Study on the Benefit Analysis and Configuration of Submarine Search and Rescue Related Equipment

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Abstract. This paper primarily investigates the challenges associated with underwater search operations, particularly focusing on the accurate location of distressed submersibles using advanced modeling techniques. Utilizing environmental data and considering factors such as ocean currents, sea densities, and seabed geography, a comprehensive location model has been developed. Initially, the study introduces a basis function to address the uncertainties inherent in the marine environment, enhancing the model's accuracy and fault tolerance. Subsequently, it adopts Voronoi partitioning to allocate search tasks among Unmanned Underwater Vehicles (UUVs), followed by an equilibrium-based algorithm for task redistribution, ensuring balanced workload distribution. The implementation of an improved raster confidence function facilitates efficient UUV path planning, optimizing search operations. The findings indicate significant improvements in the probability of target containment within planned search areas, although limitations exist due to the omission of biological factors and the efficiency constraints imposed by the discrete grid method. The study concludes with suggestions for future enhancements, particularly in addressing biological influences and optimizing grid size for improved model efficiency.

Keywords: Underwater Search, Submersible Location, Voronoi Partitioning, Raster Confidence Function.

1. Introduction

Ocean exploration, a critical component of human curiosity, has been revolutionized by the advent of submersibles, providing tourists with unprecedented access to the ocean's depths and its mysteries, such as historical shipwrecks. Despite their allure, the safety of these underwater voyages remains a pressing concern, underscored by incidents like the Titan disaster in June 2020, which highlighted the grave risks posed by communication failures and mechanical malfunctions. Considering such challenges, there is an urgent need to refine our ability to precisely locate submersibles and to devise efficient search and rescue strategies, thereby enhancing the safety and reliability of underwater travel for tourists. This paper aims to address these issues by proposing a comprehensive model that predicts the location of submersibles over time, considering the inherent uncertainties. Furthermore, it seeks to recommend cost-effective search equipment for host ships, establish optimal initial deployment points and search patterns to expedite the discovery of lost submersibles, and evaluate the model's adaptability to various marine environments and scenarios involving multiple submarines [1].

2. Submersible location predicting model

2.1. The Establishment of submersible location predicting model

In this study, the dynamics of ocean currents are characterized to understand their impact on submersible navigation. The current is assumed to have a constant speed and direction to facilitate a simplified approach to the model. The aerodynamic drag, represented as a vector quantity, is determined by the relative velocity of the submersible to the current, considering the drag coefficient, submersible's surface area, and seawater density. The drug can function as a resisting or accelerating force contingent on the alignment of the current's direction with the submersible's trajectory.

$$\overrightarrow{F_c} = -\frac{1}{2}\rho|\overrightarrow{v_r}|^2 C_D A \cdot \frac{\overrightarrow{v_r}}{|\overrightarrow{v_r}|} \tag{1}$$

The buoyancy force, a crucial factor for the submersible's vertical positioning, is calculated based on the sea water's density. This density is subject to variations due to depth, temperature, salinity, and pressure, governed by the EOS80 equation of state within the submersible's operational parameters. For depths exceeding one kilometer, a quadratic relationship is utilized to approximate the seawater density, ensuring accuracy in the buoyancy estimations, which are pivotal for the submersible's stability and control.

$$\rho = -4.2336048 \times 10^{-8} d^2 + 4.8000 \times 10^{-3} d + 1027.80 \tag{2}$$

Having obtained the resultant force acting on the submersible, the application of Newton's second law gives the second derivative of the submersible's location with respect to time:

$$ma = F_b + F_c + G \tag{3}$$

$$m\frac{d^2\vec{r}}{dt^2} = m\vec{g} + \rho\vec{g}V_s + \frac{1}{2}\rho|\overrightarrow{v_r}|^2 C_D A \cdot \frac{\overrightarrow{v_r}}{|\overrightarrow{v_r}|} \tag{4}$$

2.2. Application of submersible location predicting model

Assuming that the direction of the ocean current and the initial velocity of the submarine are both in the horizontal plane, they have a certain initial angular separation.

