Insurance Risk Assessment Model based on Game Theory-Combined Weighting-TOPSIS

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Abstract. With the increasingly severe climate change and intensifying severity of weather events and natural disasters, ensuring the sustainable development of the property insurance industry becomes particularly important. The policy distribution of insurance companies has become a key issue. This paper proposes a novel Game Theory-Combined Weighting-TOPSIS model, which weights indicators using The Entropy Weight Method and the CRITIC method, combines the weights of the former two through game-based combined weighting, and leverages the advantages of the TOPSIS method in handling multi-criteria decision-making problems. Among them, taking China as an example and applying the model, it is found that southeastern China and coastal areas have higher scores and are suitable for insurance. Through this approach, we can scientifically and reasonably assess the risk levels in different regions, provide decision support for property insurance companies to optimize policy distribution, and promote the healthy development of the insurance industry.

Keywords: Risk Assessment, Game Theory-Combined Weighting, TOPSIS.

1. Introduction

With the intensification of global climate change, the severity of climate disasters caused by extreme weather events is increasingly alarming. This not only burdens the national economy and property insurance companies but also, due to the high cost of preventive measures, makes property insurance increasingly difficult to afford, surpassing the payment capabilities of ordinary individuals. Consequently, property insurance companies have to frequently adjust the underwriting strategy and geographical layout, which further aggravates the insurance risk of insurance companies.

Ward P.J. summarized and categorized previous risk assessment models for natural disasters, finding that most studies focused on establishing models for individual types of natural disasters [1]. Jiang Feng proposed a comprehensive risk assessment model, which evaluates disaster-prone environmental sensitivity, vulnerability of the exposed elements, and risk degree of causative factors through subjective weighting of each factor, thereby establishing the model [2]. Yang Wei Chao established a risk assessment model called MRAM based on grey relational analysis and geographic detector method [3].

Although various risk assessment models for natural disasters have been proposed in previous studies, these studies focus on a single type of disaster or lack objectivity and comprehensiveness in assigning weights. Considering that the actual situation of a region's property insurance is affected by many factors, this paper innovatively considers all-natural disasters faced by the region and integrates the three dimensions of society, nature, and insurance to comprehensively analyze the problem of property insurance in the region. Using the Game Theory-Combined Weighting method, the influence of each factor on property insurance is objectively and effectively determined, which provides a practical evaluation tool and theoretical basis for the insurance decision-making of property insurance companies.

2. Model Establishment

This paper first establishes a risk assessment index system, combined with CRITIC and Entropy Weight methods, uses the Game Theory-Combined Weighting method to obtain the weight of each index, and then uses the TOPSIS comprehensive evaluation method to obtain the insurance risk level of the region. The overall process is shown in Figure 1.

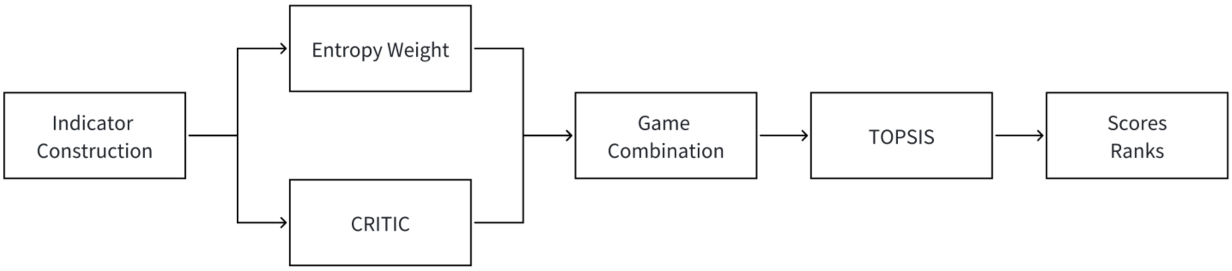

Figure 1. Flow chart of the model

2.1. Establish an Evaluation System

Insurance risk assessment not only considers the occurrence rate of natural disasters but also pays attention to the economic level of the region and the current status of the insurance industry. Through reviewing existing literature and research, the article selects disaster risk indices from three aspects: insurance, nature, and society. Eight secondary indicators are identified as shown in Table 1, including the Insured total amount and GDP [4].

