Research on sustainability of property insurance based on insurance stochastic model

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Abstract. This paper analyzes the sustainability of property insurance under extreme weather events using an insurance stochastic model in Europe and South America. First, disaster risk losses in Estonia and Mexico City were assessed and modeled using the Kramer-Lundberg risk model. Then, the approximate result of bankruptcy probability under specific initial capital distribution is obtained by parameter adjustment and calculation. The study found that in order to maintain a 10% profit, premiums in high-risk areas would have to be 4.6 times higher than those in medium-risk areas. Therefore, the price of buying insurance in Estonia must be 4.6 times higher than in Mexico City to remain profitable. This paper provides decision support for insurance companies in changing environment to ensure long-term stable development.

Keywords: Insurance stochastic model; Probability density function; Grey level analysis.

1. Introduction

At a time when climate change and extreme weather events are becoming more frequent, this article explores how insurance companies are responding to this challenge, particularly in developing risk assessment and premium pricing strategies. We discussed how to balance sustainability and profitability of insurance products, while also focusing on how to protect community heritage with cultural and historical value. By analyzing insurance demand and pricing strategies in different risk areas, and conducting sensitivity analysis to understand the impact of key variables on insurance products, this article aims to provide decision-making support for insurance companies in a changing environment and ensure their long-term stable development.

This paper aims to solve the following problems: Two continents, Europe and South America, are selected as two regions experiencing extreme weather events to demonstrate our model. The disaster risk loss map of Estonia and the loss of Mexico City are then evaluated using the Cramr-Lundberg risk model.

2. Main modules and financial perspectives of modeling

2.1. Definition

All these definitions are very important to master the basics of modeling

Average Annual Loss (Pure Premium) - The average of the distribution of losses or expected annual losses is called the average annual loss. It is estimated that annual premiums will be required to cover losses from simulated risks over time.

Probable Maximum Loss (PML) - The maximum value of loss in the event of a disaster event is called the Possible Maximum Loss. Assume that all active protection functions fail (e.g., in an earthquake, failure of a sprinkler system may cause greater damage than its availability).

Return Period - In very common terms, the return period is the inverse of the probability and explains that the event will be exceeded in any year. It is a statistical measure of historical data that represents the average recurrence interval over a period of time.). For example, a 10-year flood has a $0.02\ \ 1/10 = 0.1$or 10%chance of being exceeded in any year, while a 50-year flood has a 0.02 or 2%chance of being exceeded in any year.

$$T = 1/p = (n+1)/m \tag{1}$$

Among them, T =return period, p =probability of event occurrence n =, number of recording years, and m =number of recorded events

Excess Probability (EP) - It explains the probability that different levels of losses will be exceeded. The excess probability curve is called the EP curve. For example - The probability of a storm exceeding the standard is 2%. So this means that there is 2%a possibility that the loss will exceed a certain level.

Aggregate Excess Probability (AEP) - AEP shows the probability that total annual losses will reach a specified amount or greater.

It provides information assuming one or more losses occur during a year.

It is useful for aggregate-based structures such as stop loss, recovery, etc. AEP(>= OEP)

Out-of-Occurrence Probability (OEP) - OEP shows the probability of any single event occurring within a given period for a specific loss size or greater.

It provides information on losses assuming a single event occurs in a given year.

It is useful for event-based structures such as quota shares, excess work, etc.

Event Loss Table (ELT) - ELT generates raw data that can be used to construct EP curves and calculate other risk measures. Generally speaking, an ELT is a set of events and the estimated modeling loss for each event.

Deductible - The portion of an insurance claim paid by the insured is called the deductible or the insured's retainer.

Base Loss - Total loss before taking into account any retention, deductible or reinsurance. The basic loss is the policyholder's loss.

Total Loss - The total financial losses of an insurance company.

2.2. Input and output

Enter:

Input data of a building is required to estimate its damage. These given information is limited as data requirements may vary depending on the risk (e.g. flood, storm, and earthquake).

Geocoding data - street address, zip code, county/CRESTA area, etc.

Key property information (exposed physical features) Construction, occupancy, year built, number of floors

Secondary Attributes - Roof Type, Building Square Feet (Area)

Hazards - eg - soil type, distance from coast (for flood insurance)

Coverage limits or policy conditions - deductibles, sum assured, levels, limits and reinsurance covenants

Coverage - building, content, time elements (business interruption and expense coverage)

Hazards - Floods, storms (hurricanes or storms), earthquakes, tornadoes, winter storms (snow, ice, freezing rain), wildfires, and tsunamis

Output (financial outlook)

Insurance and reinsurers are interested in excess probabilities (AEP and OEP) and event loss tables to calculate different perspective levels (e.g. ground, gross, net preclaims and net postclaims). They also want to know the probable maximum loss (PML) and average annual loss (AAL) of their

portfolio. It can be calculated based on the loss distribution. It helps insurance companies collect premiums and underwrite their premiums in high-risk areas.

