

Portfolios Optimization Among Technology, Consumer Defensive, Consumer Cyclic and Entertainment

Xiangyu Cao *

Faculty of Natural, Mathematical & Engineering Sciences, King's college London, London, The United Kingdom

* Corresponding Author Email: k22000723@kcl.ac.uk

Abstract. Nowadays in financial field, portfolio optimization is playing a key role in making decision of investment. This article taking about the asset allocation in seven companies in Technology, Consumer Defensive, Consumer Cyclic and Entertainment. The Markowitz and Index models are the primary tools utilized in this study to analyze the portfolio data. Then, three constraints will be set to see the difference between them. In more detail, comparing two different models in same constraint and find difference in same model when setting different constraints. The results shows that McDonald's takes the highest Weight in two models when setting different constraints on models. In the meanwhile, compared to when there are no constraints, the performance of the minimal variance frontier and the capital allocation line is inferior. The research in this paper plays a reference role for investors to construct investment portfolios.

Keywords: Portfolios optimization; Markowitz model; index model.

1. Introduction

In the last four decades, many alternative theories have been constructed to describe the choice under risk or modify the Expected Utility Theory (EUT) [1]. After American economist Markowitz proposed the portfolio theory in 1952 [2] which to optimize the portfolio by maximize returns while minimize the risks [3], the portfolio selection has becoming famous and important in the financial market. In order to find the low risk but high return portfolio, the Capital Asset Pricing Model (CPAM) was introduced by William Sharpe in 1964 based on Markowitz's theory [4]. The index model is widely used in field of econometries as compromise between two restrictive parametric models [5]. Following Markow's idea of index for producing covariance terms, in 1963, William F. Sharpe developed the more straightforward Single Index Model (SIM) for portfolio analysis. This model gives us an assessment of value of the index as well as the return on a security. Sharpe added to the Markowitz model by introducing the CPAM [6].

However, after doing some research in portfolio optimization, most of the articles focusing only on Markowitz model but not comparing with index model and they didn't set any constraint to compare the differentials between them. For example, Armeanu and Balu evaluate the effectiveness of the Bucharest Stock Exchange's use of the Markowitz Model [7]. In order to lower risk and raise the average return of the minimal risk portfolios created, they analyze the data. Ivanova and Dospatliev offer a useful analysis of the Markowitz model conducted on the Bulgarian stock market between 2013 and 2016 by choosing 50 different stocks [8]. Kamil, Abdulbasah, Yew Fei, and Kin Kok selected 15 stocks in KLSE using Markowitz model to assemble the ideal portfolio in order to fulfill the goal of minimizing risk and maximizing return [9].

Therefore, in this article it will focusing on the allocation of different asset in four different areas which are Technology, Consumer Defensive, Consumer Cyclic and Entertainment in seven different companies. First, gathering the prices of seven companies in recent six years. Second, filter the prices into monthly data and plotting them into the Excel. Third, using the equipment of Excel to calculate the annualized average return, beta, annualized standard deviation, alpha and annualized residual standard deviation with correlations between the seven stocks. Fourth, comparing their minimal variance, Capital allocation line and maxima Sharpe Ratio with efficient frontier in The Index and

Markowitz models when setting three different constraints. Fifth, plotting the graph of these data in excel and analyze the performance of these seven stocks in constraints.

This paper is organized as follows: The data for seven stocks are displayed in Section 2, the methodology is explained in Section 3, results of this research will be presented in Section 4, and lastly the conclusion is provided in Section 5.

2. Data

The data from this paper have been derived from Yahoo! finance (<https://finance.yahoo.com/>). The prices were chosen from seven companies. i.e. AT&T, MCD, KO, DIS, SBUX, NIKE, WMT from January 1, 2015, to December 1, 2020. The daily data is aggregated to monthly data to reduce the non-Gaussian effects. The Annualized average returns (AAR), standard deviation (ASD) as well as Beta and Annualized alpha and residual standard deviation (RSD) of the index model are calculated as shown in the Table 1. The correlation coefficient is shown in Table 2 calculated from previous data.

The two tables clearly show that NKE has the highest annualized average return while AT&T has the lowest. Meanwhile KO has the lowest annualized standard deviation and DIS has the largest. In addition, AT&T, KO and DIS have negative alpha. The correlation between the AT&T and WMT is the smallest and it is the biggest between SPX and DIS.

