiang*
Department of Mathematics, University of Manchester, Manchester, UK
chenye.jiang@student.manchester.ac.uk

Abstract. Due to the global recession caused by the effects of pandemics, the Russia-Ukraine war, and the Fed's interest rate hike, investors are wary of the U.S. stock market. In this paper, we will focus on how to find quality portfolios by using Harry Markowitz's variance portfolio model. Firstly, the data is retrieved to see the return distribution of each stock in this article four stocks are used which are Amazon (AMZN), Apple (AAPL), Microsoft (MSFT) and Tesla (TSLA), the expected return and standard deviation of the portfolio are modelled using the Monte Carlo model, and finally Scipy is used to solve for the minimum variance frontier and the most assets portfolio to obtain the optimal asset portfolio return case, the global minimum variance portfolio point, and finally bring it into the CML market for display, the chart can clearly observe the most assets portfolio and the distribution of the asset portfolio, which plays a key role in the later market practice of the investor.

Keywords: Sharp ratio; us stock; portfolio；Monte Carlo simulation.

1. Introduction

The U.S. stock market is in tatters. Bond markets are in turmoil, real (or inflation-adjusted) interest rates are back up: and the dollar is back up, which is why investors are wary of U.S. stocks [1]. As of July 2023, the Federal Reserve has raised interest rates to between 5.25% and 5.5%. Jerome Powell's cautious comments on interest rates foreshadowed future tightening, which negatively impacted investor sentiment [2]. The S&P 500 fell 1.4% and second quarter earnings fell 5.2%.2023 Economic data for August was mixed, with the risk of a recession further clouding the outlook [2]. For example, the Russell 3000 Index rose only 6.3% in three months, while the Sunglasses World Index outside the U.S. rose 22% [3].

Based on this, it is important for researchers to study financial data and portfolio optimisation because financial markets are full of uncertainties such as government policies, global economic changes and social activities which are challenging for researchers. Stock markets are affected by political and economic markets which makes it challenging to predict future markets [4], so how can we as US stock investors conduct research on US stock portfolio optimisation?

The Markowitz model, which can also be referred to as the mean-variance model, is the core of current portfolio theory. The Markowitz model explains well the detailed process of how to arrive at the optimal portfolio and its principles [5]. Markowitz's model provides a mathematical approach to investment decision-making, through which investors can make asset allocation in a more objective and systematic way.

In this paper, we mainly use Markowitz's mean-variance model for the study of investment portfolios. In this paper, we have selected four stocks as the research objects, and their closing prices from 9/2020-9/2023 are used to calculate the expected return of each stock, to set the weights of the portfolios, and then to calculate the expected return and risk of the portfolios, and to optimise the weights by using mathematical methodology (Quadratic Programming Methodology), and finally to go to the evaluate these portfolios. This paper is applicable to the following groups: for investors, it can help them better understand and quantify the relationship between risk and return, and make asset allocation more reasonable and scientific; for portfolio managers, they can use the model to customise the best asset portfolio strategy for their clients and make investment decisions based on expected return and risk. For financial analysts, the model allows them to more accurately assess the performance and risk of various investment products; for academics, this paper provides a solid foundation for further research and development in financial economics.

In this paper, we will first introduce the methodology, then introduce the data using the Scipy software package to solve for the minimum variance frontier and the optimal asset portfolio, and finally conclude.

2. Methodologies

2.1. Mean-variance model

Expected Portfolio Return: This event is an estimate of the likelihood of an investment return. For portfolios with multiple investments, the expected return is the weighted average of the expected returns of each investment:

$$rn = \sum_{i=1}^{n}(w_i \times r_i) \tag{1}$$

wi is the weight of the ith investment, ri is the expected return of the ith investment, n is the number of investments in the portfolio.

Portfolio Variance: It measures the degree of dispersion of returns and, simply put, the risk associated with returns. When there are two or more assets in a portfolio, the variance takes into account both the variance of the individual assets and the correlation between them [6].

$$e = \sum_{i=1}^{n}\sum_{j=1}^{n}(w_i \times w_j \times \sigma_i \times \sigma_j \times \rho_{ij}) \tag{2}$$

σ_i and σ_j are the standard deviations of returns for assets i and j, ρ_{ij} is the correlation coefficient between assets i and j. Portfolio Standard Deviation: This is simply the square root of the portfolio variance and is an indicator of the level of portfolio risk.