2.3. Basic concepts of pricing

The basic concept and process of pricing is shown in Figure 1.

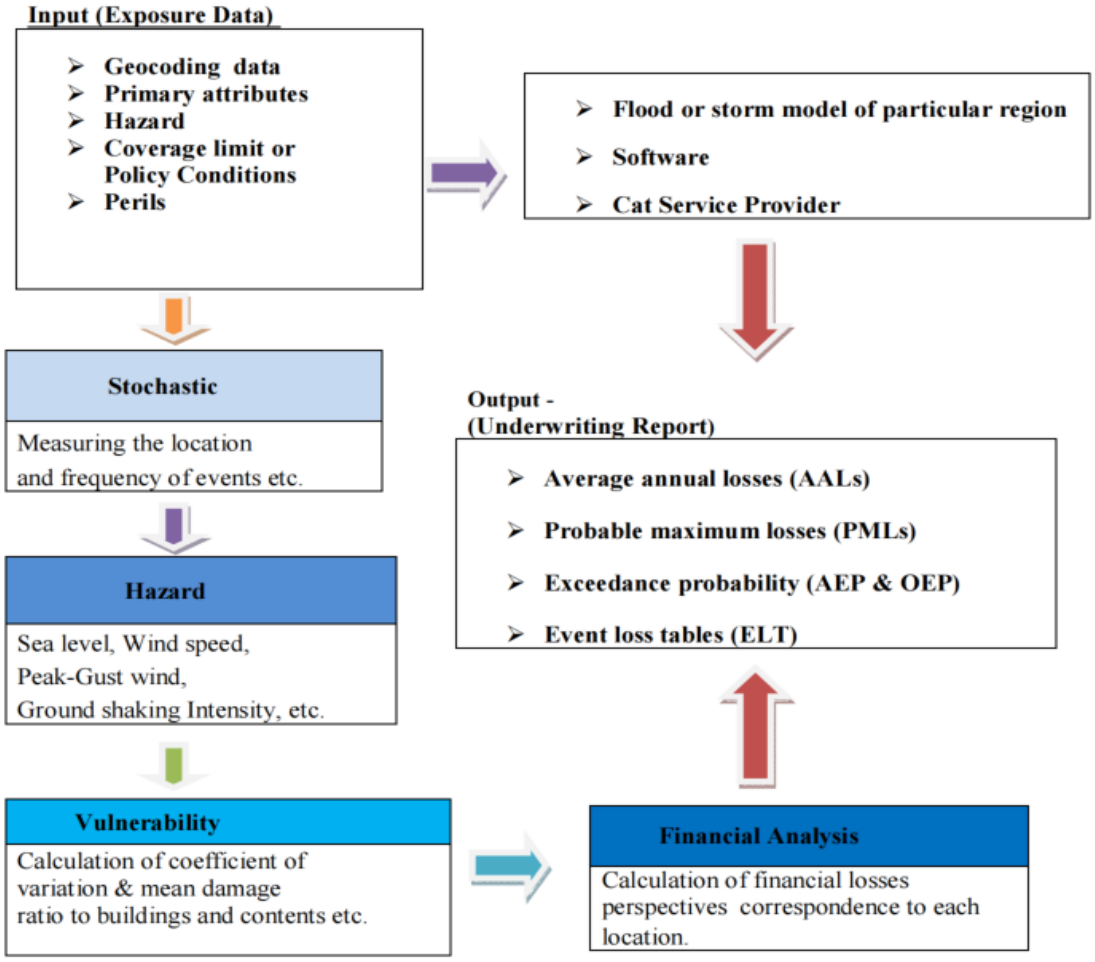


Figure 1. Basic concepts and processes of pricing.