Table 1. Descriptive statistics of the selected stocks

	SPX	AT&T	MCD	KO	DIS	SBUX	NKE	WMT
annualized average return	13.842 %	- 0.647 %	15.714 %	5.941 %	15.054 %	17.373 %	21.310 %	10.558 %
annualized st dev	14.884 %	18.110 %	16.987 %	14.789 %	26.499 %	21.096 %	21.413 %	18.036 %
beta	1.000	0.654	0.631	0.600	1.256	0.822	0.777	0.370
annualized alpha	0.000 %	- 9.703 %	6.974 %	- 2.361 %	- 2.332 %	5.999 %	10.551 %	5.436 %
annualized residual st dev	0.000 %	15.269 %	14.151 %	11.791 %	18.780 %	17.189 %	18.018 %	17.174 %

Table 2. Correlation matrix

correlations	SPX	AT&T	MCD	KO	DIS	SBUX	NKE	WMT
SPX	1.000	0.538	0.553	0.604	0.706	0.580	0.540	0.305
AT&T	0.538	1.000	0.262	0.395	0.369	0.367	0.304	0.118
MCD	0.553	0.262	1.000	0.666	0.410	0.625	0.259	0.308
KO	0.604	0.395	0.666	1.000	0.550	0.557	0.256	0.277
DIS	0.706	0.369	0.410	0.550	1.000	0.555	0.474	0.183
SBUX	0.580	0.367	0.625	0.557	0.555	1.000	0.278	0.150
NKE	0.540	0.304	0.259	0.256	0.474	0.278	1.000	0.221
WMT	0.305	0.118	0.308	0.277	0.183	0.150	0.221	1.000

3. Method

Markowitz model was introduced in by Harry Markowitz. This model analyzes different portfolio combinations based on the anticipated returns called means and standard deviations called variance of the assets in order to determine the most efficient portfolio in terms of return to risk [10]. Markowitz gathering material and putting them together discussed about the mean and variance of

the portfolio. He then researches the efficiency of mean-variance when utility functions are not quadratic. Nowadays many theories and Model are being created using in the field of asset allocation, but the Markowitz model can still be described as the “ancestor” of the asset allocation.

The Standard Deviation of financial portfolios can be calculated as below:

$$\sigma(w) = \sqrt{\sum_{i,j} [(w_i * s_i) * C_{ij} * (w_j * s_j)]} \tag{1}$$

Where W_i means the same and S_i means the standard deviation of stock i and C_{ij} stands for the correlation between stock i and stock j . In this paper, two portfolio models are selected, i.e., Marowitz model and Index model. The method of calculating the returns in Markowitz model and he Index model are different. For Markowitz model:

$$r_M = \sum_i r_i * w_i \tag{2}$$

Where r_M represents for the return and r_i represent the annualized average return of stock i . For Index model:

$$r_i = \sum_i w_i * (\beta_i * r_{Market} + \alpha_i + R_i) r_M = \sum_i r_i * w_i \tag{3}$$

Where r_i represents for the return, α_i and β_i are annualized alpha and beta shown before, r_{Market} is the S&P 500 Index. R_i is the residual standard error of stock i and R_i will be considered as zero in this article. The Sharpe ratio contrasts the return of an investment with its risk, it can be calculated as follows:

$$SharpeRatio = \frac{E(R_p) - R_f}{\sigma_p} \tag{4}$$

To be more specific, the $E(R_p)$ and R_f at molecule represent the expected return and risk-free rate of investment portfolio. The σ_p at denominator means the standard deviation of the portfolio.

The Minimal Risk or Variance Frontier is made up of the Minimal Return Frontier and the Efficient Frontier together. In follows:

Minimal Risk or Variance Frontier:

$$\begin{cases} \sigma(w) \rightarrow \min \\ \text{subject to: } r(w) = \text{const} \end{cases} \tag{5}$$

Minimal Return Frontier:

$$\begin{cases} r(w) \rightarrow \min \\ \text{subject to: } \sigma(w) = \text{const} \end{cases} \tag{6}$$

Efficient Frontier:

$$\begin{cases} r(w) \rightarrow \max \\ \text{subject to: } \sigma(w) = \text{const} \end{cases} \tag{7}$$

Two unique points also need to be found on the Efficient Frontier are of special interest:

Minimal Risk Portfolio:

$$\sigma(w) \rightarrow \min \tag{8}$$

and

Efficient Risky Portfolio:

$$\frac{r(w)}{\sigma(w)} \rightarrow \max \tag{9}$$

The list are the constraints been set in producing the result.

1. The first constraint can be represented as a “free” case which means without any additional constraints, to show how portfolios performed in this case with efficient frontier. It can be a standard with the following two constraints.

2. The second constraints is set the first item, the S&P 500 index in portfolios to be zero. After that to see how the portfolios will perform with any positive or negative effect. The additional optimization constraint:

$$w_i = 0 \tag{10}$$

3. The requirement that any weight in the portfolios be more than or equal to zero is the third constraint. The constraint represents as follow:

$$w_i \geq 0, \text{ for } \forall i \tag{11}$$

4. Result

The following Fig. 1, Fig. 2, Table 3 and Table 4 report the results.

