Comparison of Markowitz Model and Single Index Model with ESG Factors Involved

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Abstract. ESG considerations have become increasingly prominent within the realms of investor and corporate decision-making, underscoring the importance of employing robust methodologies for incorporating ESG factors into portfolio construction. This study aims to leverage the Markowitz Model and Single Index Model to construct portfolios utilizing a selection of 10 stocks, evaluating their applicability across various conditions, particularly investors' risk tolerance levels. Utilizing the solver function within Microsoft Excel, graphical representations will be generated under four distinct constraints, facilitating comparisons based on both graphical depictions and accompanying data analysis. The findings reveal that, under specified conditions, the two models exhibit negligible differences, while highlighting the significant role of ESG constraints in portfolio construction. Nonetheless, given the distinct assumptions and conditions underlying each model's calculations, investors retain the flexibility to choose either approach for portfolio analysis. This research underscores the impact of ESG factors on asset allocation outcomes within both models, offering valuable insights into ESG investing practices and the comparative performances of the Markowitz Model and the Single Index Model.

Keywords: ESG investing, Markowitz Model, Index Model, Portfolio analysis.

1. Introduction

ESG has been a popular topic discussed in corporations and among investors. ESG is a three-dimensional measurement of corporations' performances. The first dimension is the environment, which measures how well companies doing in protecting the environment. The second dimension is social, which measures how companies manage relationships with stakeholders like investors, employees, and related communities. The last dimension is governance, which measures how transparent corporations are governed [1, 2]. ESG matters encompass business concerns related to sustainability that have a direct influence on a company's performance, risk assessment, and ability to secure capital. In recent years, corporations have focused more on practicing sustainable business activities, and investors also taken ESG factors into account in stock investments. Investors are growing more convinced that companies excelling in ESG performance are not only less risky but also better equipped for long-term success and more resilient in the face of uncertainty [3]. For individual investors, there are multiple models to perform portfolio analysis. Markowitz Model and Index Model are more helpful in building investment portfolios. Much research conducted has shown that the Markowitz Model and Index Model have close relationships [4, 5]. Some argue that two models produce the same result since the graphs closely resemble [5]. Others reject it by demonstrating that investors risk tolerance determines which model they will utilize risk-averse investors often use Index Model, whereas risk-seekers use Markowitz Model [6]. In addition, characteristics of different stock markets come into play in determining the effectiveness of the models [7]. However, as ESG factors gradually become a crucial in investments, it is necessary to perform portfolio analysis with ESG factors included. This article will demonstrate how Markowitz Model and Index Model perform with the ESG scores of 10 company stocks included. The research provides insights for investors in choosing models to perform portfolio analysis. Additionally, the research functions as a bedrock for investors to engage factors other than ESG criteria based on their interests.

2. Method

2.1. Data

In this passage, there will be a portfolio constructed by 10 U.S. stocks from different industries, typically internet-retail, technology, financial service, and logistics. These stocks are traded on the NYSE and NASDAQ, which are AMZN, AAPL, FFIV, JPM, BRK/A, PGR, UPS, FDX, JBHT, and LSTR. Table 1 below shows the detailed information for the 10 stocks selected. The portfolio will be using historical closing prices of the stocks between 2003 to 2023, according to Yahoo! finance, in calculating annualized average returns, annualized standard deviations, beta, annualized alpha, and residual standard deviation. Weights of the stocks in the portfolio will be allocated using two models, Markowitz Model and Index Model under 2 constraints each. Additionally, ESG scores data from Bloomberg will come into play as a constraint in each of the 2 constraints for both models.

Table 1. The detailed information for the 10 stocks selected

Stock	Full Name	Sector (Yahoo!finance)
AMZN	Amazon.com, Inc.	Consumer Cyclical
AAPL	Apple Inc.	Technology
FFIV	F5 Networks, Inc.	Technology
JPM	JPMorgan Chase & Co.	Financial Services
BRK/A	Berkshire Hathaway Inc.	Financial Services
PGR	The Progressive Corporation	Financial Services
UPS	United Parcel Service, Inc.	Industrials
FDX	FedEx Corporation	Industrials
JBHT	J.B. Hunt Transport Services, Inc.	Industrials
LSTR	Landstar System, Inc.	Industrials

2.2. Constraints

For each model, there will be 4 optimization constraints.

