Application of mathematical model in the field of financial investment

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Abstract. With the vigorous development of the international financial industry, more and more new financial products and investment strategies have emerged. Mathematical models with their rigorous logic and accuracy, through in-depth analysis of various data in the financial market, can reveal the laws hidden behind the data and help investors make more intelligent decisions. The wide application of mathematical models not only enriches the theoretical system of finance, but also plays an important role in the actual financial market. It is increasingly recognized that the study of mathematical model has become a key technology in the study of finance, and has played a positive role in promoting the stability and development of financial market. This paper will analyze the classical mathematical model and its application in the field of financial investment.

Keywords: mathematical model, financial markets, asset pricing and valuation.

1. Introduction

In the modern financial market, it has become the main development trend of modern financial analysis to quantify the objects studied, establish appropriate mathematical models, and apply modern mathematical knowledge to analyze the pricing of financial assets and their derivative assets [1]. With the acceleration of global economic integration and the rapid development of science and technology, the application of mathematical models in the field of financial investment has become more and more extensive and in-depth. Mathematical model through the use of statistics, probability theory and other methods to establish analytical models, these models to explain and predict various phenomena and data in the financial market, provide investors with effective decision-making tools, help financial institutions to develop more scientific investment strategies.

2. Application value of mathematical model in financial investment field

The core issue of financial theory is to evaluate the value and risk of financial assets in an uncertain environment [2]. Due to time factor, uncertainty factor and their interaction, financial behavior presents extreme complexity. Dealing with this complexity requires the introduction of mathematical tools. For example, uncertainty requires the introduction of probability, statistics and stochastic process theory, such as the allocation of resources in time and space requires the introduction of optimization models. The application of mathematical models in finance is very common and of great significance for investment decision-making. The application value of mathematical models in the field of financial investment is mainly reflected in the following aspects:

2.1. Risk assessment and management

In the financial market, risk is an inescapable problem for investors. By constructing the corresponding mathematical model, the risk of investment projects can be quantitatively analyzed, to help investors understand the potential risk factors and formulate the corresponding risk control strategy. This not only helps reduce investment risks, but also improves the accuracy and reliability of investment decisions.

2.2. Asset pricing and valuation

The price and valuation of assets are the focus of investors' attention. Mathematical models can reasonably price and value assets based on market data and asset characteristics. This not only helps
investors to understand the real value of assets, but also provides an important reference basis for investment decisions.

2.3. Optimize the investment portfolio

In financial investment, investors usually need to allocate their funds among several different investment projects to achieve diversification and diversification of assets. The mathematical model can provide the optimal investment portfolio scheme for investors through comprehensive analysis of the income, risk and other factors of investment projects, to maximize the return and minimize the risk of assets.

2.4. Provide effective risk management tools

As an important participant in the financial market, financial institutions are faced with complex and changeable risk challenges. Through the application of mathematical models, financial institutions can conduct quantitative analysis of their own risk exposure and formulate corresponding risk management strategies, to ensure their stable operation.

3. Application of mathematical model in the field of financial investment

3.1. CAPM Model

The Capital Asset Pricing Model (CAPM) is a cornerstone of modern financial market pricing theory [3], widely used in investment decision-making and corporate finance. The primary significance of this model lies in establishing an equivalent relationship between capital risk and return, clearly stating that the expected return of a security is the sum of the risk-free rate and the risk premium, and revealing the internal structure of securities returns. Another important aspect of the CAPM is that it classifies risk into non-systematic risk and systematic risk. Non-systematic risk is unique to a specific company or industry and can be diversified through asset diversification. Systematic risk, on the other hand, refers to the risk caused by factors that affect the entire market, which are inherent in the stock market and cannot be eliminated through diversification. The role of the Capital Asset Pricing Model is to diversify non-systematic risk through investment portfolios, leaving only systematic risk.

The CAPM model is specifically expressed as follows:

$$ R_a = R_f + \beta (R_m - R_f) $$

In this formula, $R_a$ represents the return on the asset portfolios; $R_f$ represents the risk-free rate of return; $R_m$ represents the return on the market portfolios, and $\beta$ represents the orthogonal covariance between the return on the asset portfolios and the return on the market portfolios.

As an essential theory in financial economics, the CAPM model provides scientific basis and guidance for hedgers by optimizing asset portfolios and market portfolios, laying a solid foundation for more advanced models.
3.2. Portfolio Model

Portfolio theory studies how to allocate resources among several competitive options with uncertain future outcomes (such as stocks and bonds) [5]. Portfolio allocation exists in many decision-making fields. Investors in financial markets must determine the composition of their portfolios, which include stocks, bonds, and their derivatives.

