Research on cryptocurrency portfolio based on Markowitz model

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Abstract. In recent years, the booming growth of the cryptocurrency market has garnered significant attention from a diverse range of investors with the overall market capitalization experiencing continuous expansion. This surge in popularity, however, is accompanied by challenges arising from the market's inherent high volatility and complexity. The increasing diversity of available assets presents investors with a complex landscape to navigate. The quest to build an optimal investment portfolio has become a focal point, particularly as market participants seek strategies that not only capitalize on the market's potential for substantial returns but also effectively manage the inherent risks. In this paper, nine cryptocurrencies have been chosen to construct the portfolios, which include BTC, Ether, Tether, Binance, ripple, USDC, ADA, DOGE and TRX. The data selected comprises the daily price of the above assets from November 1, 2020, to October 9, 2023. This paper presents a comprehensive framework for constructing and optimizing cryptocurrency investment portfolios, by employing the Markowitz model and Sharpe ratio analysis. We examined these nine selected digital currencies to determine the optimal investment allocation. Our findings suggest that the highest sharpe ratio is achieved when each investment cryptocurrencies (according to the order above) allocates 0, 0.003, 0.762, 0.038, 0.005, 0.172, 0, 0.014 and 0.007, respectively, to each cryptocurrency. And the lower risk portfolio is obtained when each cryptocurrencies accounted for 2.740e-20, 6.315e-20, 0.5, 0, 0.001, 0.499, 0, 1.419e-04 and 4.210e-20, respectively. This study contributes to the sustainable development of the cryptocurrency market by providing valuable insights for financial institutions, policymakers, and other stakeholders. However, limitations of this research come from the small number of selected digital currencies and the exclusion of risk-free assets such as government bonds. Future research should consider expanding the sample size and incorporating risk-free rates to enhance the applicability and reliability of investment strategies. In conclusion, this research offers investors a practical framework to navigate the complexities of the cryptocurrency market and optimize their investment decisions for long-term value appreciation.

Keywords: Cryptocurrency, digital currency, investment.

1. Introduction

1.1. Introduction to cryptocurrency

Cryptocurrency is defined as a form of digital currency utilized for secure transactions, underpinned by the blockchain technology. Unlike conventional currencies governed by governmental or central banking authorities, cryptocurrency operates within a decentralized network of computers, which is known as nodes, affording it the trait of global accessibility. This decentralized structure extends opportunities to individuals who may not have access to traditional banking services. Cryptocurrency functions on blockchain technology, wherein transaction data including time and amount is securely recorded on a central ledger, forming an unalterable and sequential list of records, known as blocks. These blocks are interconnected through hash pointers, facilitating the verification of any alterations to preceding blocks. Every transaction which intends to be done on the blockchain requires to be verified by several nodes before it has been added to the entire blockchain. Cryptography is essential for the cryptocurrency transaction, mainly applying the utilization of the public and private key. Each user in cryptocurrency network posses a pair of keys, a public key and a private key, for its account. When a user initiates transaction, the private key will give the permission to this user to generate the digital signature, and the others will use the public key of this user to ascertain the validity of the signature. These inherent features ensure the immutability of the transaction history, bolstering the security of cryptocurrency against tampering by external entities.

Furthermore, the issuance of new units of cryptocurrency often follows a predetermined schedule or algorithm, engendering a transparent and predictable supply mechanism. Additionally, the highly divisible nature of many cryptocurrencies permits transactions involving fractional units, enhancing their utility and facilitating micro-transactions. Moreover, the pseudonym of users within the cryptocurrency ecosystem enhances privacy, although this varies across different cryptocurrencies [6]. Finally, the decentralized and open-source nature of many cryptocurrencies fosters innovation and collaboration, allowing for the continuous development of the cryptocurrency landscape.

1.2. History of cryptocurrency

Before Bitcoin, there was no "real" cryptocurrency in the world. In 1983, American cryptographer David Chaum introduced the concept of a cryptocurrency called ecash, marking the first introduction of the idea of digital currency. He subsequently executed ecash transactions using an electronic trading system known as Digicash, which he had created earlier. Digicash ensured that third parties could not track the digital currency. In 1998, computer engineer Wei Dai published an paper titled "B-Money," and concurrently, Nick Szabo proposed the idea of Bit Gold. Although these two electronic currencies were not launched into the market, they played essential roles in the birth of Bitcoin. Several years later, the world witnessed the creation of the first genuine cryptocurrency, Bitcoin, in January 2009. Developed by the anonymous individual or group Satoshi Nakamoto, Bitcoin utilized blockchain technology, preserving high security and incorruptibility. It has since expanded and evolved. In the subsequent years, the development of cryptocurrencies has undergone periods of growth and setbacks, leading to the creation of numerous other cryptocurrencies.

1.3. Background and significance

The emergence of cryptocurrencies introduced features such as decentralization, security, transparency, and fast transactions, which had a profound impact on the traditional financial system. At their core, cryptocurrency can represent a form of decentralized and encrypted medium of exchange that relies on blockchain techniques for secure financial transactions. This paper aims to provide a comprehensive exploration of the multifaceted aspects of cryptocurrency, traversing their historical evolution, current status, and the pivotal role they play in investment portfolios.

The journey of digital currencies traces back to the inception of Bitcoin in 2008 [2], marking the genesis of a new era in financial technology. Over the past years, the cryptocurrency market has undergone substantial transformations, witnessing the emergence of various novel digital assets. In 2021, most of the cryptocurrencies peaked at their highest price through the chosen time period. While most of them showed a downward tendency within 2021. Currently, there are a total of 8888 types [1] of digital currencies in the market, with a cumulative market capitalization reaching $1.63 trillion.

The cryptocurrency market has undergone rapid changes in recent years, cryptocurrency investment has become an integral part of investors' diversified asset allocation. Due to the highly dynamic nature of this market, timely and thorough research into issues related to cryptocurrency investment portfolios is crucial and have direct practical values for investors to understand market trends, addressing risks associated with emerging digital assets and researching the construction and management of cryptocurrency investment portfolios [3].