We use data from the Titan and the Ionian Sea to make our calculations, where Initial location (0, 0, -2000), initial velocity (1, 1, 0), $g = 9.8m/s2$, $CD = 0.8$, $v = 46.9km/h$, $A = 18.76km2$, $vm = 0.09km/h$

Expressing the vector equation in component form:

$$\begin{cases} \frac{1}{2}\rho C_D A\sqrt{(x-v)^2 + V_y^2} = m\ddot{x}(t) \\ \frac{1}{2}\rho C_D A V_{y0}\sqrt{(V_x - V)^2 + V_y^2} = m\ddot{y}(t) \\ \rho g V_0 - mg = m\ddot{z}(t) \end{cases} \tag{5}$$

Utilize the ode45 function in MATLAB to numerically solve the system of differential equations, subsequently visualize the obtained numerical solutions in Fig. 1.

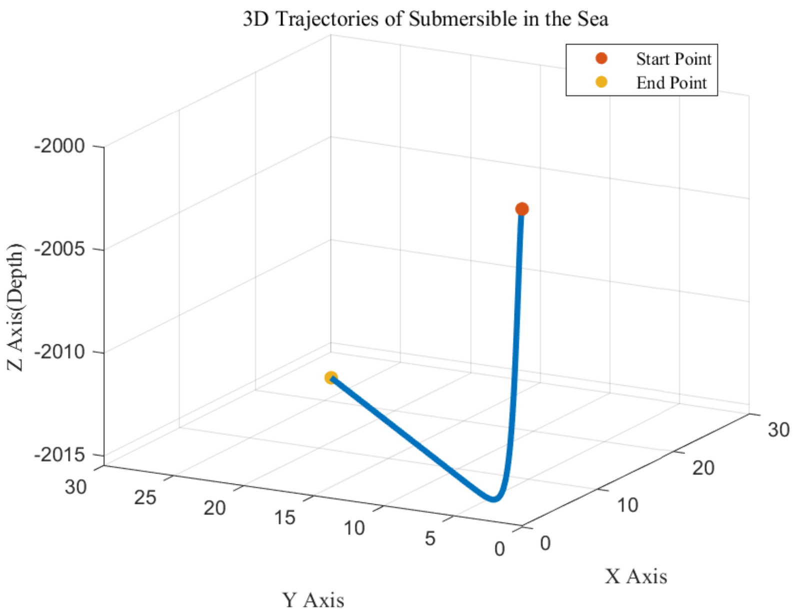

Figure 1. 3D Trajectories of submersible in the Ionian Sea

In the initial phase after the submersible loses propulsion, buoyancy is less than gravity, resulting in a net downward force and downward motion.

The buoyancy of seawater increases in depth. Under the assumed conditions, the underwater vehicle will reach the neutral buoyancy point after 0.82 seconds, at which time the depth is -2015.12 meters.

Ignoring the influence of terrain, if the malfunctioning underwater vehicle does not reach the seabed and encounters no obstacles, it will experience slight oscillations in the vicinity of this depth.

In assessing the uncertainties within the submersible location prediction model, this paper examines the horizontal plane, denoted as region S, where the submarine's projected trajectory lies. These uncertainties arise from a myriad of factors including the seabed environment, uncharted obstacles, and external disturbances, all of which can be represented as a linear aggregation of basic functions that adhere to a Gaussian distribution, as evidenced by prior research. The seabed environment introduces uncertainties through spatial and temporal variations in ocean currents, water density, and seabed topography, which necessitate the incorporation of real-time data for accurate predictions. Similarly, the presence of unidentified obstacles—ranging from inanimate underwater structures to marine life—poses navigational risks, demanding up-to-date sonar or laser-derived obstacle detection information. Moreover, the possibility of external interference from nearby vessels or submarines further complicates the prediction model, requiring the integration of precise self-location and movement data to mitigate potential trajectory disruptions [2].

3. AHP and Cost-benefit evaluation model

3.1. Factor weight determination based on AHP.

In this paper, we will analyse several major types of ocean search equipment, several types of search equipment are shown in Fig. 2.