2.4. Hazard module

We will choose two continents, Europe and South America, as regions that experience extreme weather events to demonstrate our model. The hazard module estimates the potential disasters and their frequency. Whenever wind speed reaches to its heavy level and getting ($\geq 33m/s$or ≥ 74mph) a form of hurricanes. In this case intensity parameters (wind speed, pressure, forward velocity, radius of maximum wind etc.) are modeled using complex mathematical equations. Windstorm model will not simulate just only historical windstorms already occurred but also simulate a much larger number of storms. Mostly windstorm models are derived from 10,000 of stochastic storms. Each event is modeled using the exposure data. Basically, it depends on the location of building (ie a hurricane occurring in Pärnu does not impact on the building situated in Harju County) no impact in this case. However, it may have some impact if the building is close to the hurricane path. The windstorm model equations allow the model to estimate the wind speed and a frequency at the building location, for each windstorm and its intensity parameters. Intensities from all computed events give the probabilistic distribution of wind speeds at the structure location. This is sent to the engineering module where the probability distribution of the corresponding damage will be derived. Distribution of wind speed probability in Estonia (Figure 2).

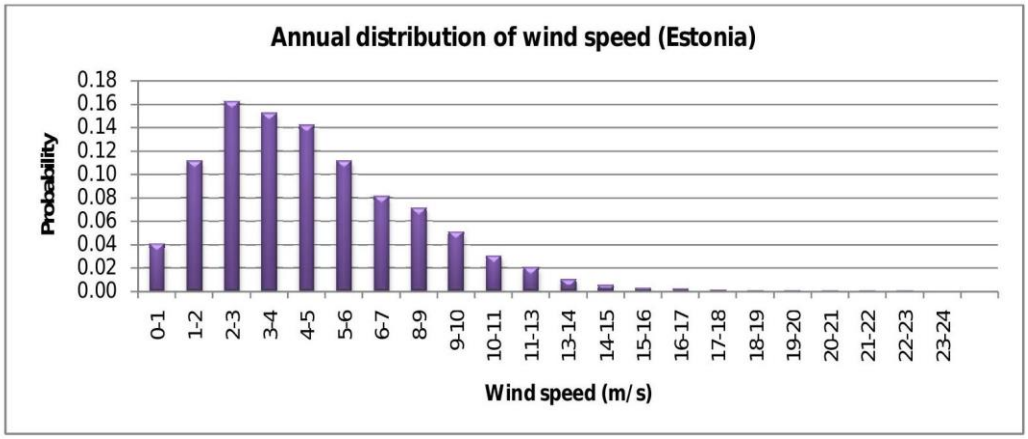


Figure 2. Output of hazard module.

We have updated the disaster risk loss maps for different regions as follows (Figures 3 and 4).

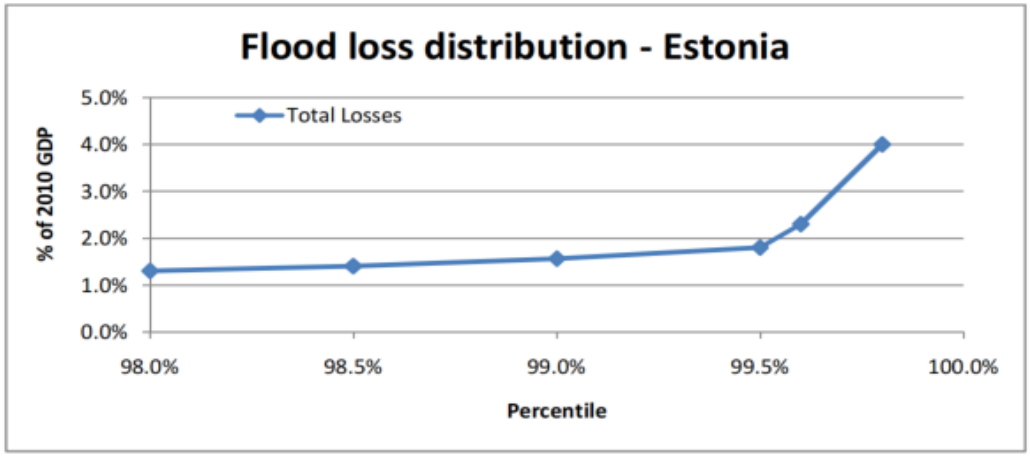


Figure 3. Flood losses suffered by Es tonia.