This research aims to emphasize on the background of cryptocurrency investment portfolios, emphasizing the developmental trajectory of digital currencies, the interrelationships among various digital assets, and methodologies for constructing effective investment portfolios. Through random weight allocation, coupled with the calculation of the Sharpe ratio, the study seeks to furnish investors with a comprehensive and practical framework for managing cryptocurrency investment portfolios.

As we delve into this exploration, it is crucial to reiterate the overarching purpose of this research. Beyond providing insights into digital currencies and their investment portfolios, this paper seeks to respond promptly to market changes. It endeavors to offer a comprehensive framework for constructing and optimizing cryptocurrency investment portfolios, addressing the practical needs of investors. Simultaneously, this research aims to contribute to the sustainable development of the cryptocurrency market, serving as a reference for financial institutions, policymakers, and other stakeholders. The concluding section will further emphasize the significance of informed decision-making in cryptocurrency investments and explore potential future avenues for research in this dynamic and evolving field.

2. Overview of our work

2.1. Research contents and methods

This paper focuses on nine prominent cryptocurrencies, namely Bitcoin (BTC), Ethereum (Ether), Tether (USDT), Binance Coin (BNB), Ripple (XRP), USD Coin (USDC), Cardano (ADA), Dogecoin (DOGE), and TRON (TRX). These cryptocurrencies represent the top nine positions [1] in terms of market capitalization within the digital asset market, with market capitalizations of $818.45 billion, $266.88 billion, $90.65 billion, $38.15 billion, $33.55 billion, $24.19 billion, $21.11 billion, $13.64 billion, and $9.25 billion, respectively.

The reasons behind selecting these specific cryptocurrencies lies in their collective dominance, both in market capitalization and trading volume, thereby ensuring that the study encompasses entities that significantly influence and reflect the overarching dynamics of the entire digital asset market. These cryptocurrencies possess substantial market capitalizations and high liquidity, characteristics that contribute to the reliability and stability of market data. This is of paramount importance for robust data analysis and modeling, particularly when addressing variables such as price movements and trading volumes.

This study employs the Markowitz theory and model [4] as the primary model for investigation, utilizing the Python programming language for computational tasks such as data analysis and model solving. The Markowitz Model is known as the mean-variance model, which was introduced by the American economist Harry Markowitz in 1952 . The methodology of this model aims to construct efficient investment portfolios by considering the covariance and correlation between assets to maximize expected returns while minimizing risk. The main idea of the Markowitz Model is to reduce the overall portfolio risk by diversifying investments among assets, allowing investors to choose optimal portfolios at different risk levels. The model introduces the concept of the efficient frontier, through mathematical optimization methods, investors can find portfolios that offer the maximum returns at a given risk level and the minimum risk at a given return level.

To study cryptocurrency portfolios, we combine the Markowitz Model with data analysis techniques to form our research method. The method includes data collection, calculation of daily returns, calculation of annual returns and covariance matrix, random allocation of weights, calculation of Sharpe ratio, and determination of optimal portfolio by efficient boundary. The model describes a portfolio that maximizes expected returns while minimizing risk. The effective frontier represents the minimum level of risk for a given level of expected return. These results enable investors to quantitatively evaluate and compare the potential risks and returns of different investment strategies, providing a powerful tool for making rational investment decisions.

2.2. Paper framework

This rest of the paper is divided into three parts:

Section 3 is to introduce the basic information of the selected cryptocurrency types, price trends, yield comparisons, etc.;

Section 4 describes the research method in detail: the Markowitz model;

Section 5 presents results of our analysis;

Section 6 includes the conclusions and reflections, summarizing the content characteristics of the whole paper and possible directions for improvements.

3. Cryptocurrency data analysis

3.1. The price changes of nine cryptocurrencies

For viewing a comprehensive price tendency of cryptocurrencies, we choose the time period from December 6, 2018, to October 9, 2023 to do the analysis of price changes, according to the daily returns data obtained from the financial website https://www.investing.com.

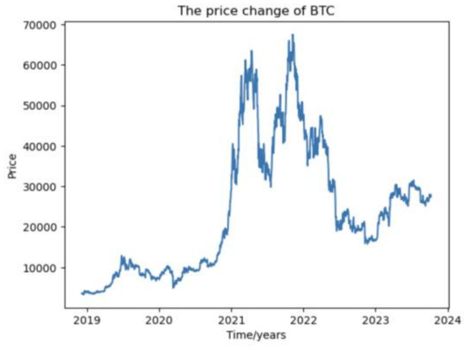


FIG.1 The price change of BTC

The price of bitcoin start at $3531.30 in December 6th, 2018, which is the highest initial price compared to the rest cryptocurrencies in this paper. And it peaked at $67526 in November 8th, 2021, which has approximately $64000 difference between the highest and the lowest in the chosen time period.

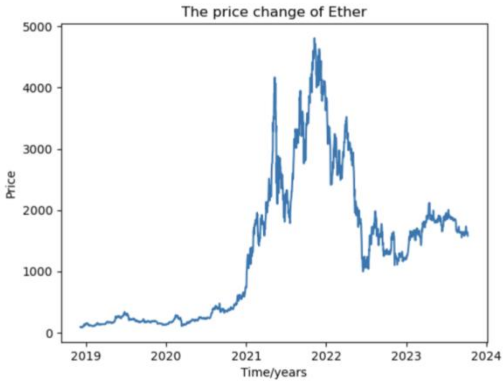


FIG.2 The price change of Ether

The price of Ether start at $90.68 in December 6th. And it peaked at $4808.38 in November 8th, 2021, which has approximately $4700 difference between the highest and the lowest in the chosen time period.

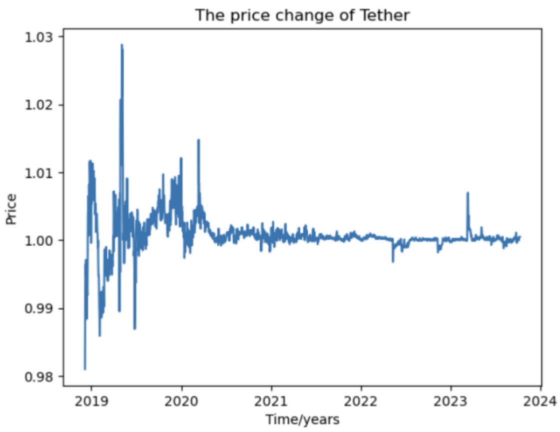


FIG 3 The price change of Tether

The price of Tether start at \$0.981 in December 6th, 2018. It peaked at \$1.0288 in May 5th, 2019, which has no much difference between the highest and the lowest in the chosen time period.