In 1952, Markowitz published an article titled *Portfolio Selection--Efficient Diversification of Investments* in the *Journal of Finance* [6], which is considered a milestone in the history of modern financial theory and marks the beginning of modern portfolio investment theory. Investors invest a given amount of funds for a certain period, purchasing securities at the beginning and selling them at the end. Therefore, at the beginning, investors need to decide which securities to purchase and how to allocate their funds among these securities. In other words, investors have to select an optimal portfolio from all possible portfolios at the beginning. In this case, investors have two decision-making goals: achieving the highest possible return and minimizing the uncertainty risk. The best objective is to strike the best balance between these two conflicting goals. The investment model established from this is known as the Markowitz Portfolio Model.

In Markowitz’s theory, a sound investment considers not only the expected rate of return but also the risk associated with it. The Markowitz Portfolio Model balances both the expected return and risk, which is actually a quadratic programming problem. Specifically, given a certain rate of return, investors tend to favor portfolios with lower risks. Conversely, for a given level of risk, investors prefer portfolios that offer higher returns. The illustrative chart of the Markowitz model reveals that as the number of stocks in a portfolio increases, the risk associated with the portfolio investment decreases, indicating a more significant effect of diversification in reducing risk.
The expected rate of return in the Markowitz Portfolio Model:

\[ E(R_p) = \sum_{i=1}^{4} \omega_i E(R_i), \]

In the formula, \( E(R_p) \) is the expected return of the portfolio; \( \omega_i \) is the investment proportion of the \( i^{th} \) asset, and \( E(R_i) \) is the expected return of the \( i^{th} \) asset.

Portfolio risk

\[ \sigma_p^2 = \sum_{i=1}^{4} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{4} \sum_{j=1}^{4} \omega_i \omega_j \sigma_{ij}, \]

In the formula, \( \sigma_p^2 \) is the variance of the portfolio; \( \sigma_i^2 \) is the variance of the \( i^{th} \) asset, \( \omega_i \) is the investment proportion of the \( i^{th} \) asset, and \( \sigma_{ij} \) is the covariance of asset \( i \) and asset \( j \).

The problem now is transformed into the expected return of a given combination such that the combination variance is minimized.

\[ \min \sigma^2 = \sum_{i=1}^{4} \sum_{j=1}^{4} \omega_i \omega_j \sigma_{ij}, \]

S.T.

\[ \begin{align*}
E(R_p) &= \sum_{i=1}^{4} \omega_i E(R_i), \\
\sum_{i=1}^{4} \omega_i &= 1.
\end{align*} \]

### 3.3. Risk-utility function model

Financial investment is an economic activity with risks. How to avoid risks is a problem that investors need to consider. In the process of investment, it is necessary to evaluate the future income situation, in which there is a high uncertainty. There are many factors affecting financial investment risk, such as time, policy, market environment and other uncontrollable factors. To accurately assess investment risk, it is necessary to apply the risk-utility function model and judge according to the deviation degree of return and expected value. The greater the deviation degree, the higher the uncertainty of return and the higher the investment risk will be. In financial investment, there are many types of risks, such as market risk, financial risk, interest rate risk and capacity risk. In the process of participating in financial investment, there are different types of investment due to different preferences. Individual investors have their own preferences, which can be roughly divided into three categories:

1. Risk Aversion
2. Risk Seeking
3. Risk Neutral

The three types of investors have different Risk Tolerance, and the higher the risk tolerance, the higher the risk investors are willing to bear to achieve their goals.

The reason for this discrepancy leads to Utility Theory [7], which quantifies the expectations and acceptable risks of such investors:

\[ U = E(r) - 0.5A\sigma^2 \]

\( U \): The Utility of assets to investors

\( E(r) \): The investor's expected return

\( \sigma^2 \): Variance of assets

\( A \): a measure of risk aversion
Figure 3. Measure of risk aversion

The risk-utility function model provides us with a powerful tool to quantify the relationship between risk and return. As a classic framework in the financial field, it helps investors to find the best balance point between risk and return. But it also has certain limitations. First, it assumes that investors' risk aversion is constant, whereas in reality, investors' risk appetite may change as external circumstances or personal financial circumstances change. Second, the model simplifies the measurement of risk by measuring it only by variance and does not adequately consider other factors such as skewness or kurtosis of the return distribution. By understanding and applying the risk-utility function model, investors can make investment decisions more scientifically. Rational investors should use these theoretical tools flexibly, considering actual circumstances and personal preferences. Make more scientific judgments about investment in future projects.