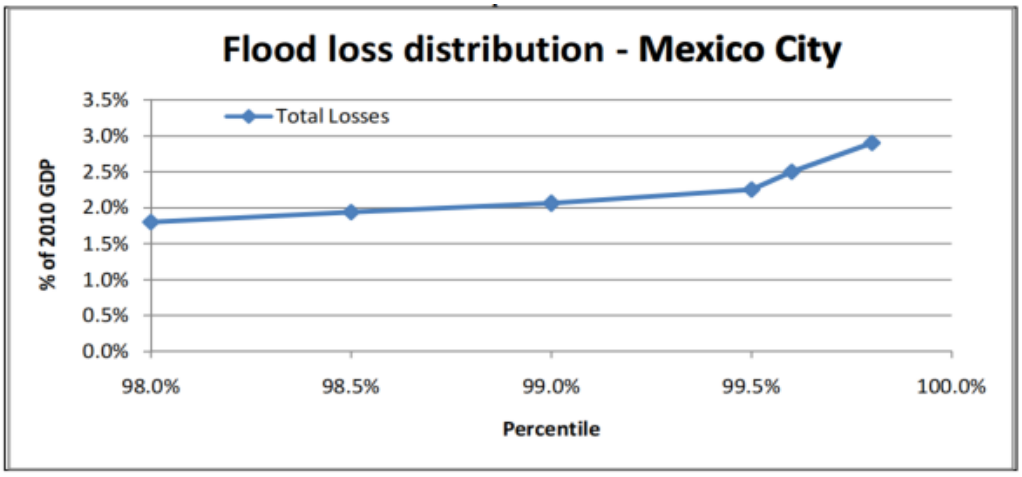


Figure 4. Flood loss suffered by Mexico City _

We can be seen from the comparison of pictures, Estonia is a place with a higher probability of disaster than Mexico City. However, there are many factors which can cause damage to the building but the main feature of a building proves to be a good indicator of its vulnerability and damage ratio. The ratio of the cost to repair a building or content, to the cost of rebuilding it, is known as damage ratio. Damage ratio of a building is a function of wind speed (v).

Damage ratio

$$\left(\mathrm{DR}_B(v)\right) = \text{Cost to repair of damaged building/replacement cost of building} \quad (2)$$

Where, Replacement cost of building = Replacement value is the actual cost to replace an item or structure at its pre-loss condition

$\boldsymbol{B}$ =Building which we are analyzing

$\boldsymbol{v}$ =Wind speed

As quiet often, all the buildings have small differences in construction, occupancy, number of stories and local site. So, when the same intensity of wind speed hit to two identical buildings. It faces different levels of damage and major differences in losses [1-3]. To find this variability in damage and losses, it is better to concentrate on the whole distribution of possible values of the damage ratio not only a single value. The mean of this distribution is called as the mean damage ratio. Mean damage ratio is expectation of damage Ratio $\left(\mathrm{DR}_B(v)\right)$.

$$\text{Mean damage ratio}\left(\mathrm{MDR}_B(v)\right) = \text{Average loss}/\text{Replacement value} \tag{3}$$

Uncertainty in building damage ratio is a reflection of the variance of Damage ratio

$$[\sigma_{\mathrm{DR}}(v)]^2 = \mathrm{Var}[\mathrm{DR}_v] \tag{4}$$

Where, $\sigma_{\mathrm{DR}}(v)$ =standard deviation

2.5. Financial Module

To compute the loss distribution of damage, which is done to the building by windstorm, is a part of financial module. While doing all the calculation in this module all the policy conditions of insurance should be remembered because it is also incorporated in it. The damage ratio distribution calculated from the vulnerability module for a windstorm is multiplied by the building replacement value to compute the loss distribution. Sometimes convolution proves the key to compute financial loss distribution [4]. The combined loss distribution of all buildings can be calculated by convolution method. Let us assume that two locations A and B, for each event has loss distributions l_i and l_j respectively. So all the possible combinations of loss distributions $1_i + l_j$ to their correspondence probabilities, given the probability distributions of 1_i and l_j separately can be calculated by convolution method. Let, L defines the total loss for two locations then probability distribution for two locations can be shown as

$$P(L) = \sum_{P_1(l_i)\times P_2(l_j)} P_1(l_i) \times P_2\left(1_j\right) \tag{5}$$

Where, $P(L)$ =Total Probability distribution of both the locations

$P_1(l_i)$ =Probability distribution of location A

$P_2\left(l_j\right)$ =Probability distribution of location B

In this way by using convolution method, if we find two loss distributions for the two locations then the range of the resulting loss distributions is equal to the sum of the ranges of loss distributions separately.