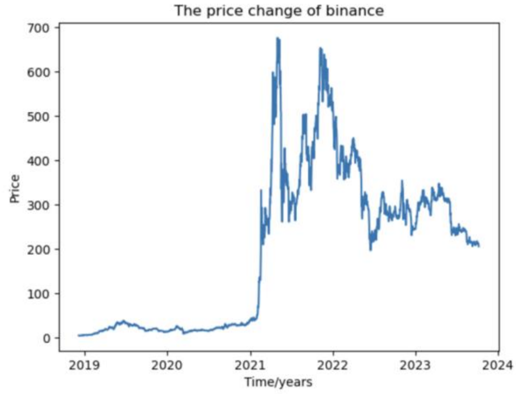


FIG 4 The price change of binance

The price of binance start at \$4.9 in December 12th, 2018. It peaked at \$676.56 in May 3th, 2021, which has approximately \$670 difference between the highest and the lowest in the chosen time period.

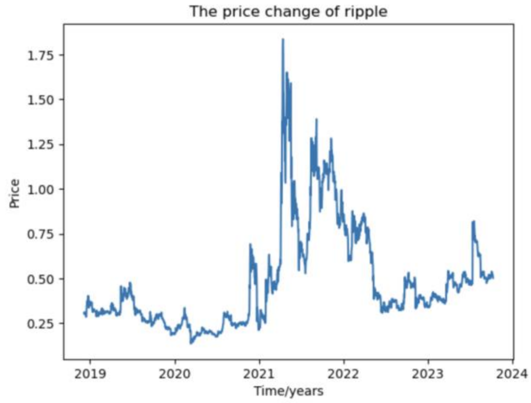


FIG. 5 The price change of ripple

The price of ripple start at \$0.30879 in December 12th, 2018. It peaked at \$1.83625 in April 14th, 2021, which has no much difference between the highest and the lowest in the chosen time period.

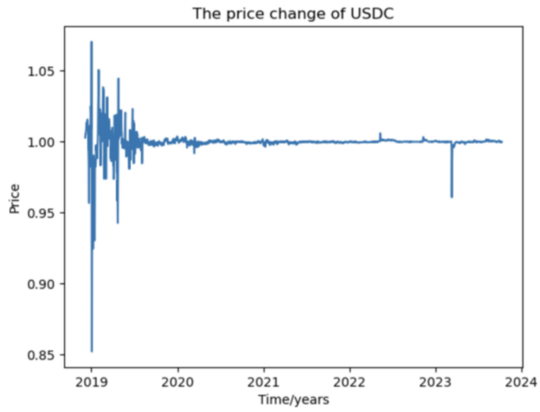


FIG. 6 The price change of USDC

The price of USDC start at \$1.0028 in December 6th, 2018. It peaked at \$1.0704 in January 2th, 2019, which has no much difference between the highest and the lowest in the chosen time period. Even the negative growth were more than the positive growth.

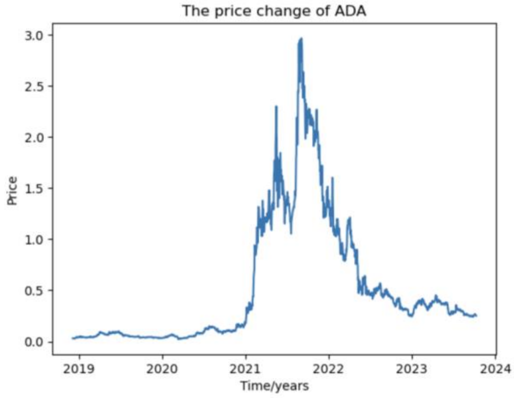
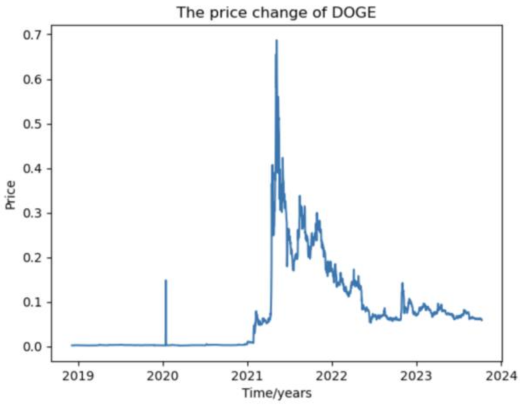


FIG. 7 The price change of ADA

The price of ADA start at $0.0307 in December 12th, 2018. It peaked at $2.9652 in September 3th, 2021, which has nearly $3 difference between the highest and the lowest in the chosen time period.



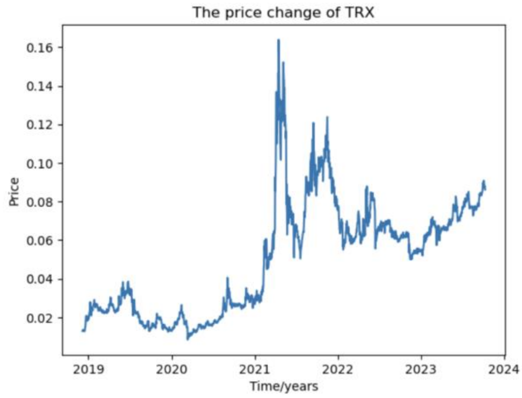


FIG.8 The price change of DOGE **FIG.9** The price change of TRX

The price of DOGE start at $0.002111 in December 6th, 2018, which is the lowest initial price. It peaked at $0.68688 in May 7th, 2021, which has no much difference between the highest and the lowest in the chosen time period.

The price of TRX start at $0.012849 in December 12th, 2018. It peaked at $0.1639 in April 15th, 2021, which has no much difference between the highest and the lowest in the chosen time period.