2.2. The Sharpe ratio

The Sharpe ratio, proposed by economist William Sharpe, is an indicator that assesses the risk-return relationship of an investment. Specifically, it measures the performance of an investment relative to a risk-free asset, such as a U.S. Treasury bill. The formula for calculating the Sharpe ratio is:

$$Sharpe\ Ratio = \frac{(R_i - R_f)}{\sigma_i} \tag{3}$$

R_i is the expected return of the investment or portfolio, R_f is the risk-free rate, often represented by the yield on U.S. Treasury bills. σ_i is the standard deviation of the investment' sreturns, representing the risk or volatility. For the key points First, Excess returns: numerator (R_i – R_j) Represents the "excess return" an investor can expect to earn relative to a risk-free asset. Second, Risk measurement: denominator σ_i Quantifying investment risk. The higher the standard deviation, the higher the risk and vice versa. Third interpretation the higher the Sharpe ratio, the higher the risk-adjusted return of an investment. Conversely, the lower the Sharpe Ratio, the lower the risk-adjusted return. Ratios below 1 are considered suboptimal or poor, Ratios above 1.0 indicate good performance, Ratios above 2.0 denote very good performance, Ratios 3.0 and above are seen as excellent [7].

3. Data

3.1. Distribution of individual stock returns

Calculation of Yield on US Stocks. The yield on US stocks usually refers to the rate of return on the stock, which can be calculated based on the price movement of the stock as well as dividends and other factors. Usually, the formula for calculating the yield is: (ending price - starting price + dividends) / starting price.

Calculation of standard deviation. First calculate the average return on the stock, denoted x_0.

Then calculate the square of the difference between each return and the average return as $(x_1-x_0)^2$.Add the squares of all these differences.Finally take the square root of this sum to get the standard deviation of stock returns.

This paper selects four very famous stocks, Amazon (AMZN), Apple (AAPL), Microsoft (MSFT) and Tesla (TSLA), and obtains data from yahoofiance for the period 9/2022-9/2023 (See Figure 1). Use (Amazon (AMZN), Apple (AAPL), Microsoft (MSFT) and Tesla (TSLA)) to plot a graph from the closing prices of 9/2022-9/2023.

Fig. 1 Returns on (AMZN,AAPL,MSFT,TSLA)

Next, let's look at the histogram of the distribution of returns for the following currencies (See Figure 2):