Usually, insurance risk stochastic models mainly consist of three basic stochastic processes[5-7]: (i) insurance revenue process $\{U(t), t \geq 0\}$, U_t which represents the total premium received during the period (a stochastic process with time parameters); (ii) counting process of claim arrivals $0, t\{N(t)\}$, N_t represents the total number of claims that have occurred (a random process of time parameters); (iii) the claim amount sequence $0, t\{X_i\}_{i=1}^{\infty}$, X_i represents i the amount of the first claim. In theoretical research and practical applications, people are concerned about the surplus process:

$$S(t) = x + U(t) - \sum_{i=1}^{N(t)} X_i, t \geq 0 \tag{6}$$

It represents the insurance company's t surplus or accumulated capital at a time, where x is the insurance company's initial reserve, the insurance fee income process is $\{U(t), t \geq 0\}$ a decisive function of $\{X_i\}_{i=1}^{\infty}$ time, the claim amount sequence t is an independent and identically distributed

random variable sequence describing catastrophe claims, and the general assumption Independent from the claims arrival counting process $\{N(t)\}$.

This article assumes that the insurance revenue process$U(t) = ct$, the constant $c$represents the insurance rate; the claim amount $\{X_i\}_{i=1}^{\infty}$has a common distribution function $F$and finite expectation; $\mu > 0$the number of claims in $N(t)$the time interval $[0, t]$is a $\{X_i\}_{i=1}^{\infty}$homogeneous Possion process with independent intensity.$\lambda > 0$

In addition, since insurance operations will be affected and interfered by various factors during the insurance operation process, this article adds an interference process to the model (6), $\{W(t)\}_{t\geqslant 0}$and assumes that the interference process is a standard Brownian motion, and its distribution for

$$\mathrm{P(W(t) \leq u)} = \Phi\left(\frac{\mathrm{u}}{\sqrt{2\mathrm{Dt}}}\right) \tag{7}$$

Among them $D > 0$, Φ is the standard normal distribution. Therefore, the surplus process (8) becomes

$$\mathrm{S(t)} = \mathrm{x} - \sigma(\mathrm{t}) \tag{8}$$

In

$$\sigma(t) = \sum_{i=1}^{N(t)} X_i - ct - \alpha W(t) \tag{9}$$

$\alpha \geq 0$Is a constant and is called the Cramr-Lundberg risk model with interference. $\{\sigma(t)\}_{t\geq 0}$In particular, if$\alpha = 0$, the model is the standard Cramr-Lundberg risk model, which is a spectrally positive process with stationary independent increments and non-negative jumps. Without loss of generality$c = 1$, it is further assumed that relative safe load conditions

$$\rho = (\lambda\mu)^{-1} - 1 > 0 \tag{10}$$

Established. Under this condition, it is generally believed that when the surplus capital of an insurance company becomes negative at a certain moment, the insurance company is considered bankrupt. Therefore, $x$the final bankruptcy probability of an insurance company under the condition that the initial capital is can be defined through the risk process $\{\sigma(t), t \geq 0\}$as

$$\Psi(x) = P(\sigma(t) > x, \text{for } t) = 1 - R(u) \tag{11}$$

Here $R(u)$ represents u the final survival probability of the insurance company under the condition that the initial capital is.

We note the exponential distribution function

$$E(x) = \begin{cases} 1 - \exp\left(-\frac{x}{Da^2}\right), & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{12}$$

And use the convolution $A(x)^{*}B(x)$to $B(x)$represent $A(x)$the sum, $A^{*n}$and $n$the reconvolution $f(u) \sim g(u)$ to represent the notation: A represents the real function $f(u)$, $g(u)$ which satisfies $\lim_{u\to\infty} f(u)/g(u) = 1$.

If the distribution of catastrophe claims $F$is a standard lognormal distribution, that is, the density function of its distribution is

$$f(x) = \frac{1}{\sqrt{2\pi x}}\exp(\ln^2 x), x > 0 \tag{13}$$

This distribution$F \in S^{*}$, which is the famous large claims distribution in actuarial mathematics. After calculation: $\mu = \sqrt{e} = 1.64872$ its tail distribution $\overline{F}(x) = 1 - \Phi(\ln x)$, where Φ is the standard normal distribution function. Therefore, there is an equilibrium distribution:

$$\begin{aligned} F_e(x) &= \frac{1}{\mu}\int_0^x \left(1-\Phi(\ln t)\right)dt \\ &= \frac{1}{\mu}\int_0^x \left(1-\frac{1}{\sqrt{2\pi}}\int_\infty^{\ln t} e^{-\frac{u^2}{2}}du\right)dt \\ &= \frac{1}{\mu}\left[x-\frac{1}{\sqrt{2\pi}}\int_0^x\int_\infty^{\ln t} e^{-\frac{u^2}{2}}dudt\right] \\ &= \frac{1}{\mu}[x-x\Phi(\ln x)+\sqrt{e}\Phi(\ln x-1) \\ &= \frac{x}{\sqrt{e}}[1-\Phi(\ln x)+\sqrt{e}\Phi(\ln x-1) \end{aligned} \tag{14}$$