Table 1. Evaluation index system

Primary Indicator	Secondary Indicator	Indicator Nature
Insurance (I)	loss ratio (LR)	+
	Insured total amount (ITA)	+
	Claimed total amount (CTA)	-
	Claim Loss Rate (CLR)	-
Nature (N)	Disaster Risk Index (DRI)	-
	Disaster Economic Loss Rate (DELR)	-
Society (S)	Gross Domestic Product (GDP)	+
	Population density (PD)	+
	Direct economic loss (DL)	-

2.2. Game Theory-Combined Weighting

To make the weighting of weights more scientific, this paper introduces a game theory-based weighting method. Combining the Entropy Weight and CRITIC method [5], it fully considers the correlation between indicators and the balance of information.

2.2.1. Entropy Weight Method

The Entropy Weight Method is an objective weighting approach that utilizes information entropy to calculate the weights of various indicators [6]. This article exemplifies the calculation process using data from 34 provinces in China.

(1) Normalize the original matrix:

$$X_{\min} = \min\left(X_{1j}, X_{2j}, \ldots, X_{nj}\right) - 0.0001 \tag{1}$$

$$X_{\max} = \max\left(X_{1j}, X_{2j}, \ldots, X_{nj}\right) + 0.0001 \tag{2}$$

For positive indicators:

$$z_{ij} = \frac{X_{ij} - X_{\min}}{X_{\max} - X_{\min}} \tag{3}$$

For negative indicators:

$$z_{ij} = \frac{X_{ij} - X_{\min}}{X_{\max} - X_{\min}} \tag{4}$$

(2) Calculate the entropy value of each indicator:

$$e_{ij} = -\frac{1}{ln56}\sum_{i=1}^{56} p_{ij} ln(p_{ij}) \tag{5}$$

(3) Calculate the redundancy of the entropy values of various indicators:

$$dj = 1 - ej \tag{6}$$

(4) Calculate the weight of each indicator:

$$w_j = \frac{d_j}{\sum_{i=1}^{56} d_j} \tag{7}$$

The weights of each indicator calculated using this method are denoted as W_{j1}.

2.2.2. CRITIC

For comprehensive evaluation issues involving multiple indicators and objects, the CRITIC method can mitigate the impact of indicators with strong correlations, reducing informational redundancy among indicators, and thereby facilitating more reliable evaluation outcomes [7]. The steps are as follows:

(1) Find the correlation coefficient r_{ti} between the evaluation indicators, and construct the correlation coefficient matrix $R = (r_t)_{m\times n}$, where r_{ti} is:

$$r_{tj} = \frac{\operatorname{cov}(d_t, d_j)}{\sqrt{Dd_t}\sqrt{Dd_j}} \tag{8}$$

$$\operatorname{cov}(d_t, d_j) = E\Big(\big(d_t - E(d_t)\big)\cdot\big(d_j - E(d_i)\big)\Big) \tag{9}$$

(2) Calculate the information content and weights of each evaluation indicator. The objective weight of each evaluation indicator is comprehensively measured by the intensity of contrast and conflict between the evaluation indicators. Let C_j represent the information content contained by the j^{th} evaluation indicator.

$$C_j = \sigma_j \sum_{t=1}^{n}(1 - r_i), j = 1, 2, \cdots, n \tag{10}$$

Among them, σ_j is the standard deviation of the j^{th} evaluation index, and r_{ti} is the correlation coefficient between evaluation indicators. The larger C_j is, the greater the amount of information contained in the j^{th} evaluation index, and the greater the relative importance of the index.

(3) Find the weight w_j:

$$w_j = \frac{C_j}{\sum_{j=1}^{n} C_j}, j = 1, 2, \cdots, \mathrm{n} \tag{11}$$

The weight of each indicator calculated by this method is recorded as W_{j2}.