$$w_i \geq 0, \text{ for } \forall_i \tag{1}$$

The first constraint is crafted to mimic the common constraints found within the mutual fund sector in the United States, that is, the U.S. mutual fund does not allow short sale of any security. For this constraint, the weight of every stock in the portfolio is designed to be greater than or equal to 0%.

$$\sum_{i=1}^{10}(E_i + S_i + G_i)w_i \leq 0.9 \times \sum_{i=1}^{10}(E_i + S_i + G_i)\,\widehat{w}_i \tag{2}$$

The second constraint is an additional constraint built upon the first constraint. It considers the ESG performances of the ten companies. An ESG score is calculated in advance to help evaluate the performance of companies. In specific, after having the efficient risky portfolio in constraint 1, an ESG score for the portfolio is calculated by multiplying each ESG score by the weight of each stock. However, the constraint limits the new portfolio's total ESG score to be less than 90% of the ESG total score of the efficient risky portfolio in constraint 1.

$$\sum_{i=1}^{10}|w_i| \leq 2 \tag{3}$$

The third constraint excludes previous constraints. It is formulated to replicate FINRA's Regulation T, permitting broker-dealers to enable their clients to hold positions where 50% or more of the funding originates from the client's account equity.

$$\sum_{i=1}^{10}(E_i + S_i + G_i)w_i \leq 0.9 \times \sum_{i=1}^{10}(E_i + S_i + G_i)\,\widehat{w}_i \tag{4}$$

The fourth constraint is also an additional constrain built upon constraint 3. However, it uses the efficient risky portfolio calculated in constraint 3 to be the new measure.

For each constraint, a minimum variance portfolio and an efficient risky portfolio will be calculated. A minimum risky portfolio is calculated by minimizing standard deviation to allocate weights accordingly, whereas an efficient risky portfolio maximizes the Sharpe ratio.

2.3. Model Analysis

The first model, as shown in Figure 1, used in this passage is the Markowitz Model, also known as the Modern Portfolio Theory or Mean-Variance Portfolio. It is first introduced by Harry Markowitz. The primary objective of the model is to construct the most efficient portfolio in terms of return relative to risk, achieved through the analysis of various combinations of assets based on their expected returns (mean) and standard deviations (variance). Markowitz rejected the common principle that higher returns follow by higher risks, rather, he stated that investors can diversify the risks of assets to construct an optimal portfolio with the highest possible return. As a result, Markowitz developed the theory based on several assumptions. The model assumes that investors are rational and risk-averse, so they will prefer a less risky portfolio over a riskier one with a given level of return [8].

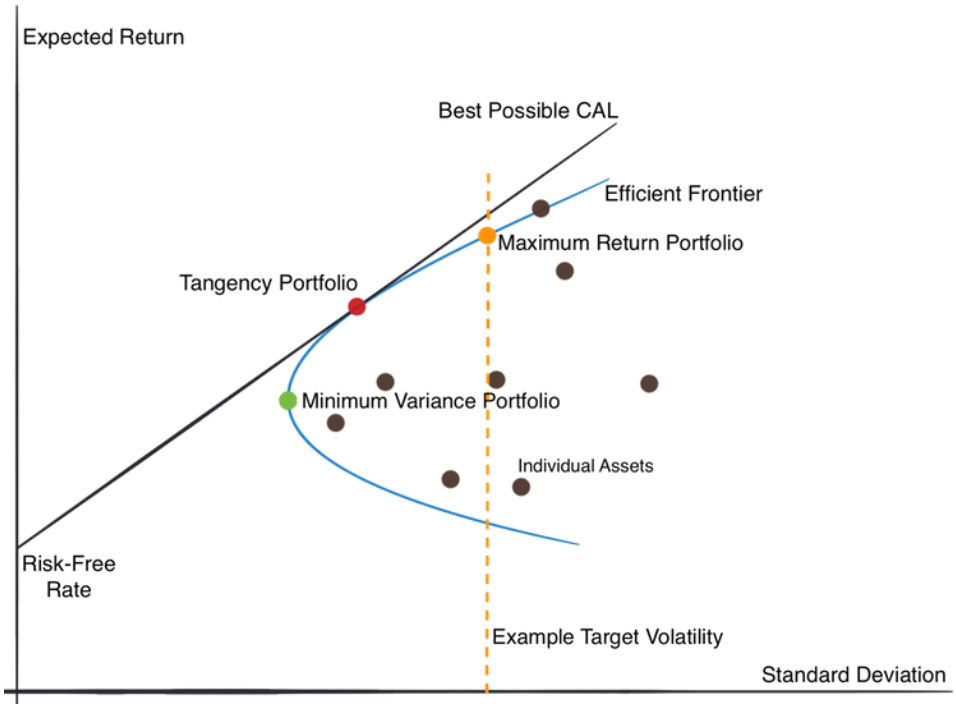

Fig. 1 Markowitz Model [9]