Therefore, get

$$\Psi(x) \sim \frac{1}{\rho}\left[1-\frac{x}{\sqrt{e}}[1-\Phi(_{\ln}x)]-\Phi(_{\ln}x-1)\right] \tag{15}$$

Below we give a specific result:

We define high-risk areas as having a 10% probability of a disaster occurring each year, and medium-risk areas as having a 5% probability of a disaster occurring each year [8-10]. Expected losses and necessary premium levels can be calculated for each risk level area. Expected losses are the average losses that are likely to occur each year and can be calculated by multiplying the probability of a disaster occurring by the average claim cost.

Next, let's calculate the expected losses for each area and determine the premium levels that insurance companies should set. We will assume that the insurance company wants to earn a profit margin of at least 10%. Assume initial capital of 1 million euros.

According to the calculation results, insurance companies should set the following premium levels for areas with different risk levels to maintain profitability and sustainability: assuming parameters $\lambda = 0.1$and 0.5 in model (10) respectively, the probabilities of no catastrophe at this time reach 0.9048 and 0.6065 respectively., so the safe load factor and 0.2131 can be calculated respectively$\rho = 5.06531$, and substituted into (4.10), the specific calculation formula can be obtained:

$$\Psi_1(x) \sim 0.197421(1-\Phi(_{\ln}x-1)-0.119742x[1-\Phi(_{\ln}x)] \tag{16}$$

And

$$\Psi_2(x)-4.692633(1-\Phi(_{\ln}x-1)-2.846277x[1-\Phi(_{\ln}x)] \tag{17}$$

Table 1 gives approximate results of the sum of $\Psi_2(x)$bankruptcy probabilities $\Psi_1(x)$for some specific initial capital distributions$x$.

Table 1. The ruin probability of different initial capital.

Initial capital (Euros)	$\ln x$	$\Phi(\ln x-1)$	$\Phi(\ln x)$	$\Psi_1(x)$Approximate result of	$\Psi_2(x)$Approximate result of
10	2.3025	0.90320	0.98928	1.78267×10^{-2}	1.49126×10^{-1}
20	2.9957	0.97725	0.998650	1.25829×10^{-3}	2.99079×10^{-2}
40	3.6889	0.996427	0.9998787	1.24397×10^{-4}	2.95664×10^{-3}
60	4.0943	0.998999	0.99997843	4.26483×10^{-5}	1.01367×10^{-3}
80	4.3820	0.9996376	0.999994066	1.47014×10^{-6}	3.49426×10^{-4}
100	4.6051	0.9998469	0.999997897	5.04341×10^{-6}	1.19870×10^{-4}
120	4.7875	0.99992468	0.999999166	2.88740×10^{-6}	6.86279×10^{-5}
140	4.9416	0.99995926	0.999999609	1.49496×10^{-6}	3.28309×10^{-5}
160	5.0752	0.99997748	0.999999860	1.19277×10^{-6}	2.83504×10^{-5}
180	5.1929	0.99998605	0.999999900	6.07288×10^{-7}	1.44342×10^{-6}
200	5.2983	0.99999106	0.999999942	3.77147×10^{-7}	8.96410×10^{-6}

It can be seen from Table 1 that the more the initial capital of an insurance company, the smaller the probability of bankruptcy in its catastrophe insurance business. In other words, the more initial capital an insurance company has, the stronger its ability to deal with catastrophe risks. It is calculated that the premium in high-risk areas must be 4.6 times that of medium-risk areas to maintain the insurance company's 10 % profit. Otherwise, it is wise to refuse insurance. In other words, when buying insurance in Estonia, the price is

The price must be 4.6 times that of Mexico City to be profitable.

3. Conclusions

This paper analyzes the sustainability of property insurance under extreme weather events using an insurance stochastic model from Europe and South America. The results show that in order to maintain a 10% profit, premiums in high-risk areas must be 4.6 times higher than those in medium-risk areas. Therefore, the price of buying insurance in Estonia must be 4.6 times higher than in Mexico City to remain profitable. This provides decision support for insurance companies to ensure long-term stability in a changing environment. The results of this study have important implications for insurance companies to develop premium pricing strategies and help strike a balance between sustainability and profitability.

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