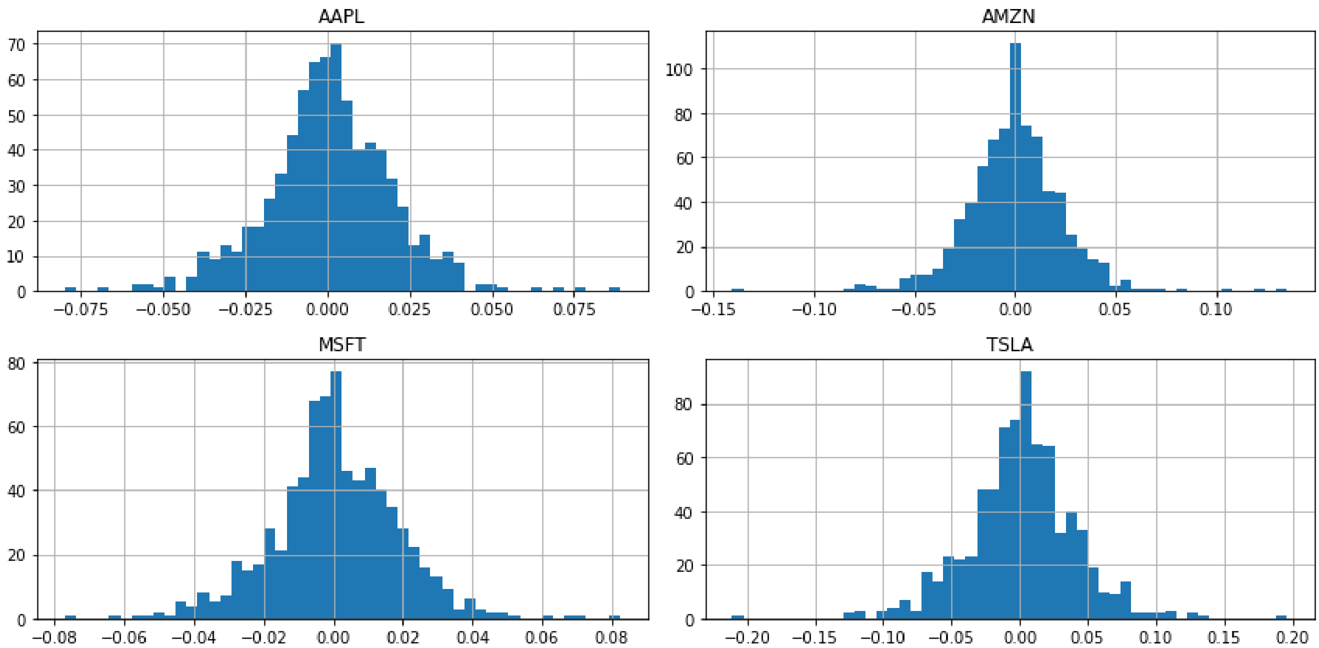

Fig. 2 Histogram of income distribution

3.2. Monte Carlo simulation of the expected return and standard deviation of the portfolio

Monte Carlo simulation is a statistical technique used primarily to approximate the solution of complex mathematical problems and to predict the behaviour of systems. Its basic idea is to use random numbers and probability statistics to approximate the solution of problems. The following are its main methods: In the first step, define the model parameters: first, determine the input parameters of the model and their probability distributions. Step 2, generate random inputs: generate a set of random inputs based on the probability distribution of each parameter. Step 3, perform simulation: use this set of random inputs to perform simulation calculations. Step 4, Record results: Record the output of the simulation. Step 5, Repeat the process: Repeat the above steps many times, usually thousands to millions of times, to generate a distribution of results.

First generate random weights and standardise the weights to compute the expected return, The result after running is shown below:

Table 1. Expected return, variance and standard deviation of the portfolio

Portfolio expected return:	0.0010
combinatorial variance:	0.1385
Combined standard deviation:	0.3722

Unsurprisingly, this stochastic portfolio constructed from these four stocks did not have a high return of 0.104%.

Monte Carlo simulation was next used to predict the combination, Monte Carlo simulation is a powerful mathematical technique for modelling the probability of different outcomes under uncertainty. The simulation can generate ranges of values for various inputs, thus enabling analysts to make more informed decisions [8-10].