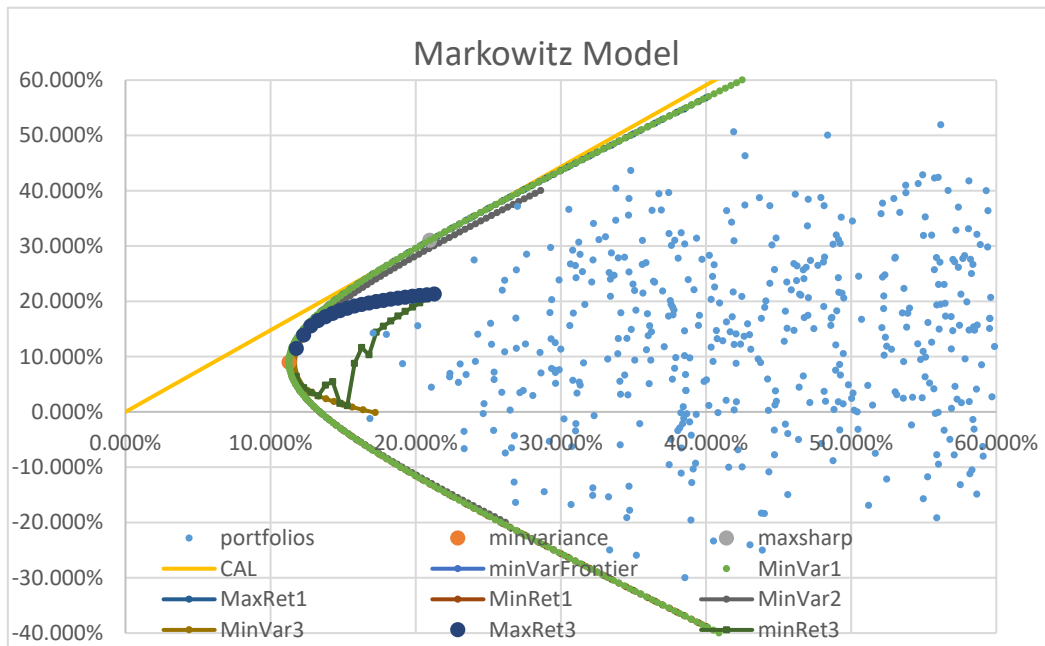


Fig. 1. Effective frontiers under different constraints for Markowitz model

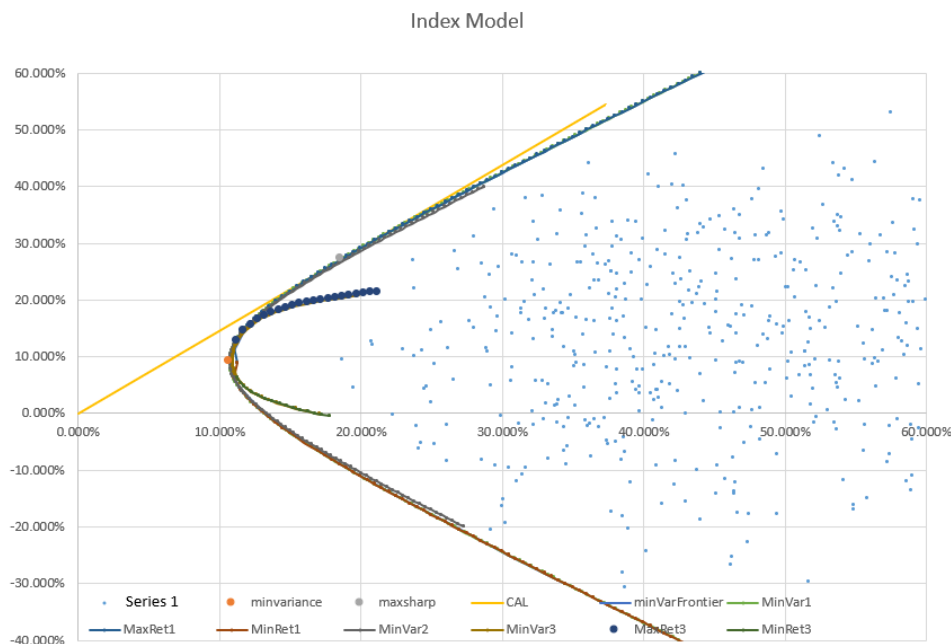


Fig. 2. Effective frontiers under different constraints for index model

Table 3. Portfolio performance of Markowitz model

	SPX	AT&T	MC D	KO	DIS	SBU X	NK E	WM T	Return	StDev	Sharpe
MM(Constr 1)											
Min variance	0.218	0.172	0.060	0.325	-0.150	0.028	0.120	0.229	8.961%	11.280%	0.794
Max sharp	0.755	-0.561	0.497	-0.506	-0.189	0.339	0.494	0.170	30.992%	20.977%	1.477
MM(Constr 2)											
Min variance	0.000	0.218	0.092	0.355	-0.111	0.041	0.154	0.250	8.389%	11.425%	0.734
Max sharp	0.000	-0.458	0.659	-0.473	-0.045	0.420	0.656	0.243	30.985%	21.957%	1.411
MM(Constr 3)											
Min variance	0.086	0.196	0.098	0.271	0.000	0.000	0.095	0.253	8.922%	11.594%	0.770
Max sharp	0.046	0.000	0.310	0.000	0.000	0.135	0.371	0.137	17.224%	13.817%	1.247

Table 4. Portfolio performance of Index model

	SPX	AT&T	MC D	KO	DIS	SBU X	NK E	WM T	Return	StDev	Sharpe
IM(Constr1)											
Min variance	-0.029	0.171	0.213	0.333	-0.084	0.070	0.079	0.247	9.053%	10.753%	0.842
Max sharp	0.478	-0.532	0.445	-0.217	-0.084	0.259	0.415	0.235	27.230%	18.649%	1.460
IM(Constr2)											
Min variance	0.000	0.167	0.208	0.327	-0.089	0.066	0.076	0.245	9.110%	10.754%	0.847
Max sharp	0.000	-0.462	0.517	-0.113	0.001	0.324	0.469	0.264	26.177%	18.230%	1.436
IM(Constr3)											
Min variance	0.000	0.152	0.192	0.305	0.000	0.047	0.060	0.244	9.406%	10.926%	0.861
Max sharp	0.000	0.000	0.328	0.000	0.000	0.184	0.310	0.177	16.845%	13.049%	1.291

By the performance, it can be found clearly that the sharp ratio operates better in Index model than in Markowitz model overall. In both two model, the line represented minimum variance, minimum return and maximum return operate nearly the same when setting constraint 1 and constraint 2 on them. It means that include a broad index into the portfolios has very little impact. However, when setting the constraint 3, variance and return operate in very different ways. In both models, the return and variance drop about around 10% compare with other two constraint which means the investors should consider the lost they will have when they set this constraint. In two models, the McDonald's take the highest weight when setting three different constraints.

5. Conclusion