3.4. EVA valuation model of investment value

The intrinsic value of listed companies will be affected by many factors, such as the prospects of the industry, the stability of the capital market, the international situation and so on. It is challenging for evaluators to calculate the intrinsic value of a listed company. Therefore, some scholars choose to use different valuation methods to analyze the investment value of enterprises. EVA valuation model is one of the most widely used investment value analysis methods.

EVA model, also known as economic value added, is expressed through the difference between adjusted net income and income tax and the company's cost of capital\[8\]. EVA is derived from the earnings theory, and the biggest difference between EVA and accounting profit is that EVA adjusts some items, especially the impact on the cost of capital. EVA is an important indicator to measure corporate performance. When EVA is greater than 0, it means that the company has obtained super profit. The company's profit is higher than the cost of debt and equity, indicating that the company has brought huge benefits to investors and shareholders. When the cost of debt and equity is lower than 0, it means that the cost of debt and equity of the company is higher than the operating income, indicating that the profits generated by the company cannot make up for the cost already invested, which is the loss of the company's value.

EVA is the remainder of adjusted net operating profit after tax less cost of capital. Calculation formula:

\[ EVA = \text{NOPAT} - TC \times WACC \]

NOPAT is net income after tax,
\( TC \times WACC \) is the capital input of the company
\( TC \) is the possession of capital
\( WACC \) is a weighted average of capital.
It can be seen from the formula that the scale of EVA is related to the three factors mentioned in the formula, which will be analyzed below.

NOPAT is the portion of a company's earnings from operating activities less income tax. Accounting adjustment items include interest expense, impairment provision, etc.

Capital appropriation refers to the capital needed to maintain the normal production and operation of the company, which includes the capital of creditors and the investment of shareholders. The contribution of shareholders is the contribution of shareholders; Creditor capital refers to all kinds of loans provided by the creditor for the company, including long-term loans, short-term loans, etc. Because this money is free of charge to the company, it is not counted as capital possession.

WACC is the ratio of what a company pays to use its funds to what it can provide, which is largely determined by the company's cost of debt and equity capital.

4. Discussion

In financial investment, both return and risk are the issues that need to be paid attention to. To ensure the accuracy and robustness of investment, it is necessary to accurately evaluate financial risks. Present value formula model, linear programming model and risk utility function model are used to evaluate investors' expected returns. Using the financial mathematical model as a tool, we hypothesize and simulate different investment portfolios. In the process of investment, while seeking higher economic returns, avoid risks as much as possible to avoid economic losses. In the case of similar expected returns, people with different risk preferences hold different attitudes towards investment. To obtain a more scientific and accurate reference basis, it is necessary to use financial mathematical models for evaluation. In the actual investment risk assessment, aiming at improving the expected return and reducing the investment risk, the financial mathematical model is established, questions are raised specifically, and parameters are clarified. On this basis, model assumptions are made. A portfolio model is established, and then numerical simulation is carried out to calculate the maximum expected return, which is used as a reference to formulate a more perfect investment strategy. According to the results of income and risk assessment, we can judge whether the financial investment is safe and feasible, and provide a reliable reference for investment decision.

The flexibility of capital operation in the financial sector, to some extent, can also be regarded as a major factor in whether the investment is reasonable. However, if the analysis is blindly carried out according to the stage of financial investment, it is very likely to be biased. At this time, based on the financial mathematical model and using professional data framework to study the fund model structure, such problems can be solved well to ensure the flexibility of asset investment funds. For example, in the process of real estate investment, investors first need to assess the development of the financial sector of the industry from a macro perspective, determine whether the investment direction is tourist vacation housing or daily leasing, and then use the different investment methods of financial funds to calculate from the aspects of housing loan purchase, repayment forms, and the relationship between principal and loan. If investors can achieve budgetary access to the amount of funds in the process of discount transformation, investors can obtain profits in the process; On the contrary, investors will not be able to realize the investment robustness returns in the field of financial asset application, which is the exploration operation form of financial mathematical model in the capital operation degree in the field of financial economic investment.

5. Conclusion

To sum up, mathematical knowledge not only provides rich theoretical knowledge for financial research, but also is an indispensable tool in financial research. The application of mathematical knowledge can express the knowledge of economic theory and finance more visually, and its application value needs to be explored deeply. At present, with the continuous development and progress of information technology, we actively use the relevant contents of mathematical algorithms
to carry out the control of financial target law, which can effectively discover the inherent law of financial market, and promote the digitalization and scientific of financial theory through mathematical modeling, theoretical analysis, numerical calculation, quantitative analysis and other means.

References