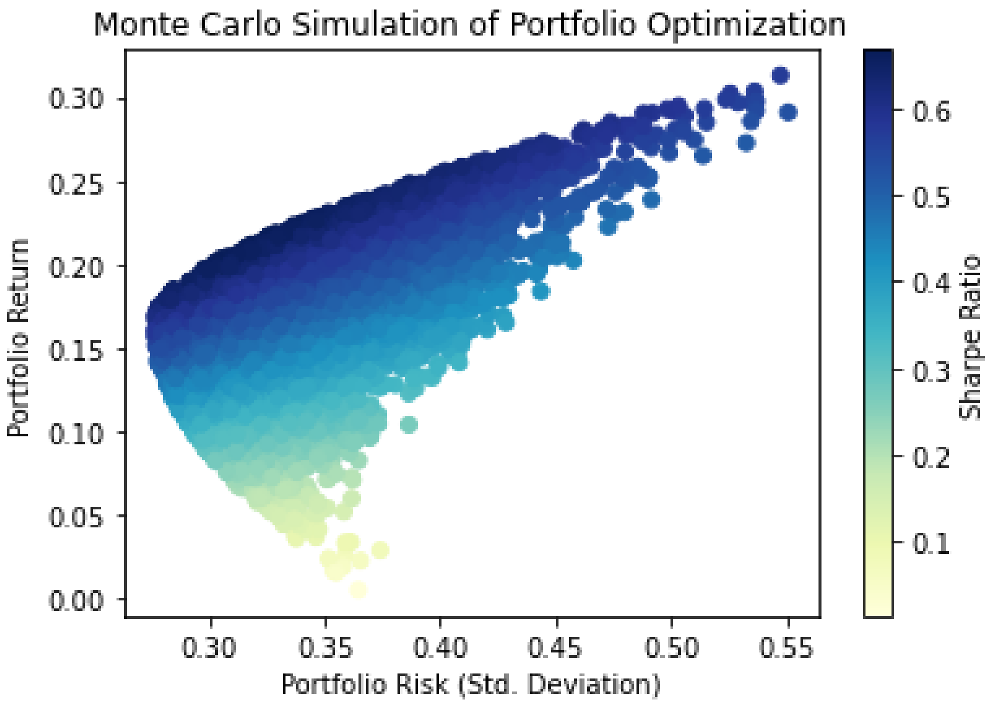

Fig.3 Monte Carlo Simulation of portfolio Optimization

Figure 3 Scatter plot using Matplotlib, where the x-axis represents the portfolio risk (standard deviation). The y-axis represents the portfolio returns. The colours correspond to the Sharpe ratios of each portfolio.

4. Solving for minimum variance frontiers and optimal asset portfolios using the Scipy package

4.1. Solve for the optimal asset mix, i.e. maximise the Sharpe ratio

Objective Function for Portfolio Optimisation: This section defines the objective function for calculating the negative Sharpe ratio. The Sharpe ratio is a metric that assesses investment performance by adjusting for investment risk. It is the ratio of returns to their standard deviation (See Table 2):

Table 2. Optimal portfolio returns

Optimal Weight:	[3.3082e^-01 ,2.7756e^-17 ,3.3948e^-01 3.2969e^-01]
Optimal Return:	0.2602
Optimal volatility:	0.3305
Optimal Sharpe ratio:	0.7873

4.2. Solving for Global Minimum Variance Combination Points

Optimize: scipy.optimize.minimize is used to find the optimal portfolio weights to maximiser the Sharpe ratio.

The constraint ensures that the weights sum to 1.Constraints ensure that the weights are between 0 and 1.The initial assumption is the average distribution of stocks (See Table 3).

Table 3. Returns on minimum variance portfolio points

Minimum variance weights:	[4.3603e^-01, 0.0000e^+00 ,5.6397e-01, 1.7347e-17]
Minimum variance return:	0.2038
Minimum variance volatility:	0.2677
Minimum variance Sharpe ratio:	0.7613

4.3. Visualisation (Extension)

This section extends the initial scatterplot by highlighting the following: The location of the minimum variance portfolio (lowest risk).The location of the optimal portfolio (best Sharpe ratio).

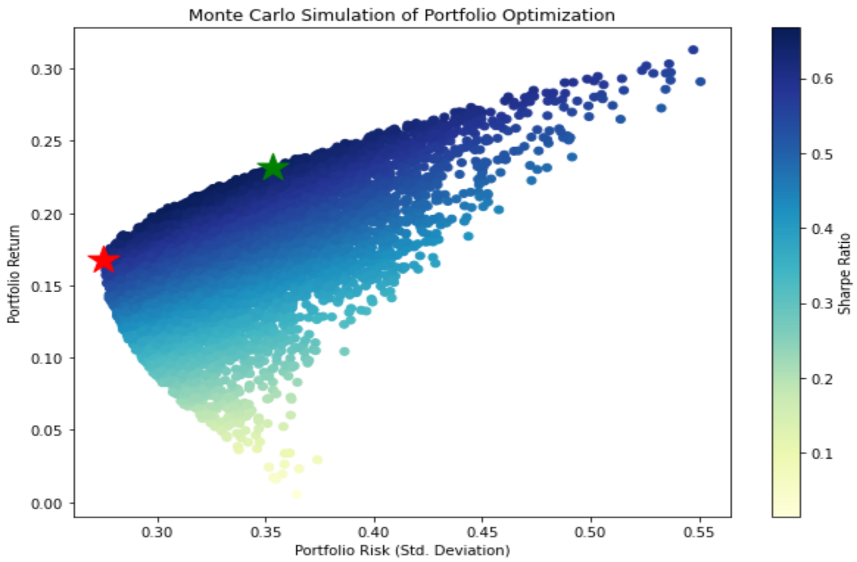

Fig.4 Monte Carlo Simulation of Portfolio Optimization with points

The red star is the global minimum variance portfolio point and the green star is the maximising Sharpe portfolio point (See Figure 4).