Figure 2. Five search devices

Because of their relatively high cost, optimal selection is critical, aimed at improving the cost-effectiveness of equipping the primary ship. Indicators such as service life, search range, accuracy, reliability and number of operators significantly influence the effectiveness of search equipment, thus guiding strategic choices.

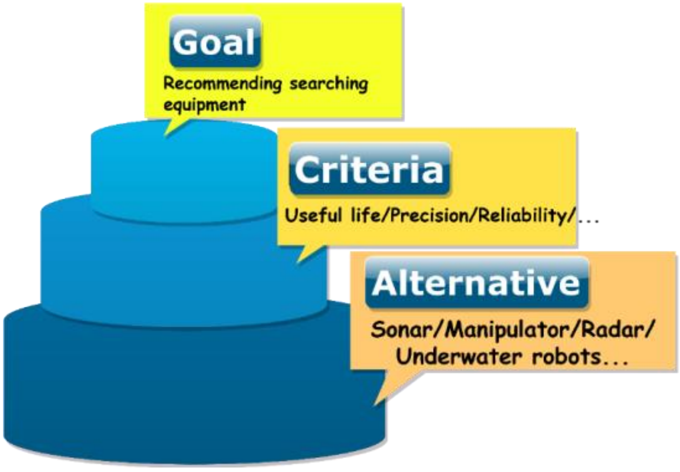

Figure 3. Schematic diagram of AHP.

The relative weights of each impact indicator were calculated based on the AHP schematic in Fig. 3.

A preliminary comparison matrix was created:

$$W = (a_{ij})_{5\times5} \tag{6}$$

Where ij denotes the importance index of i relative to j.

To mitigate the subjectivity of the Analytic Hierarchy Process, word frequency, serving as a reflection of the significance of each Indicator to a certain extent, was employed [3].

We conducted a frequency analysis of the appearance of each indicator in literature related to ocean search devices. Subsequently, this information was utilized to determine the relative importance of various indicators in the matrix as shown in Fig.4 and Fig. 5.

Figure 4. Word Frequency Clouds of equipment indicators

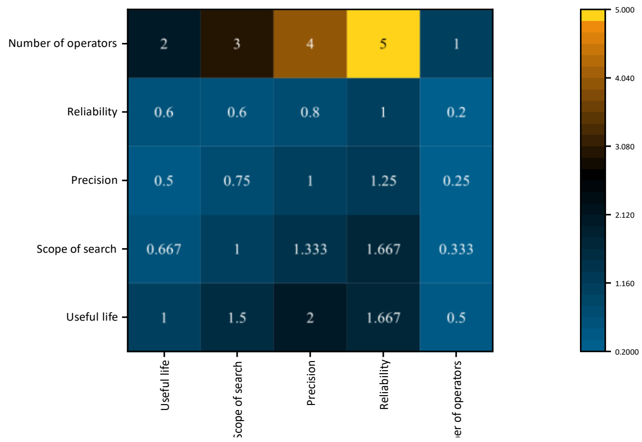

Figure 5. Relative weight heat map

The characteristic vector of the weights is denoted as $W_0 = (0.737, 1.012, 1.35, 1.562, 0.337)$ as shown in Fig. 6.

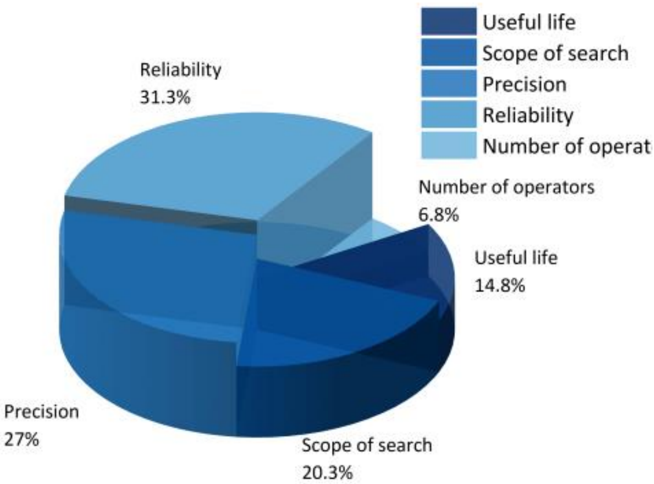

Figure 6. Indicator weights pie chart

3.2. Cost-benefit analysis model

After consistency checks and normalization, we quantified effectiveness. Utilizing the weight vector w, the efficiency scores for search equipment were computed using the formula:

$$E = w_1 \cdot S + w_2 \cdot P + w_3 \cdot R + w_4 \cdot N + w_5 \cdot Y \tag{7}$$

Where S represents scope of search, P denotes precision, r signifies reliability, N stands for number of operators, and U represents Useful life.

