Application Of Markowitz and Index Models to Real Markets

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Abstract. After Markowitz put forward the portfolio theory in the 1950s, the modern investment theory was developed, and more related theoretical models were created based on it. To find the optimal portfolio in the market, this paper randomly selects 7 stock types, uses three different constraint methods to show the investment characteristics of different group of investors, and uses Index model and Markowitz model to find efficient frontier and the proportion of asset allocation under the condition of maximum Sharpe ratio and minimum variance. The results show that to achieve the maximum Sharpe ratio and find the best combination, it is necessary to purchase AAPL primarily. Secondly, it chooses to purchase assets with less correlation and relatively large return rate, such as MSFT and WMT. The results in this paper shed lights for investor in selecting potential assets and may provide helpful insight for future investments.

Keywords: Modern portfolio theory; Markowitz model; Index Model; constraints.

1. Introduction

Modern investment theory (MTP) was developed in the 1950s by economist Harry Markowitz. Since then, it has changed the way of investors' investment and the way they combine assets and occupy a permanent place [1-2]. By constructing a proper investment portfolio, investors can easily achieve the goals in financial activities.

By reading the relevant literature of portfolio theory, this paper finds that most of they are focus on analysis of the model theory and optimization of the models. Myles uses the simple word form to describe how to use excel to implement Markowitz model. He thought Markowitz model has some uncertainty in efficient market, but risk of the combined portfolio always less than the not combined one [3]. Francesco Cesarone set some constraints in Markowitz model and presented that the best combination of stock will change in different constraints [4]. In additional, to study the accuracy of the Markowitz model, Ertugrul and Ayşe determined Markowitz model through historical price of stocks, and then used the Markowitz model to obtain more accurate data [5]. Sanjiv Das, on the other hand, combined Markowitz model and behavioural portfolio theory to get a new strategy model which connect the investors' consumption goals and portfolio production [6]. Most of these related articles are based on the MPT for further research and exploration but lacks intercomparison between different models.

In this paper, seven different types of stocks in the market and their stock prices in the past 19 years are randomly selected, and the stock price on the first day of each month is selected as the stock price of each month. Then the average annual return rate, Correlation index between different stocks and other desired data characteristics will be obtained by analysis the monthly prices. Finaly, using Excel data analysis tool to obtain the effective frontier and each asset allocation at maximum Sharpe ratio and minimum variance under three different constraints, and make a comparison between Markowitz Model and Index Model.

This paper's structure will be shown as below. The second part shows the method of data processing. The third part describes the theory of Markowitz model and Index model and three different constraint methods. The section four is devoted to presenting the results. Section 5 shows the relevant conclusions.

2. Data Processing

In this paper, the stock prices, which on the first day of each month from 2004 to 2023 of seven stocks SPX, Pfizer (PFE), Microsoft (MSFT), Apple (AAPL), Procter & Gamble (PG), Walmart (WMT), and Coca-Cola (KO) are selected from Yahoo website (https://finance.yahoo.com/). By analysing the stocks' historical price data, the annualized return rate, annualized variance, and annualized residual of each stock are obtained and displayed in Table 1.

Table 1. Data Characteristics

	SPX	PFE	MSFT	AAPL	PG	WMT	KO
ANNUALIZED AVERAGE REUTRN	6.598%	0.655%	14.153%	34.658%	4.930%	5.075%	3.882%
ANNUALIZED STDEV	15.419%	21.448%	22.670%	33.481%	15.423%	16.818%	15.801%
ANNUALIZED RESIDUAL STDEV	0%	18.118%	17.012%	27.073%	13.667%	15.643%	13.236%

The data in the table shows that the annual return rate of AAPL is the highest, which also has the largest risk. SPX has the least risk and more satisfactory returns. In contrast, PFE has the lowest return and the third highest risk. However, the returns and variances of PG, WMT and other stocks have little difference in specific numerical performance. The variances of them are significantly larger than the returns.