The high price volatility proves that the cryptocurrency has a wide range of fluctuations in historical prices, with a high risk, but at the same time, it will also have a high return. For example, Bitcoin has about $64,000 from the beginning to the highest point, which proves that Bitcoin has high volatility and high risk, but it can also gain a lot. However, like tether and USDC are basically in a flat line in the later period, the fluctuation is not very large, which also proves that their risk is not big, but also based on his small price base, only less than $1. And based on the historical price changes of tether and USDC, it can be roughly concluded that their appreciation probability is not high, and if you want to invest in it, the probability of getting a decent return is not high. On the contrary, other digital currencies have had higher prices within 2021, which represents that these digital currencies have been considered by the market to have high value or potential growth room. After 2021, the collective has a downward trend in price, rather than only one or two have a downward trend, which represents the decline of the overall market, assuming that the market returns to the situation in 2021, these digital currencies will also show their larger appreciation trend. For instance, the Bitcoin, Ether and Binance got a relatively high price base, which normally reach $20000, $1500 and $200 respectively nowadays, but they peaked at nearly $70000, $5000 and $700 in the past.

However, the conclusion only got from the historical data. More specific prediction needs to be done by analysis of the company's fundamentals, industry prospects and market environment.

4. Cryptocurrencies portfolio based on Markowitz model

4.1. Markowitz and related mathematics models

4.1.1 Markowitz model

We applied the Markowitz model as the main tool to figure out the best portfolio in the paper. This model provides a powerful basis for the portfolio selection.

Markowitz model is an optimization model. The objective function includes

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{w_i w_j cov(i,j)},$$

$$\max \sum_{i=1}^{n} \mathrm{ER}_i \mathrm{w}_i.$$

The constraints includes

$$\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{w_i} = 1,$$

$w_i \geq 0$, i=1,2,...,n, j=1,2,...,n,

where

W_i represents the weight of ith asset;

W_j represents the weight of jth asset;

Cov(i,j) represents the covariance between returns of the ith asset and jth asset;

ER_i represents the expected return of ith asset.

The constraints mean that the sum of the portfolio weights must be 1, and the weights for each asset should not be negative.

At the central theory of the Markowitz model is the reduction of overall portfolio risk through a specific strategic diversification of different assets. This strategic diversification allows investors to find out the best portfolio for different risk assets. This model introduces a concept called efficient boundary. Using sophisticated mathematical optimization techniques, investors can systematically determine portfolios that provide the greatest return at a specified level of risk, or the least risk at a specified level of return. The aim of this module is to maximize its return and minimize the risk.

4.1.2 Sharpe ratio

$$\boldsymbol{sharpe\ ratio} = \frac{\mathrm{E(R_p) - R_f}}{\boldsymbol{\sigma_p}},$$

where

$E(R_p)$ represents the expected return of the portfolio;

R_f represents the risk-free rate of the portfolio;

σ_p represents the standard deviation of returns of the portfolio, which also refers to the risk of the portfolio.

In the paper, we assume the risk-free rate will be 0, so the formula could be transformed into a simple form:

$$\boldsymbol{sharpe\ ratio} = \frac{\mathrm{E(R_p)}}{\boldsymbol{\sigma_p}}.$$

The Sharpe Ratio, developed by William F. Sharpe in 1966, is widely utilized as a financial metric, presenting the relationship between expected return and expected risk. It serves as an essential and valuable tool for investors, helping them assess whether the anticipated return will sufficiently cover the associated risk. A higher Sharpe Ratio signifies better risk-adjusted performance, indicating that the portfolio generates a superior return per unit of risk. Conversely, a lower Sharpe Ratio implies the opposite phenomenon.

4.1.3 More mathematical models

More mathematical models are applied in the paper：

The calculation of annual expected return of a asset:

$\boldsymbol{E(R_i) = u_i\, t},$

where

$E(R_i)$ means the annual expected return of the ith asset;

u_i means the past average return of the ith asset;

t represents the total number of trading days.

To visualize the trend of individual cryptocurrencies under the same initial conditions, we use the Pandas library in Python to deal with our data, which is stored in an excel file named “data1.xlsx”. The data is read into pandas as a dataframe whose name is “data2”. Then in python we use the following sentence to compare the price of an individual cryptocurrency at each point in time with the initial price and then multiplying by 100 to get a standardized price as a percentage relative to the initial value.

```
(data2/data2.iloc[0,:]).plot(figsize=(10,8),grid=True)
```

This helps to see how the price of an individual stock of each cryptocurrency changes relative to its initial level. The sentence can be transformed into the mathematical formula:

$$\frac{a_{ij}}{a_{1j}},\ i=1,2,3...,m,\ j=1,2,3...,m.$$

4.1.4 Sco.minimize

The minimize function is a tool in scipy.optimize library in python that can find the minimum value of a function in multiple data or under constraints. So initially we're going to set a function to go in, and then we're going to set a minimum value to start the search, and then the minimize function is going to start the search until it finishes the search and returns the minimum.

This is the example of how we use in the python process to figure out the optimal sharpe ratio:

```
opts=sco.minimize(min_func_sharpe,number_of_assets*[1./number_of_assets,],method='SLSQP',bounds=bnds,constraints=cons)
```

4.1.5 Covariance and variance

Covariance formula:

$$cov(i,j) = \frac{1}{n}\sum_{k=0}^{n}(i_k - \bar{\imath})(j_k - \bar{\jmath}),$$

where

i_k and j_k represent the the price of i and j at time k;

$\bar{\imath}$ and $\bar{\jmath}$ represent the the mean of i and j;

N represents the number of samples.

Variance formula:

$$\boldsymbol{\sigma^2} = \frac{1}{n}\sum_{k=0}^{n}(i_k - \bar{\imath})^2.$$

Covariance is a measure used to display the relationship between two random assets. It indicates whether these two assets change together and whether they are highly correlated. It is commonly utilized to measure the diversification of a portfolio.

Variance describes the degree of deviation of an asset from its mean. It is commonly utilized in measuring the risk or volatility of assets.

5. Results

The paper selects nine representative cryptocurrencies, including BTC, Ether, Tether, Binance, ripple, USDC, ADA, DOGE and TRX, collecting the data between November 1, 2020, to October 9, 2023 and including 1669 trading days data.