The cost-benefit ratio CER is defined as:

$$CER = \frac{E}{C} \tag{8}$$

Where C represents the cost value.

The cost-benefit ratios for each piece of equipment are shown in Fig. 7.

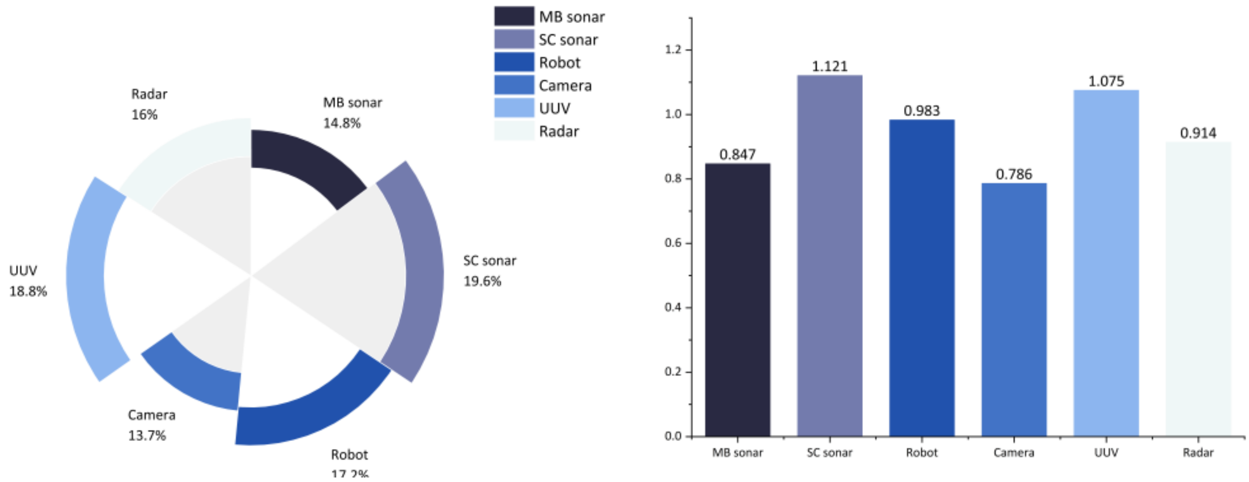

Figure 7. cost-benefit ratios for search equipment

Considering various indicators including overall cost and usage, the most recommended search equipment to be continued the main vessel consists of UUV fleet and side-scan sonar. Additionally, fundamental search devices such as emergency beacons, satellite communication equipment, and meteorological monitoring devices should also be equipped on the main vessel to ensure basic safety [4].

4. Lost submersible search model

4.1. Modelling

This paper presents a methodological approach for submersible search operations, utilizing a fleet of Unmanned Underwater Vehicles (UUVs) equipped with side-scan sonar, as determined by cost-benefit assessments. The operation requires a minimum of five UUVs to effectively scan a 250-square-kilometer area, based on the specification that each UUV maintains a speed of 5 knots over a 10-hour period. The search strategy employs raster-based spatial modeling to delineate a 20-kilometer square search region, enhancing computational efficiency and navigation precision. The selection of raster cell size is critical, as it is influenced by the sonar's resolution and the UUVs' operational capabilities, directly affecting the accuracy of the submersible's location estimation, as shown in Fig. 8 and 9.