The portfolio optimization become more and more important in financial market. This research focus on using Markowitz model and Index model to analysis seven different companies in four different fields which are Technology, Consumer Defensive, Consumer Cyclic and Entertainment. Meanwhile, three constraints which have significant infect are added to two models to see how these factors influence the results by comparing different companies in same constraints and different constraints in same company. Overall, the McDonald's take the highest weight when setting three different constraints and models perform worse when adding constraints on them. Investors should weight the benefits and lost when they are setting the constraints on their portfolios.

For deficiencies, this research focus on constant correlation on calculation by choosing data online but in financial market data are fluctuating and dynamic so an alternative method deserves further investigations.

References

- [1] Georgalos, K., Paya, I., Peel, D. A. On the contribution of the Markowitz model of utility to explain risky choice in experimental research. *Journal of Economic Behavior & Organization*, 2021, 182: 527-543.
- [2] Pace, R. K. A simple exposition of the Markowitz model. *Journal of Financial Education*, 1996: 65-70.
- [3] Mangram, M. E. A simplified perspective of the Markowitz portfolio theory. *Global journal of business research*, 2013, 7(1): 59-70.
- [4] Elbannan, M. A. The capital asset pricing model: an overview of the theory. *International Journal of Economics and Finance*, 2015, 7(1): 216-228.
- [5] Hristache, M., Juditsky, A., & Spokoiny, V. Direct estimation of the index coefficient in a single-index model. *Annals of Statistics*, 2001: 595-623.
- [6] Mandal, N. Sharpe's single index model and its application to construct optimal portfolio: an empirical study. *Great Lake Herald*, 2013, 7(1): 1-19.
- [7] Armeanu, D., Balu, F. O. Testing the efficiency of markowitz model on bucharest stock exchange. *Economic computation and economic cybernetics studies and research / Academy of Economic Studies*, 2008, 42(1-2): 201-217.
- [8] Ivanova, M., Dospatliev, L. Application of Markowitz portfolio optimization on Bulgarian stock market from 2013 to 2016. *International Journal of Pure and Applied Mathematics*, 2017, 117(2): 291-307.
- [9] Kamil, A. A., Fei, C. Y., Kok, L. K. Portfolio analysis based on Markowitz model. *Journal of Statistics and Management Systems*, 2006, 9(3): 519.
- [10] Vidal-García, J., Vidal, M. A note on Markowitz model. Available at SSRN 3461392, 2021.