5.1. Trends of nine cryptocurrencies:

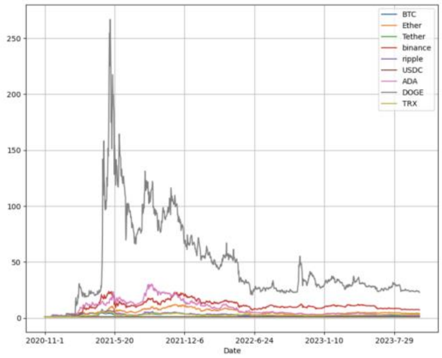


FIG.10 The trends of nine cryptocurrencies in under the same initial conditions

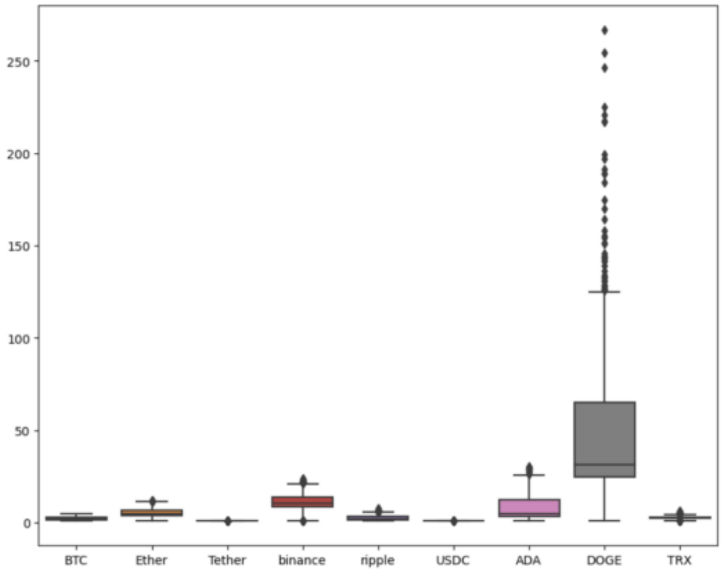


FIG.11 The trends of nine cryptocurrencies in under the same initial conditions

In FIG.10 and FIG.11, the graphs illustrate the performance trends of nine cryptocurrencies under identical initial conditions from November 1, 2020, to October 9, 2023. It is evident from the chart that the gray line (DOGE) exhibited a remarkable growth trajectory in the early stages, surpassing other cryptocurrencies with multiple instances of significant increases. However, in the later period, it was also the highest one, its place was not overtaken by any cryptocurrencies. Conversely, the purple line (Ripple), blue line(BTC), yellow line(TRX) and brown line(USDC) showcased the least pronounced development trend, maintaining a relatively stable curve with minimal fluctuation. Also in the box plot, DOGE showed the greatest development trend, and it had a huge number of outliers.

DOGE has the largest price fluctuation among the nine digital currencies, which means its trend is the most volatile and unpredictable compared to other digital currencies. Conversely, if volatility is low, like ripple, BTC, TRX, and USDC, the price moves are relatively small and the trend may be more stable and predictable. Similarly, DOGE has a large appreciation space among the nine digital currencies, because assuming that they are all under the same initial conditions, DOGE has the greatest price increase ability, which means that it has a greater growth potential, in order to show an upward trend. On the contrary, if the room for appreciation is limited, it may limit the room for its price to rise, which may lead to a relatively stable or declining price trend. In turn, DOGE,ADA, and binance are riskier, their prices may be more volatile, and their trends may be more volatile and riskier.

5.2. Rate of return

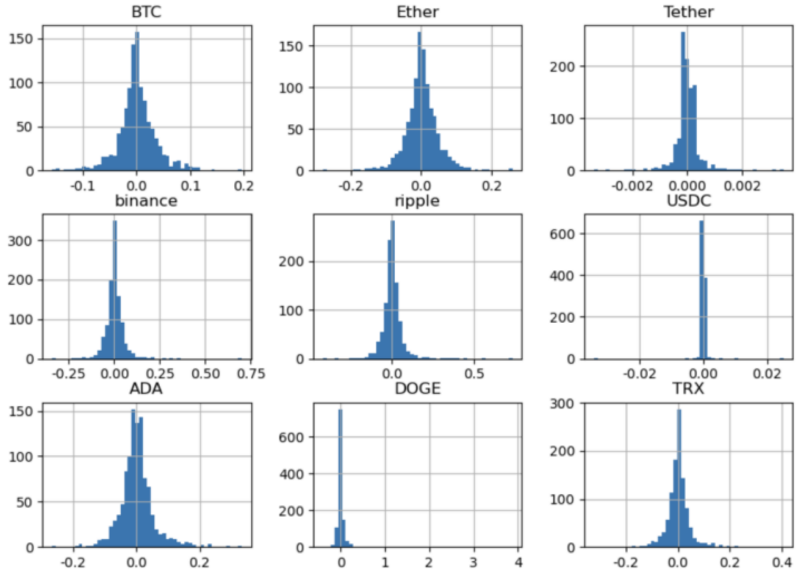


FIG.12 The return histogram of nine cryptocurrencies

In FIG.12, the graph plots the logarithmic return histogram of nine cryptocurrencies between November 1, 2020 and October 9, 2023. In these graphs, the horizontal axis represents the rate of return, while the vertical axis represents the frequency. It is worth noting that all distributions are very similar in shape and are normally distributed (high in the middle, low on both sides), with most return values show between -0.2 and 0.2, and each distribution exhibiting a clear concentration of values near zero. In this case, the extreme high or low returns of the cryptocurrency are relatively less. This is due to the fact that the price movements of most assets are relatively stable, while extreme price movements are relatively rare. In a more detailed analysis, the three digital currencies of Tether and USDC are the most extreme distributed near 0, proving that the risk and volatility of the three of them are smaller than the remaining six and relatively stable. While in the rest, Ether and ADA show a relatively high unstable price movements, which can be seen in the figure that there are more rate of return of Ether and ADA close to 0.2 or -0.2.