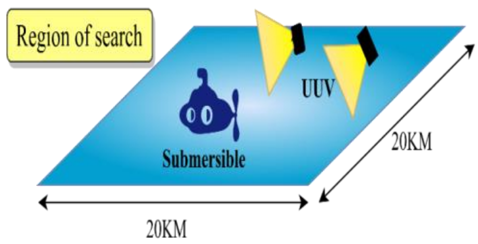

Figure 8. Schematic representation of the task area

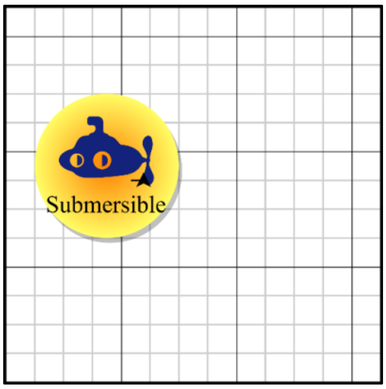

Figure 9. raster representation of the task area

The environment representation function K of the region where the submersible is located, derived from the Submersible Location predicting Model, yields the probability of the submersible's existence at any location r within the region:

$$exist(r) = \frac{1}{2\pi} exp\left[-\frac{1}{2}(r - r_0)^T K_i (r - r_0)\right] \tag{9}$$

Where r_0 represents the location where the probability of the distressed submersible's existence is high, as determined by the location Ing model in the first question. K_i is a diagonal matrix representing the intensity of existence at r_i.

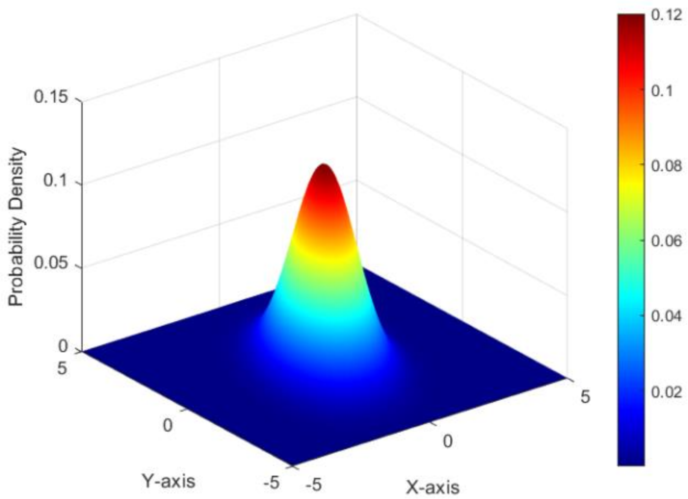

Figure 10. Probability density function of the submersible's location on the S plane

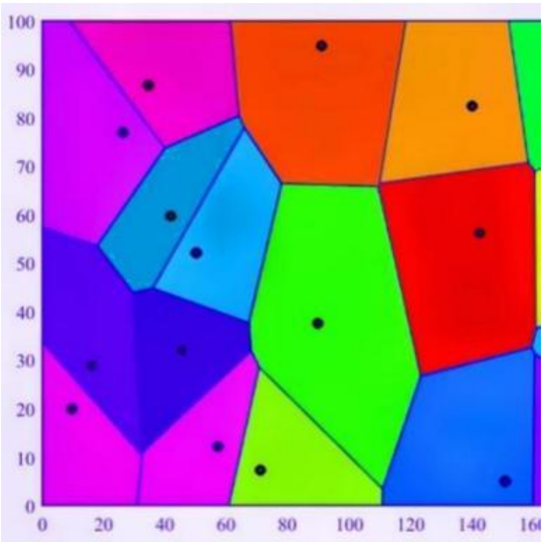

Figure 11. Schematic of Voronoi partitioning principles

The partitioned coverage search by a fleet of Unmanned Underwater Vehicles (UUVs) is modeled using Voronoi diagrams, which serves to divide the search area into regions based on proximity to each UUV, treated as a node. Each region contains only one UUV node, and any given point within a region is closer to its own node than to any other. The borders of these regions are equidistant from the neighboring nodes. This method of spatial division allows for the target area to be segmented into sub-regions corresponding to the number of UUVs in the fleet, optimizing the initial allocation of the search task. Despite the efficiency in coverage, this Voronoi-based partitioning may lead to an uneven distribution of search areas and does not consider the variable probabilities of finding distressed submersibles [5]. Consequently, it can result in disproportionate task loads for the UUVs and potentially irrational resource allocation. Therefore, maintaining uniformity in UUV task loads is essential for the rationalization of search operations as shown in Fig. 10 and 11.