The second model used in this passage is the Single Index Model, introduced by William Sharpe. He avoids the calculation of the correlation matrix in the Markowitz Model by introducing systematic risk factor and firm-specific risk factors [10].

$$r_j = \alpha_i + \beta_i r_I + e_j \tag{5}$$

The single Index Model can be demonstrated by this equation, where
α_j= the expected return when the market is neutral
r_I= random return on the index (the common factor),
e_j= the random firm-specific component of the return,
$\beta_j$is the sensitivity of r_j to r_I, $\beta_j = Cov[r_j, r_I]/Var[r_I]$
As a result, $\alpha_j + e_j$is the return part independent of the market return, $\beta_j r_I$ is the return part due to market fluctuations.

In both models, the efficient frontier is plotted by maximizing return with the 4 given constraints in the solver for different level of standard deviations range from 10% to 50% with an increment of 0.5%. An inefficient frontier is similar to an efficient but minimizing returns. A minimal variance

frontier is plotted by minimizing standard deviation with given constraints in solver for different levels of returns ranging from -10%-50% with increment of 0.5%.

3. Results

Figure 2 represents the result of returns and risks with 4 constraints, where standard deviation and expected return are set on the x-axis and y-axis. The model also depicts three portfolios, they are minimum variance portfolio (big red dots) showing as Min Var portfolios in Table 2, maximum return portfolio (big green dots) showing as Max Sharpe portfolios in Table 2, and tangency portfolio. A minimum variance portfolio represents a portfolio with the least risk. A maximum return portfolio represents a portfolio with the highest return. Tangency portfolio represents a portfolio that lies at the point where the efficient frontier is tangent to the highest possible capital market line (CML) in the risk-return space, which is an optimal mix of assets that maximize an investor's risk-adjusted returns [11]. Besides, the red parabola represents the minimal variance frontier for constraints 1&2, and green squares and red squares lie above the minimum variance portfolios representing the efficient frontier. The green parabola represents the minimal variance frontier for constraints 3&4, and all green dots the lie on the frontier constructed an efficient frontier. Investors can choose any portfolio lies on the efficient frontier according to their risk tolerance level. All points lie under minimal variance portfolios are considered as inefficient portfolios since risks increase but returns decreases.