2.2.3. Game Theory

Game Theory-Combined Weighting involves seeking a game equilibrium solution among the weights obtained from different singular methods, thereby minimizing the discrepancy between the measurement weights of each method and the game-theoretic combination weighting [8]. The specific steps are as follows:

(1) Use the index weights obtained by the entropy weight method and CRITIC method to establish a weight vector set $Q_1 = \{q_1, q_2\}$, let $a = \{a_1, a_2\}$ is the linear combination coefficient, and any linear combination of these vectors be:

$$Q = \alpha_1 q_1^T + \alpha_2 q_2^T \tag{12}$$

(2) According to the idea of game theory, the two linear combination coefficients a_1 and a_2 in the above formula are optimized, to minimize the deviation, and the most reasonable weight can be obtained. This method determines the objective function as:

$$\min \left\| Q - q_i \right\|_2, i = 1, 2 \tag{13}$$

(3) Use matrix differential properties to equivalently transform the above equation into a system of linear equations with optimized first-order derivative conditions:

$$\begin{pmatrix} q_1 q_1^T & q_1 q_2^T \\ q_2 q_1^T & q_2 q_2^T \end{pmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} q_1 q_1^T \\ q_2 q_2^T \end{bmatrix} \tag{14}$$

(4) The optimal linear combination coefficients a_1 and a_2 are obtained from the above formula and normalized, and the comprehensive weight Q based on game theory combination weighting is obtained:

$$Q = \alpha_1' q_1^T + \alpha_2' q_2^T \tag{15}$$

Where $\alpha_1' = \frac{\alpha_1}{\alpha_1 + \alpha_2}; \alpha_2' = \frac{\alpha_2}{\alpha_1 + \alpha_2}$.

2.3. TOPSIS Comprehensive Evaluation Score

The TOPSIS algorithm is a method for multi-attribute decision analysis that can be used to evaluate a set of candidate solutions and determine the best solution [9]. The above steps have determined the weight through game combination weighting. Next, calculate the optimal solution and the worst solution:

Optimal solution:

$$\left(Z_1^+, Z_2^+, \ldots, Z_m^+\right) \tag{16}$$

Worst solution:

$$\left(Z_1^-, Z_2^-, \ldots, Z_m^-\right) \tag{17}$$

Then, calculate the difference between each evaluation index and the optimal and worst vectors:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} w_j \left(Z_j^+ - z_{ij}\right)^2}, \quad D_i^- = \sum_{j=1}^{m} w_j \left(Z_j^- - z_{ij}\right)^2 \tag{18}$$

Where w_j is the weight (importance) of the j^{th} attribute.

Finally, calculate the overall score:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{19}$$

The larger the value of C_i, the higher the comprehensive score and the better the evaluation object.

3. Results

3.1. Weight Result Analysis

Through calculations, the game coefficients are obtained as $\alpha_1^{'} = 0.632, \alpha_2^{'} = 0.378$. The weight results obtained from the Entropy, CRITIC, and Game Theory-Combined Weighting comprehensive weighting method are shown in Table 2:

Table 2. The weights of indices

	Entropy Weight Method	CRITIC	Weight
Insured total amount	21.134	16.446	19.573
GDP in 2022	17.195	7.984	13.885
Claimed total amount	14.235	12.922	13.881
Disaster Risk Index	12.949	8.447	11.377
Population density	6.899	18.347	11.295
loss ratio	10.169	11.997	10.961
Direct economic loss	5.165	13.112	8.221
Disaster Economic Loss Rate	6.067	6.783	6.399
Claim Loss Rate	6.187	3.962	5.408

The above weight data reflects the importance of different indicators in the assessment model, where the Insured total amount, GDP, and total claim amount are the three indicators with the highest impact on the insurance risk level. The high weight of these indicators indicates that an increase in the total insurance amount reflects the healthy development of the insurance industry and market penetration in the region, as well as a high level of risk prevention awareness among the people. An increase in the total claim amount suggests that the risks undertaken by the insurance industry are increasing, which could affect profitability. A high GDP value indicates a developed economy and higher income levels among residents, providing a larger market space for the development of the insurance industry.

3.2. Results evaluation——China as an example

This paper utilizes the quintile method from statistics to segment the scoring intervals of the evaluation model and has established an insurance assessment grade evaluation table [10]. According to Table 3, we categorize the risk levels into five tiers: High Risk, Medium-high Risk, Medium Risk, Medium-low Risk, and low risk. Each corresponds to different scoring intervals. A higher score indicates lower risk, making the area more suitable for insurance coverage; conversely, a lower score implies higher risk, suggesting a more cautious approach to insurance coverage.