4.2. Solution of the model

Synthesizing the analysis of Voronoi partitioning, the task load of the UUV is expressed as the sum of the probability of target existence within the designated region:

$$task(i) = \int_{r\in V(p_i)} exist(r)dr \tag{10}$$

The crucial aspect of the task allocation problem is to ensure the consistency of UUV load tasks, namely achieving:

$$task(1) = task(2) = \cdots = task(n) \tag{11}$$

In Nature, individuals often compete for resources, aiming to maximize resource occupancy while obeying certain rules and maintaining relative equilibrium. This is consistent with our research focus: UUV competition seeks balance, not elimination.

When UUV resources are fixed and the initial task distribution is unbalanced, the challenge is to reallocate resources to achieve equilibrium. In our work, UUV nodes represent individuals and search tasks represent total resources.

Inspired by biological competition mechanisms, mathematical modelling achieves balanced task distribution. Using a probability function for the location of distressed submarines and task load definition, individual UUV task loads can be derived.

We use $\{com_1, com_2 \cdots com_n\}$ to denote the competitiveness of each UUV, representing its capability to perform search tasks, where$\{com_1, com_2 \cdots com_n\}$ are constants. The resource ratio is defined as:

$$pro_i = \frac{task_i}{com} \tag{12}$$

In the context of the UUV fleet system, if the resource ratios differ between any two UUV, with the UUV having a higher resource ratio indicating an excessive task load (e.g., if $pro_i < pro_j, i \neq j$), UUVi will adjust its location towards the region where UUVj is located, moving along the line connecting the locations of the two UUVs.

This action by UUVi will prompt UUVj to reallocate a portion of its assigned region to UUVi. Assuming the locational movement characteristics of UUV are represented by a first-order integral system:

$$\dot{p}_i = u_i \tag{13}$$

$$u_i = -k_i \sum_{jeN_i} \vec{n}_{ij}\, \alpha_{ij}\left(pro_i - pro_j\right) \tag{14}$$

$$\alpha_{ij} = \int_{r\in V(p_i)V(p_j)} e\,xist(r)dr \tag{15}$$

Through the task allocation process, UUVs achieved a balance in task load.

The assignment of values to raster cells in the environmental raster map is based on the sonar confidence function obtained from the UUV forward-looking sonar, consisting of both a location confidence function and a direction confidence function.

The rules for assigning values to the location function of raster cells are as follows:

$$x_i = \begin{cases} \int_{r\in V(p_i)} exist(r)dr & unsearched \\ 0.5\int_{r\in V(p_i)} exist(r)dr & searched \\ -\infty & obstacle\ covered \end{cases} \tag{16}$$

To minimize the coverage of raster cells, it is assumed that each time a raster cell is covered, the location confidence function of the raster cell decreases by 0.5.

Another component of the confidence function is the direction confidence function value, introduced to reduce the repetition of search paths and prevent UUV from getting stuck in dead zones when encountering obstacles. The assignment rules are as follows:

$$y_i = \begin{cases} 1 - \frac{\Delta\theta_i}{\pi} & UUV\ is\ not\ in\ a\ dead\ zone \\ co\ s\ \Delta\varphi_i & UUV\ is\ in\ a\ dead\ zone \end{cases} \tag{17}$$

When not in a dead zone, $\Delta\theta_i$represents the difference between the current UUV direction angle θ_ε and the next location UUV direction angle θ, with $\Delta\theta_i$ given by:

$$[\Delta\theta_i = arctan\,2\,(ypp - ypc, xpp - xpc) - arctan\,2\,(ypi - ypc, xpi - xpc)] \tag{18}$$

The coordinates of the UUV at the previous, current, and subsequent moments are denoted by (x_{p_p}, y_{p_p}), (x_{p_c}, y_{p_c}), (x_{p_i}, y_{p_i}). The term $\Delta\theta_i$, where $\theta_i \in [0,\pi]$, signifies no angular change when equal to zero, indicating linear navigation as shown in Fig. 12 and 13.