3. Method

3.1. Markowitz Model

The default rational investor in the model will choose the most efficient portfolio type, which means maximize the return with the given risk [7]. Markowitz believed that it is possible to determine the optimal portfolio of risky securities given the forecast of future returns and the covariance matrix of stock returns [8]. So Markowitz model uses the mean of portfolio variance to define return and risk respectively and calculates the "mean-variance efficient portfolio" [9]. By calculating and measuring the risk and return of different asset portfolios, and then imposing three different constraints on this, the efficient frontier model of the maximum return rate portfolio under the same risk can be obtained. The variance's equation is shown as below:

$$Var(r_p) = \sum_{i=1}^{n} w_i^2 Var(r_i) + \sum_{i \neq j} w_i w_j Cov(r_i, r_j) \tag{1}$$

The standard deviation is expressed as follows:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 Var(r_i) + \sum_{i \neq j} w_i w_j Cov(r_i, r_j)} \tag{2}$$

The expected return on the portfolio is:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \tag{3}$$

Where r_i and w_i are the average annual return rate and weight of the selected assets.

3.2. Index Model

The Index model was proposed by William Sharp. The author believed that stock returns were only affected by a single factor. And when the market valuation index rises, the price of most stocks also rises, and when it falls, the price of most stocks also falls. Compared with the Markowitz model, this model greatly reduces the number of covariances, which greatly reduces the data and computation requirements [10].

The basic formula of Index model show as follows:

$$r_i = \alpha_i + \beta_i F + \varepsilon_i \tag{4}$$

F is the predicted value of the common factor, which is expressed by the average annual return of SPX stock in this paper. β_i is the sensitivity of security i to the factor, and in this paper, this is the sensitivity of security i to SPX stocks. The factor α_i uses SPX monthly stock prices as X-axis data and class i stock prices as Y-axis data, and the y-intercept of the function is obtained after linear regression. Since random errors are not considered in this paper, $\varepsilon_i = 0$. So, the return of the Index model can be transformed as follows:

$$E(r_p) = w_i \beta_i F + w_i \alpha_i \tag{5}$$

The variance is:

$$\sigma_p = \sqrt{(w_i \beta_i F)^2 + (w_i \delta_i)^2} \tag{6}$$

Where δ_i denotes the residual error. The residual rate of return of the corresponding stock each month is obtained by the formula $r_i - r_1 \times \beta - \alpha/12$. Then the annualized residual of each stock is obtained by using the residual return of each month.

3.3. Constraints

Constraints represent the different choices and views of different investors on the overall asset allocation when investing. Three different portfolios are adopted in this paper.

The first constraint means that the absolute value of the weight of each asset in portfolio cannot be greater than 1.

$$abs(w_i) \leq 1 \tag{7}$$

The second constraint means that all assets are not allowed to have short sales.

$$w_i \geq 0 \tag{8}$$

The third constraint is that there is no constraint.

4. Result

The correlation coefficients between the seven stocks are obtained, and the results are shown in the Table 2 below.

Table 2. Correlation coefficients between stocks

CORREL	SPX	PFE	MSFT	AAPL	PG	WMT	KO
SPX	1	0.5352	0.6610	0.5883	0.4634	0.3672	0.5462
PFE	0.5352	1	0.3054	0.1589	0.4145	0.2261	0.3646
MSFT	0.6610	0.3054	1	0.4718	0.2863	0.2404	0.3883
AAPL	0.5883	0.1589	0.4718	1	0.2685	0.1442	0.2516
PG	0.4634	0.4145	0.2863	0.2685	1	0.3408	0.5332
WMT	0.3672	0.2261	0.2404	0.1442	0.3408	1	0.3356
KO	0.5462	0.3646	0.3883	0.2516	0.5332	0.3356	1

It can be seen from the table that SPX and MSFT have the highest correlation, which means that there is some influence relationship between them, and it is possible for further research. AAPL and WMT have the lowest correlation among all assets, which means that investors can realize risk hedging by buying these two assets at the same time and reduce the risk generated when investing.

The following Fig.1, Fig. 2 and Fig. 3 are efficient frontiers of the Markowitz model and the Index model investment under three different constraints. The detailed asset allocations are shown in Table 3.