5.3. Rate of return annually

Table. 1 rate of return annually

Rate of return annually	
BTC	0.454974
Ether	0.848232
Tether	0.000139
Binance	1.152501
Ripple	1.008896
USDC	0.000532
ADA	0.847809
DOGE	2.871245
TRX	0.808686

Table 1 represents the annual returns of nine cryptocurrencies between November 1, 2020 and October 9, 2023. Using the above method of calculating the annual interest rate, the calculated number of days of continuous trading (t) per year is 365 days. It is clear from the data shown that most of the nine cryptocurrencies have an annual return of less than 1, which indicates that, on average, cryptocurrencies have had relatively modest returns during this period. It is worth noting that DOGE has the highest annual return of 2.871245, indicating a substantial increase in value compared to its initial price, followed by Binance at 1.152501, also indicating significant growth during this period. However, no one has a negative annual return, which means that none of the value of these cryptocurrencies decreases over the duration. Both Binance and DOGE achieved substantial returns compared to their respective initial prices over the selected time frame, indicating different performance and volatility in the cryptocurrency market over the specified time frame.

5.4. Year volatility

Table. 2 shows the covariance matrix for cryptocurrencies and Figure 13 is the corresponding heatmap. Covariance is a statistical measure used to evaluate the relationship between two random variables. Specifically, it represents how two variables go over time and whether they change in the same direction (positive covariance) or in the opposite direction (negative covariance). Positive covariance means that when one variable increases, the other variable also tends to increase, and vice versa, indicating a positive correlation. In contrast, negative covariance indicates that when one variable increases, the other variable tends to decrease, and vice versa, indicating a negative correlation.

Table.2 Covariance matrix

	BTC	Ether	Tether	binance	ripple	USDC	ADA	DOGE	TRX
BTC	0.439172	0.465172	-0.000378	0.427431	0.443481	0.000113	0.441727	0.563583	0.367051
Ether	0.465172	0.752117	-0.000125	0.571472	0.618055	-0.000433	0.623775	0.670782	0.496829
Tether	-0.000378	-0.000125	0.000073	-0.000300	-0.000552	-0.000093	0.000031	-0.001131	-0.000432
binance	0.427431	0.571472	-0.000300	1.015478	0.616819	-0.000142	0.606120	0.523777	0.511493
ripple	0.443481	0.618055	-0.000552	0.616819	1.626569	-0.000837	0.733878	0.681884	0.653609
USDC	0.000113	-0.000433	-0.000093	-0.000142	-0.000837	0.000719	-0.000151	-0.001268	0.000107
ADA	0.441727	0.623775	0.000031	0.606120	0.733878	-0.000151	1.079666	0.760602	0.545543
DOGE	0.563583	0.670782	-0.001131	0.523777	0.681884	-0.001268	0.760602	7.431063	0.599781
TRX	0.367051	0.496829	-0.000432	0.511493	0.653609	0.000107	0.545543	0.599781	0.808167

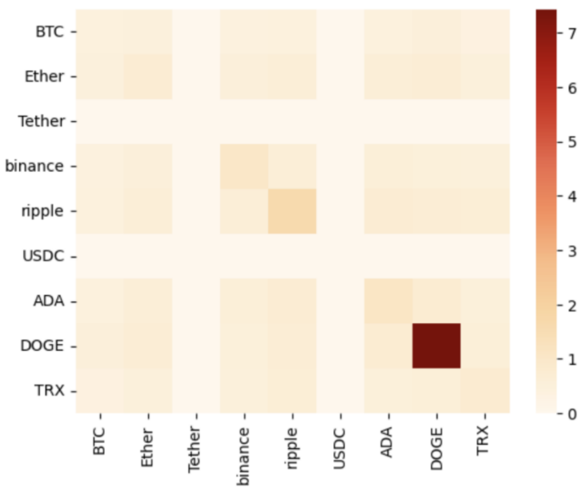


FIG.13 Covariance matrix shown in heatmap

In Table. 2, the covariance of a cryptocurrency with itself is equal to its variance. Thus, without the covariance with the cryptocurrency itself, the highest covariance is the covariance of ripple and ADA, which is **0.733878**. The data shows that the relationship between ripple and ADA is positive, which means their price will change in the same direction. More specifically, these two assets' prices tend to rise or fall at the same time. The size of the covariance indicates the degree of the correlation, while 0.733878 indicates that the correlation is strong but not complete. Moreover, if the covariance are high, then this high correlation will rise the total risk of the portfolio, because it means that these assets are likely to experience large change and same trend at the same time, increasing the overall volatility of the portfolio.

USDC and Tether are cryptocurrencies that show negative covariance with almost entire cryptocurrencies except itself, which means the price trend of Tether and the most of others are opposite. But the data are all close to 0, so it can also present that the price movements of the Tether and others are almost uncorrelated.

5.5. Portfolio return and volatility

In our analysis, we employ random numbers in Python to simulate the random allocation of weights among the nine stocks. Specifically, we simulate 50,000 portfolios, each comprising a set of randomly generated weights. In order to demonstrate the feasibility of the portfolio optimization method adopted, we selected a large number of portfolios. We use the Markowitz mean-variance model as a method for portfolio optimization. In each specific case (for example, for a given level of risk or expected rate of return), we construct the efficient frontier by calculating the covariance matrix between assets and the expected rate of return. We then choose the optimal portfolio in a way that maximizes the Sharpe ratio or minimizes the variance. For example, for a given level of risk, we can show what the optimal allocation is at a particular point on the efficient frontier to maximize the expected return or minimize the risk. Such analysis helps to demonstrate the effectiveness of our approach and reduces the risk associated with the choice of a single portfolio.

Utilizing the previously mentioned formulas, we derive 50,000 distinct investment portfolios, each characterized by unique return and volatility values. The return and volatility of these portfolios are then visualized through a scatter plot, where each point represents one of the 50,000 portfolios. Subsequently, we calculate the Sharpe ratio for each portfolio using the formula mentioned earlier. The Sharpe ratio serves as an indicator of risk-adjusted performance, allowing us to identify which portfolio allocation yields a higher Sharpe ratio. Points on the scatter plot with higher Sharpe ratios signify more optimal weight allocations.