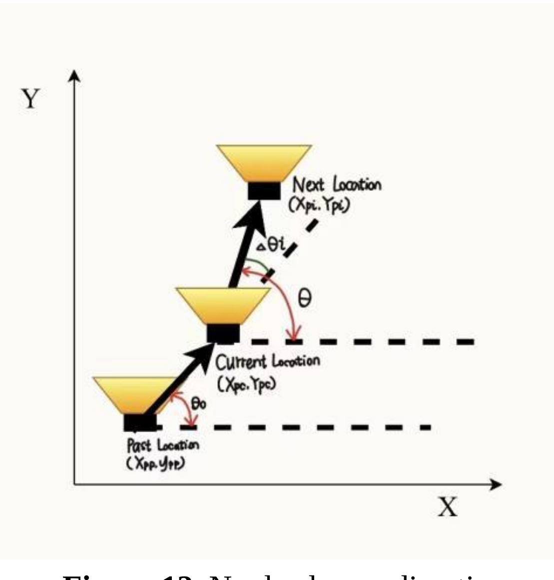

Figure 12. No dead zone direction angle change diagram.

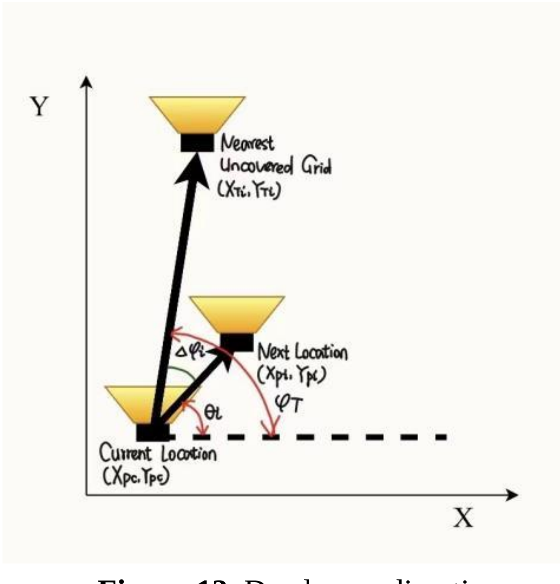

Figure 13. Dead zone direction angle change diagram.

If the UUV is in a dead zone, indicating that the surrounding raster states are already covered or obstacles, to escape the dead zone, the direction confidence function value needs to be set to the cosine of the difference between the current location and the angle of the nearest uncovered raster. $\Delta\varphi_\iota$ is given by:

$$\Delta\varphi_i = |\varphi_T - \varphi_i| = \left|atan2\left(y_{T_i} - y_{P_i}, x_{T_i} - x_{P_i}\right) - atan2\left(y_{P_i} - y_{P_i}, x_{P_i} - x_{P_i}\right)\right| \tag{19}$$

According to the definition of the direction confidence function in the previous section, if the UUV does not fall into a dead zone, then it is preferred to travel straight ahead to reduce the number of turns, reduce energy consumption, and improve the coverage rate.

This research introduces a novel task allocation framework for Unmanned Underwater Vehicle (UUV) fleets in search operations, leveraging Voronoi partitioning enhanced by biological competition principles to ensure balanced task distribution among UUVs. The methodology assigns tasks based on the aggregated probability of locating targets within designated regions, aiming for uniform workload across UUVs. To address potential imbalances, a competitive adjustment mechanism is employed, allowing UUVs to redistribute tasks dynamically, akin to natural entities vying for resources. Additionally, the study advances a raster confidence function for path planning, which assigns values to raster cells influenced by sonar data to optimize search paths and reduce redundancy. The effectiveness of search efforts is quantified through the Probability of Success (POS) and Cumulative Probability of Success (CPOS), which consider both the likelihood of the target's presence within the search area and the detection capabilities of the equipment. This probabilistic approach enables the iterative refinement of search strategies, enhancing the efficiency and success rates of underwater search and rescue missions.