4.4. Calculation of Capital Market Lines (CML) using Scipy Package Linear Interpolation

The Capital Market Line (CML) represents the portfolio with the best combination of risk and return. The code calculates the CML and superimposes it on a scatter plot of the portfolio (See Figure 5):

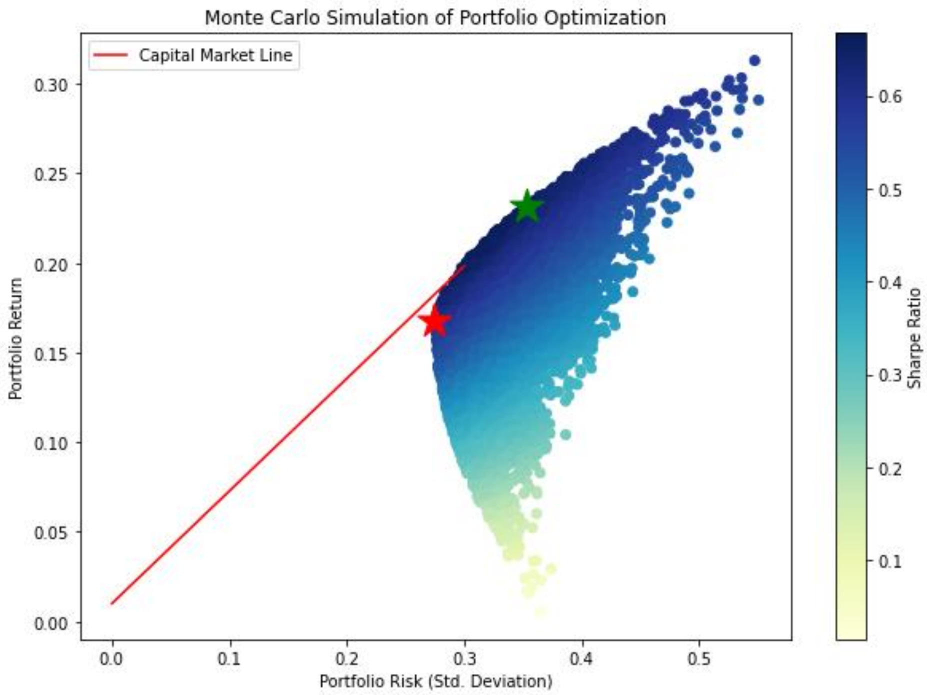

Fig.5 Monte Carlo Simulation of Portfolio Optimization with points and line

The red line is the Capital Market Line (CML) and the green star point is the Optimal Portfolio (opt). In summary, the script performs portfolio optimisation aimed at determining the asset capital allocation that maximises the Sharpe ratio and then visualises the results in conjunction with the efficient frontier and CML.

5. Conclusion

How to carry out an investment optimisation portfolio study in the US stock market and simulate the returns and standard deviation of the portfolio using Monte Carlo, and finally use the scipy package to solve for the minimum variance frontier and the optimal portfolio of assets and add it to the CLM markets.In this article Markowitz's mean-variance model was used for the portfolio, selecting Amazon (AMZN), Apple (AAPL), Microsoft (MSFT) and Tesla (TSLA). The expected return of each stock is calculated from September 2020 to September 2023 to the closing price and weights are set to calculate the expected return and risk of the portfolios and the weights are optimised using the quadratic programming method and finally these portfolios are evaluated， Before performing interpolation, ensure that the data points have been sorted by value-at-risk. Linear interpolation may not be the best way to describe the CML, especially if the effective frontier curve is non-linear. You may want to consider other interpolation methods, such as quadratic or cubic interpolation. Interpolation results are limited to the range of minimum and maximum data points. Make sure you do not exceed this range or the interpolation may be inaccurate.

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