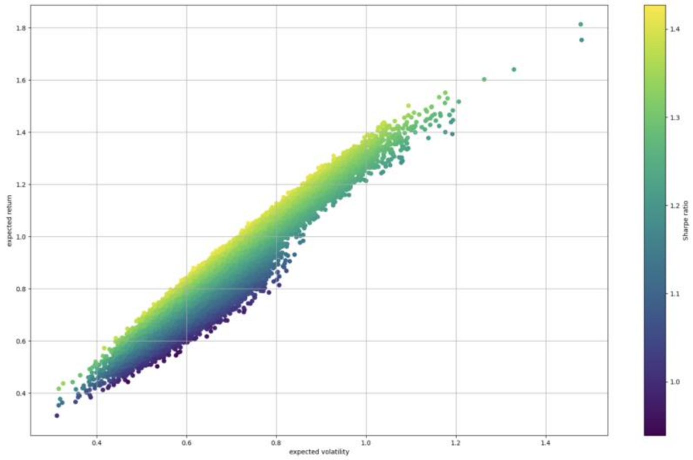


FIG.14 Sharpe ratio

This approach enables us to comprehensively assess the trade-off between risk and return across the diverse set of portfolios, ultimately pinpointing those with superior risk-adjusted performance.

5.6. Efficient frontier

Assuming that the point on the scatter plot with the highest Sharpe ratio represents the optimal portfolio, it is important to note that this point may not necessarily coincide with one of the 50,000 randomly generated portfolios. Instead, the optimal portfolio can be determined through an optimization algorithm, specifically the minimization optimization algorithm provided by the scipy.optimize package (sco.minimize).

The objective is to find the optimal set of weights that maximizes the Sharpe ratio. This optimization problem is framed as a minimization problem of the negative Sharpe ratio. By utilizing the optimization algorithm, we aim to identify the set of weights that results in the maximum Sharpe ratio output.

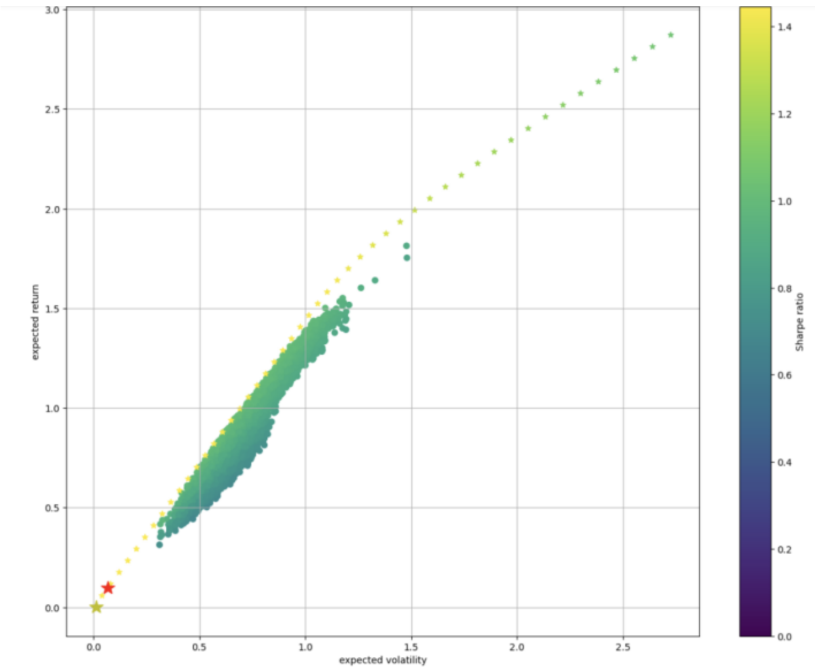


FIG.15 efficient frontier

Upon solving the optimization problem, the weights corresponding to the optimal Sharpe ratio can be obtained. This optimal Sharpe ratio is then marked with a red star on the scatter plot, denoting the portfolio with the highest risk-adjusted performance, which is at [0.097, 0.067, 1.447]. While the yellow star means the smallest variance., which is at [0.001, 0.012, 0.103].

Starting with the best Sharpe ratio (the dot shown in red), the portfolio with the highest return rate, where each cryptocurrency accounts for a percentage of the investment:

Table 3. Each cryptocurrency accounts for a percentage of the investment

	Percentage
BTC	0
Ether	0.003
Tether	0.762
Binance	0.038
Ripple	0.005
USDC	0.172
ADA	0
DOGE	0.014
TRX	0.007

Then it should be the lower variance portfolio (the yellow dot), the percentage of each cryptocurrencies:

Table. 4 The percentage of each cryptocurrencies

	Percentage
BTC	2.740e-20
Ether	6.315e-20
Tether	0.5
Binance	0
Ripple	0.001
USDC	0.499
ADA	0
DOGE	1.419e-04
TRX	4.210e-20

Each point on the efficient frontier represents a portfolio with the minimum volatility for a given level of expected return. Therefore, calculating points on the efficient frontier can be formulated as follows: Given a target level of expected return, the task is to find a set of asset weightings that minimizes the volatility of the portfolio. In other words, under the constraint of achieving a specific target return, we aim to discover the optimal combination of asset weights that results in the lowest possible volatility, thus placing the portfolio on the efficient frontier. The line marked by crosses on the graph represents the efficient frontier in this context.

6. Final conclusions

6.1. Conclusions

This study aims to respond to market changes and provide a comprehensive framework for cryptocurrency investors to build an optimal portfolio of cryptocurrency and a minimum risk portfolio to meet the actual needs of investors. Our research results show that among the nine cryptocurrencies selected, when the investment proportion of BTC, Ether, Tether, Binance, Ripple, USDC, ADA, DOGE and TRX is 0, 0.003, 0.762, 0.038,0.005, 0.172, 0, 0.014, 0.007, respectively. It can get the highest sharpe ratio. Then when they are respectively 2.740e-20, 6.315e-20, 0,5, 0, 0.001, 0.499, 0, 1.419e-04, 4.210e-20, respectively. The lowest risk rate can be obtained. This finding provides an important reference for investors to structure and optimize their portfolios in the cryptocurrency market to achieve higher investment returns, or lower risk rates. This study mainly uses Markowitz model and Sharpe ratio and other methods. Markowitz model is a portfolio optimization method, which selects the optimal portfolio allocation by maximizing the expected return or minimizing the risk. The Sharpe ratio is a measure of a portfolio's risk-adjusted return that helps investors assess its performance. This study aims to contribute to the sustainable development of the cryptocurrency market and provide a useful reference for financial institutions, policymakers and other stakeholders. We provide a comprehensive portfolio construction and optimization framework to help investors better understand the cryptocurrency market and provide support and guidance for their investment decisions. We are also aware of some limitations of this study. First, the number of digital currencies we selected is small and may not fully cover the entire cryptocurrency market. Second, our study does not take into account the effect of the risk-free rate, which can affect the risk and return assessment of a portfolio. Therefore, future studies can further expand the sample size, consider more asset classes, and fully consider the impact of the risk-free rate to improve the applicability and reliability of the study. In conclusion, this study provides a useful framework for cryptocurrency investors to better understand the cryptocurrency market and enable more effective investment decisions. By optimizing portfolio allocation, investors can better cope with market fluctuations and achieve long-term investment appreciation. We hope that this study will contribute to the development of the cryptocurrency market and the success of investors.