When there are multiple lost submarines in the same sea area, we first use the position prediction model to get the trajectory of each submarine, and then build the position-probability function of each submarine in the sea area to be searched. After superimposing each probability function, we can get the probability-position function of the submarine's existence on the plane S. Then we can use the same search model [6].

Hypothesis: there are suspicious target locations in the region. According to the environment representation function k of the target region proposed above, any position r in the region can be obtained, and the corresponding target existence probability can be expressed as follows:

$$exist(r) = \sum_{i=1}^{m} \frac{1}{2\pi} exp[-\frac{1}{2}(r - r_i)^T K_i (r - r_i) \quad (20)$$

Among them, r_i represents the position where the target has a high probability of existence, and K_i is defined as a diagonal matrix, which means the existence intensity of r_i

A schematic diagram using 4 submarines as an example is presented in Fig. 14 and 15.

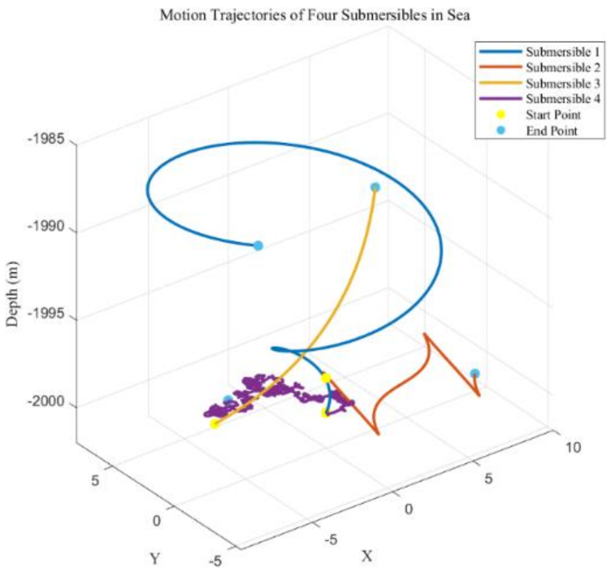

Figure 14. Predicted motion trajectories of four submarines

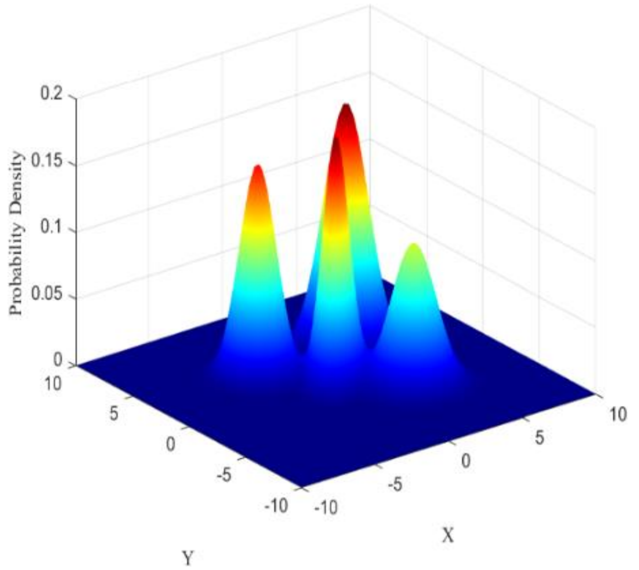

Figure 15. Location-submarine exist-once probability function on the plane S

5. Summary

The study presents a model that enhances the accuracy of underwater search operations by considering environmental factors such as ocean currents, varying sea densities, and seabed geography, ensuring robust results. The location model's universality allows for application across different marine environments, given sufficient environmental and submersible data. It introduces a basis function to quantify seabed uncertainties, improving target containment probability and model fault tolerance. The search strategy employs Voronoi partitioning for initial task allocation, followed by an equilibrium-based task redistribution and an improved raster confidence function for efficient UUV path planning, ensuring quick dead zone escape. However, the model's limitations include a lack of consideration for biological entities and constrained efficiency due to the discrete grid method limited by sonar range, suggesting areas for future enhancement.

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