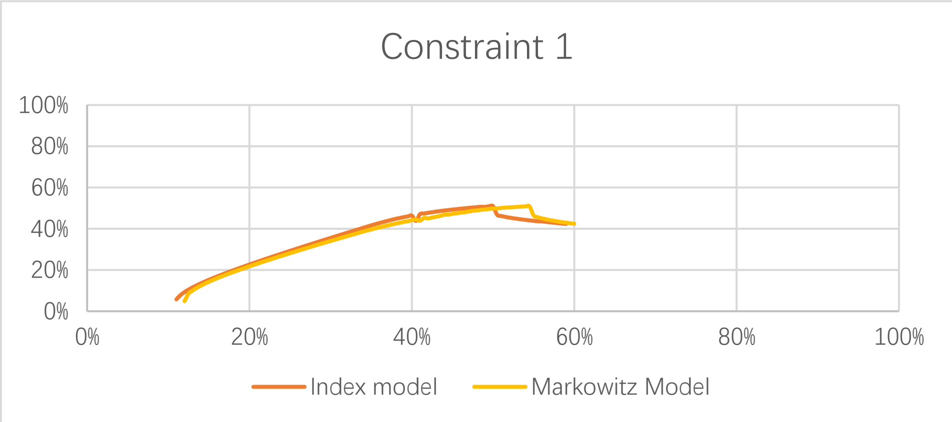

Fig. 1 Markowitz Model and Index Model with Constraint 1

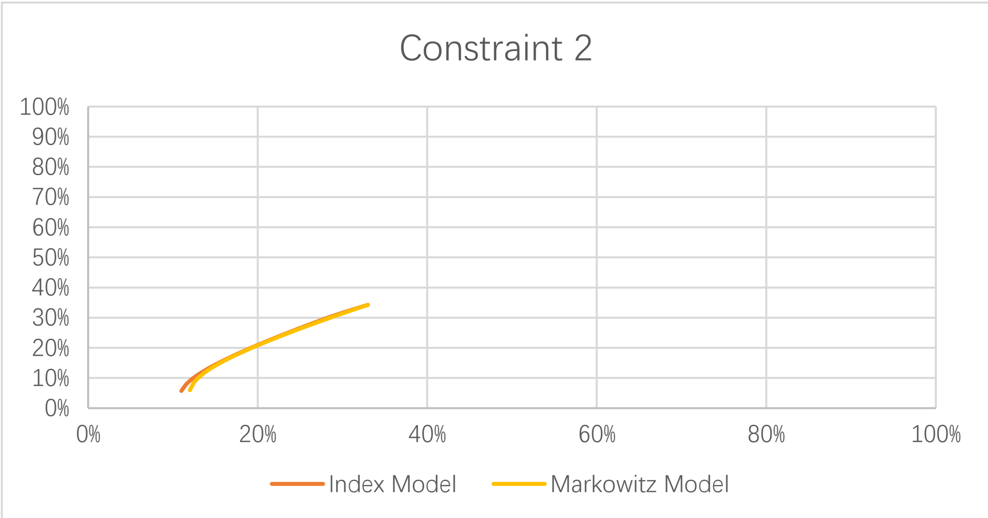

Fig. 2 Markowitz Model and Index Model with Constraint 2

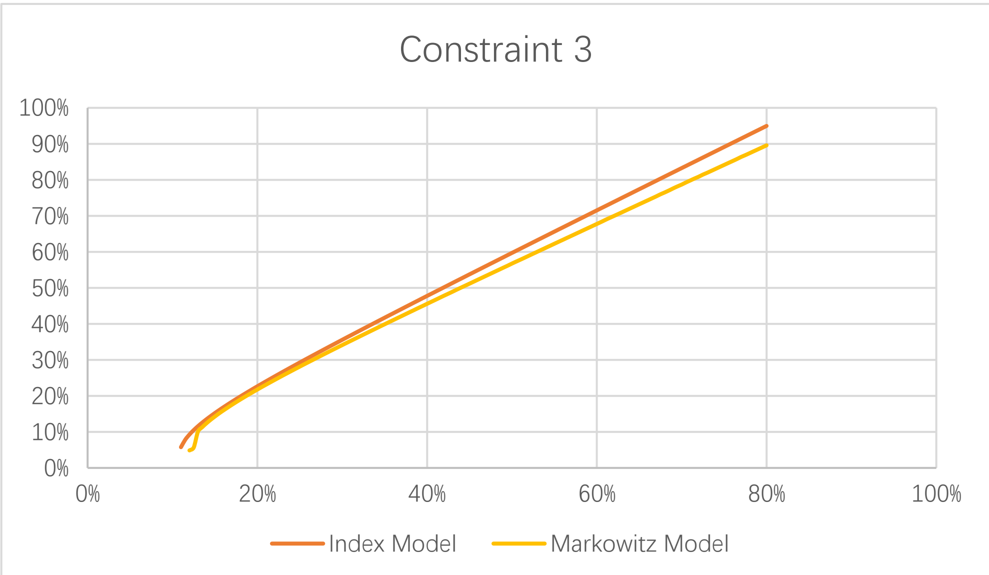

Fig. 3 Markowitz Model and Index Model with Constraint 3

Table 3. Asset allocation with maximum Sharpe ratio and minimum variance

constraint 1	MM						
	MaxSharpe=113.72%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	-1	-0.098	0.499	0.963	0.226	0.399	0.011
	MinVar=11.92%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0.245	0.036	0.023	-0.025	0.266	0.267	0.187
	IM						
	MaxSharpe=118.98%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	-1	-0.411	0.746	1	0.316	0.319	0.031
	MinVar=10.77%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0.022	0.086	0.018	-0.034	0.333	0.284	0.289
constraint 2	MM						
	MaxSharpe=105.41%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0	0	0.150	0.673	0	0.177	0
	MinVar=11.94%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0.211	0.050	0.011	0	0.260	0.272	0.197
	IM						
	MaxSharpe=106.34%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0	0	0.271	0.680	0	0.049	0
	MinVar=10.84%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0	0.080	0.018	0	0.332	0.285	0.285
constraint 3	MM						
	MaxSharpe=114.01%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	-1.399	-0.050	0.607	1.098	0.242	0.449	0.054
	MinVar=11.92%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0.245	0.036	0.023	-0.025	0.266	0.267	0.187
	IM						
	MaxSharpe=119.46%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	-1.603	-0.477	0.985	1.318	0.370	0.367	0.041
	MinVar=10.77%						
	SPX	PFE	MSFT	AAPL	PG	WMT	KO
	0.039	0.092	0.009	-0.048	0.333	0.284	0.292

From the three constraints, the efficient frontier of Markowitz model and Index model are basically same. There is a little difference between the risks and returns of the two models for investment decisions. By comparing the maximum Sharpe ratio and minimum risk under the three constraints, the investment willingness and conditions of different investors affect the investment risk and returns. The