Table 3. Insurance Assessment Grading Table

Level	High Risk	Medium-high Risk	Medium-Risk	Medium-low Risk	Low Risk
Score	[0-20]	(20-40]	(40-60]	(60-80]	(80-100]

Utilizing this evaluation system, we conducted a natural risk assessment across Chinese provinces According to the template provided by the map of China, the risk heat map of China is drawn by ArcGIS software Figure 2. The map vividly demonstrates that southeastern China and coastal areas scored higher in the comprehensive evaluation, primarily due to their higher national GDP levels, stronger awareness of property insurance among residents, and lower economic losses from natural disasters. These factors render the southeastern regions an ideal choice for property risk companies. Conversely, areas such as Tibet and Inner Mongolia, characterized by higher disaster risk indices and lower national GDP levels, present higher underwriting risks, necessitating cautious insurance underwriting by property risk companies.

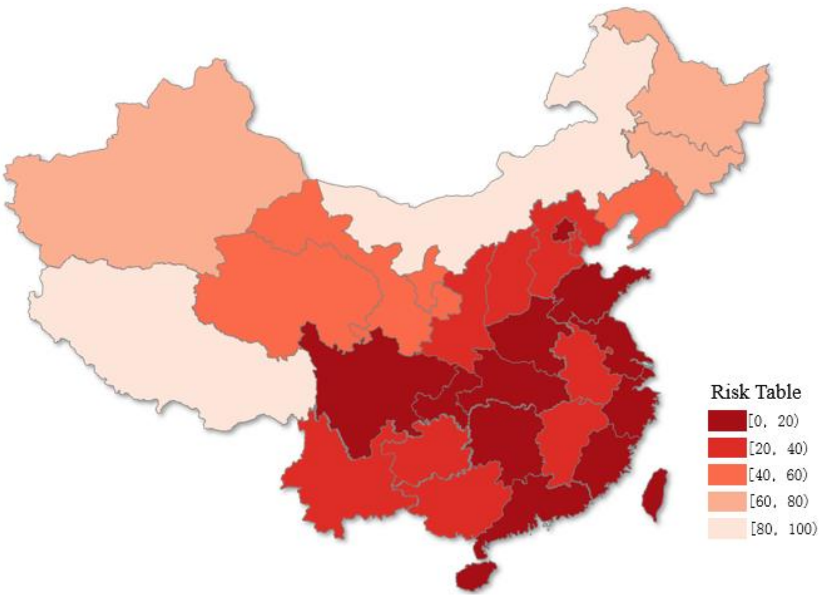

Figure 2. Risk Heatmap by Region in China

The application of this comprehensive evaluation model enables a more scientific assessment of the insurance risk levels across different regions, providing robust decision-making support for insurance companies. This, in turn, facilitates the optimization of insurance resource allocation and reduces the economic losses caused by natural disasters.

4. Conclusions and outlooks

This paper constructs three first-level indicators of insurance, nature, and society, and further subdivides them into eight second-level indicators. Using a combination of Game Theory weighting, the CRITIC method, and the entropy weight method, the comprehensive weight of each index is determined. Then, the TOPSIS method is employed to calculate the regional score. According to the five sections, the risk degree of regional insurance is judged. Among them, taking China as an example to apply the model, it is found that the southeast and coastal areas of China have higher scores and are suitable for insurance. This paper provides a novel perspective to understand the natural risk assessment problem and provides an effective decision support tool for the insurance industry. Through empirical analysis of various regions in China, the effectiveness and practicality of the model in real-world applications are demonstrated, particularly in identifying high-risk and low-risk areas.

In forthcoming research endeavors, the precision and predictive prowess of the model could be significantly enhanced by incorporating a more diverse array of indicators and corresponding datasets, such as social network data and predictions from climate change models. Investigating the integration of cutting-edge technologies like machine learning and artificial intelligence with this model could facilitate automated optimization of weights and risk assessment, thereby augmenting the model's dynamic adaptability and real-time update capabilities. With advancements in data acquisition and computational technology, there is ample reason to believe that this model will play an increasingly pivotal role in future risk management and insurance decision-making processes.

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