6.2. Directions for possible improvements

There are some areas for further improvements in the cryptocurrencies portfolio analysis in this paper.

Firstly, we have only selected a relatively small number of digital currencies, only nine, which may limit a full understanding of the entire digital asset market. By increasing the number of selected digital currencies, we can get a more complete data of the correlation and performance between different digital currencies, thus providing more information for investment decisions.

Secondly, the analysis of this paper does not take into account the effect of risk-free rate. The risk-free rate plays an important role in portfolio theory because it represents the expected return on a risk-free asset, such as a Treasury bond. Taking the risk-free rate into account can help investors more accurately assess the risk and return of a portfolio, and can provide more accurate asset allocation recommendations.

7. Appendix

```
#importing libraries
import numpy as np
import pandas as pd
import tushare as ts
import matplotlib.pyplot as plt
import scipy.optimize as sco
import scipy.interpolate as sci
import seaborn as sns
import warnings
plt.rcParams['font.sans-serif'] = ['SimHei']
plt.rcParams['axes.unicode_minus'] = False
#Reading the data
data2 = pd.read_excel('data1.xlsx', header=0, index_col=0)
#Plotting rends of nine cryptocurrencies in under the same initial conditions
(data2/data2.iloc[0,:]).plot(figsize=(10,8),grid=True)
#Setting the figure size
plt.figure(figsize=(10, 8))
# Plotting box plot
sns.boxplot(data=data2/data2.iloc[0, :])
```

```
plt.show()
#Calculating return
returns = ((data2 - data2.shift(1)) / data2.shift(1)).dropna()
#Plots histograms
returns.hist(bins=50, figsize=(10,7))
#Calculating year returns and volatilities
rets=returns
rets.mean
year_rets = rets.mean()*365
year_volatility = rets.cov()*365
#Calculating covariance
return_cov = returns.cov()*365
#Plotting heatmap
sns.heatmap(return_cov, cmap='OrRd')
#Calculating Portfolio Returns and Volatilities
number_of_assets = 9
weights = np.random.random(number_of_assets)
weights /=np.sum(weights)
portfolio_returns = []
portfolio_volatilities= []
for p in range (50000):
    weights = np.random.random(number_of_assets)
    weights /=np.sum(weights)
    portfolio_returns.append(np.sum(rets.mean()* weights)*365)
    portfolio_volatilities.append(np.sqrt(np.dot(weights.T,
np.dot(rets.cov()*365,weights))))
portfolio_returns = np.array(portfolio_returns)
portfolio_volatilities = np.array(portfolio_volatilities)
plt.figure(figsize=(20,12))
#Scatter Plot of Portfolios:
plt.scatter(portfolio_volatilities,portfolio_returns,c=portfolio_returns/portfolio_volatilities,marke
r='o')
plt.grid(True)
plt.xlabel('expected volatility')
plt.ylabel('expected return')
plt.colorbar(label='Sharpe ratio')
#Statistical Functions
def statistics(weights):
    weights= np.array(weights)
    pret=np.sum(rets.mean()*weights)*365
    pvol=np.sqrt(np.dot(weights.T,np.dot(rets.cov()*365,weights)))
    return np.array([pret,pvol,pret/pvol])
def min_func_sharpe(weights):
    return -statistics(weights)[2]
bnds=tuple((0,1)for x in range(number_of_assets))
cons=({'type':'eq','fun':lambda x:np.sum(x)-1})
#Find the optimizing point
opts=sco.minimize(min_func_sharpe,number_of_assets*[1./number_of_assets,],method='SLSQ
P',bounds=bnds,constraints=cons)
statistics(opts['x']).round(3)
def min_func_variance(weights):
```

```
        return statistics(weights)[1]**2
    optv=sco.minimize(min_func_variance,number_of_assets*[1./number_of_assets,],method='SLS
QP',bounds=bnds,constraints=cons)
    weight2=optv['x'].round(3)
    weight1=opts['x'].round(3)
    statistics(optv['x']).round(3)
    def min_func_port(weights):
        return statistics(weights)[1]
    target_returns=np.linspace(0,2.871245,50)
    target_volatilities=[]
    for tret in target_returns:
        cons = ({'type': 'eq', 'fun': lambda x:    statistics(x)[0] - tret},
                {'type': 'eq', 'fun': lambda x:    np.sum(x) - 1})
        res = sco.minimize(min_func_port, number_of_assets * [1. / number_of_assets,],
method='SLSQP',
                           bounds=bnds, constraints=cons)
        target_volatilities.append(res['fun'])
    #Plotting Efficient Frontier
    plt.figure(figsize=(15,12))
    plt.scatter(portfolio_volatilities,portfolio_returns,c=np.array(portfolio_returns)/np.array(portfolio
_volatilities),marker='o',vmin=0, vmax=2)
    plt.scatter(target_volatilities,target_returns,c=target_returns/target_volatilities,marker='*')
    plt.plot(statistics(opts['x'])[1],statistics(opts['x'])[0],'r*',markersize=15.0)
    plt.plot(statistics(optv['x'])[1],statistics(optv['x'])[0],'y*',markersize=15.0)
    plt.grid(True)
    plt.xlabel('expected volatility')
    plt.ylabel('expected return')
    plt.colorbar(label='Sharpe ratio')
```

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