Sharpe ratio of the portfolio under the third constraint has better investment return than the other two investment methods. In constraint one, the ratio of SPX stock short and AAPL' s weight is 0.96. In the case of large correlation coefficient, also the risks of the two are basically similar, but AAPL has higher returns. So little difference in risk, to maximize the return, investors need to buy more AAPL shares. The other two constraints to achieve the maximum Sharpe ratio are also to buy more AAPL and sell or not buy SPX and similar stocks. The appropriate selling the stocks with large risk, small returns and large correlation coefficients in the portfolio can bring higher returns and lower risks to the portfolio.

5. Conclusion

In this paper, Markowitz model and Index model in modern portfolio theory are selected to optimize the portfolio of the same stock with three different constraints. According to the obtained results, the efficient frontier obtained by the two models under the three constraints are mostly similar. The results of the Index model show that it can get higher returns under the same risk. But in practice, the Markowitz model is more accurate than Index model, and the Index model is easier to get relevant data than the Markowitz model. In the case of ensuring the maximum Sharpe ratio, the proportion of AAPL assets in the asset allocation obtained by the two models under three different constraints is always the highest, and the remaining two assets with large proportions are MSFT and WMT. In the correlation table, the correlation index between the three is low and annual return rate of these three assets is the highest. This proves that risk hedging can be carried out by means of asset portfolio, to further improve the expected return of assets and reduce the corresponding risk.

However, since this paper assumes that the correlation between the selected stocks is constant for analysis, but in the actual situation, the correlation between different stocks will change with time, so a more accurate portfolio model is worth further exploration.

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