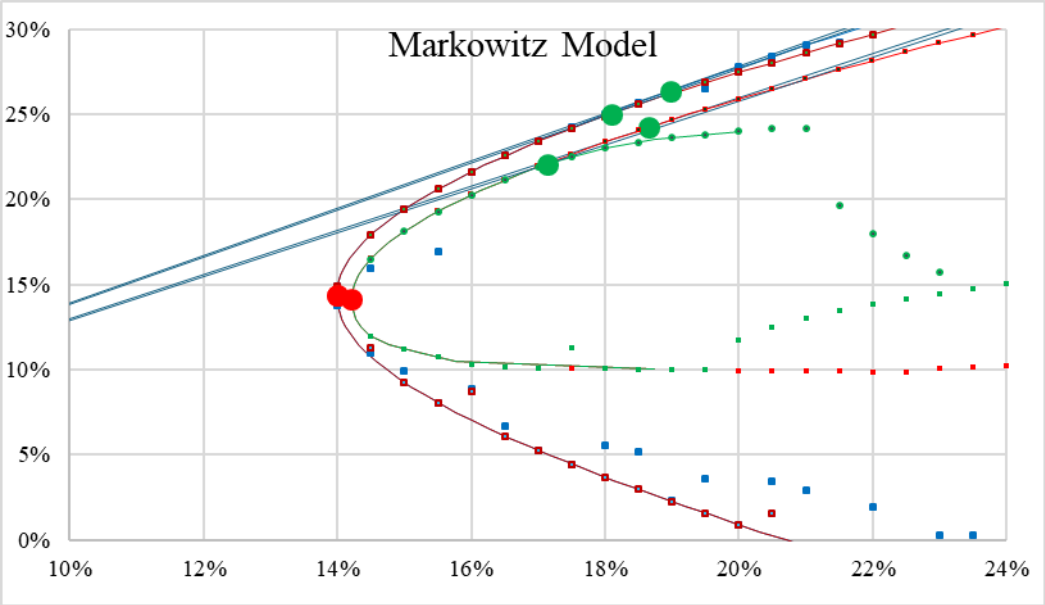

Fig. 2 Optimization Results in Markowitz Model

Table 2. Min Var and Max Sharpe Portfolio for Markowitz Model

Constraint 1	Return	StDev	Sharpe
MinVar	14.15%	14.21%	0.995
MaxSharpe	24.22%	18.65%	1.298
Constraint 2			
MinVar	14.15%	14.21%	0.995
MaxSharpe	22.07%	17.14%	1.288
Constraint 3			
MinVar	14.33%	13.99%	1.025
MaxSharpe	26.37%	18.97%	1.390
Constraint 4			
MinVar	14.33%	13.99%	1.025
MaxSharpe	25.03%	18.09%	1.383

Figure 3 is a Single Index Model that demonstrates the results with the same constraints as Figure 2. The model showed similar results to the Markowitz Model in constraints 1 and 2. For constraints 3 and 4, the Index Model demonstrates higher overall returns for efficient frontier with the same level of standard deviation. For the minimal variance frontier, which is the red parabola, the model reflects lower variance with the same level of returns.

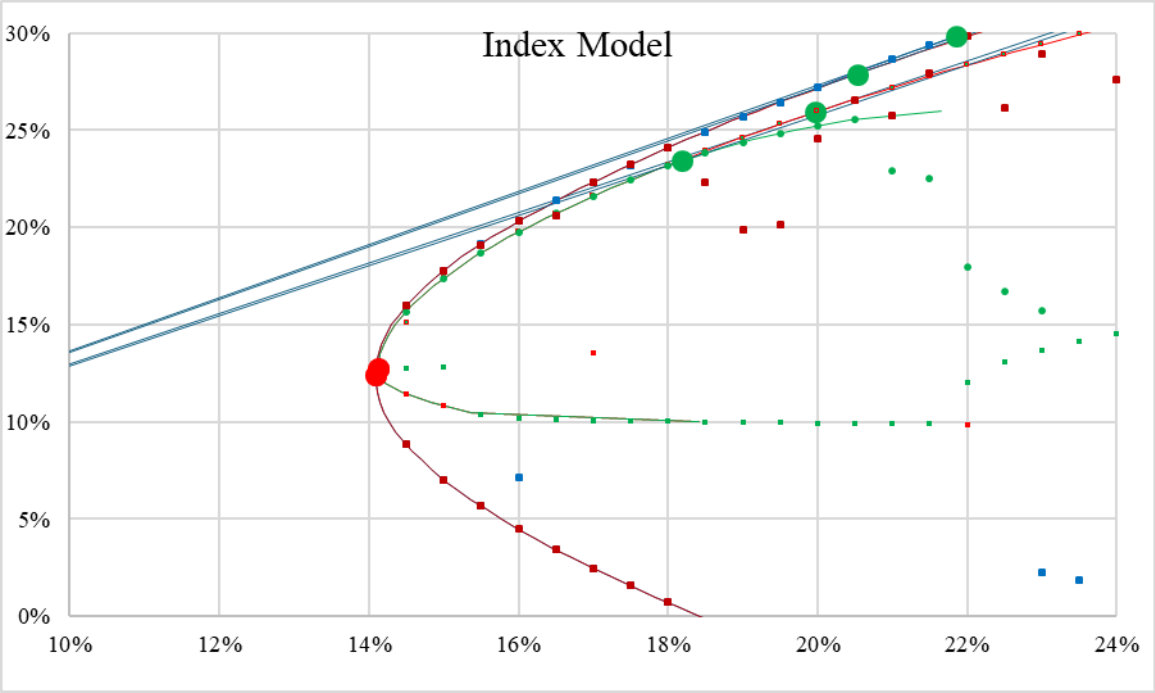

Fig. 3 Optimization Results in Index Model

However, maximum return portfolios in the Single Index Model give higher standard deviation with higher returns than in the Markowitz Model when maximizing the Shape ratio as shown in Table 3 compared to results in Table 2.

Table 3. Min Var and Max Sharpe Portfolio for Index Model

Constraint 1	Return	StDev	Sharpe
MinVar	12.77%	14.12%	0.904
MaxSharpe	25.92%	19.96%	1.298
Constraint 2			
MinVar	12.77%	14.12%	0.904
MaxSharpe	23.43%	18.19%	1.288
Constraint 3			
MinVar	12.40%	14.09%	0.880
MaxSharpe	29.84%	21.85%	1.366
Constraint 4			
MinVar	12.40%	14.09%	0.880
MaxSharpe	27.89%	20.53%	1.359

4. Discussion

The findings of the research indicate a substantial resemblance between the Markowitz Model and the Index Model, with only minor disparities evident within typical constraints. Consistent with previous research outcomes, the graphs exhibit similarity despite the differing calculations inherent to each model. Specifically, the Index Model tends to yield lower returns when minimizing variance, while higher returns are realized with increased variance when maximizing the Sharpe ratio. Conversely, the Markowitz model typically yields higher returns when minimizing variance, yet

lower returns and reduced variance when maximizing the Sharpe ratio. This discrepancy arises from the Index Model's emphasis on the correlation between returns and market risks, contrasted with the Markowitz model's foundation on various assumptions, such as investors opting for less risky portfolios at given return levels. Moreover, incorporating ESG factors into the analysis leads to a reduction in standard deviations compared to constraints lacking ESG considerations for both models. This underscores the risk-mitigating potential of constructing portfolios with ESG considerations. Some advocate that investing in ESG practices can aid in reducing systematic risk, which encompasses non-diversifiable risks affecting entire markets rather than specific industries [12]. Given the Index Model's focus on systematic risks, it proves particularly advantageous in constructing portfolios with ESG considerations, as it adeptly reflects the impact of ESG factors on returns [10]. Conversely, investors may opt for the Markowitz model to construct portfolios aimed at achieving the highest returns under a specified level of risk, leveraging calculations of mean and variance. Hence, this model is better suited for investors primarily concerned with returns and associated risks.

5. Conclusion

This research highlights the increasing significance of Environmental, Social, and Governance (ESG) considerations in both investor and corporate decision-making processes. By leveraging the Markowitz Model and Single Index Model, this study has demonstrated their applicability in constructing portfolios comprising a selection of 10 stocks under various conditions, particularly focusing on investors' risk tolerance levels. Through empirical analysis and graphical representations generated using Microsoft Excel, the research has revealed negligible differences between the two models under specified conditions, while emphasizing the significant role of ESG constraints in portfolio construction. Despite the distinct assumptions and conditions underlying each model's calculations, investors retain the flexibility to choose either approach for portfolio analysis. Moreover, this study underscores the impact of ESG factors on asset allocation outcomes within both models, offering valuable insights into ESG investing practices and the comparative performances of the Markowitz Model and the Single Index Model. The findings suggest that incorporating ESG considerations into portfolio construction can lead to risk reduction and potentially enhance portfolio performance, thereby influencing investor decision-making processes.

Both the Markowitz Model and the Single Index Model are based on certain assumptions about investor behavior and market dynamics. These assumptions may not always hold true in real-world scenarios, potentially impacting the applicability of the models and the generalizability of the findings. Furthermore, the research imposes specific optimization constraints and simplifications in the portfolio construction process, such as the exclusion of short selling and the use of particular ESG scoring methodologies. These constraints may not fully capture the complexities of real-world investment decisions and portfolio management strategies. Lastly, in the comparison of the performances of the Markowitz Model and the Single Index Model, the research does not encompass all relevant portfolio construction methodologies or alternative approaches. Other models or techniques could offer different perspectives on ESG integration and portfolio optimization. Future studies could refine ESG scoring methodologies and metrics to better capture the multidimensional nature of environmental, social, and governance factors. Incorporating dynamic ESG data sources and evolving scoring frameworks could enhance the accuracy and relevance of ESG integration in portfolio management. Incorporating additional portfolio construction models is also necessary in monitoring portfolio performances.

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