Research on the Development of Investment Portfolio Theory

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Abstract. This paper provides a comprehensive analysis of portfolio theory evolution and its practical applications, tracing its development since Harry Markowitz introduced Modern Portfolio Theory (MPT) in 1952. The urgency to enhance capital market efficiency, optimize asset portfolios, and effectively measure and manage risks has intensified due to the rapid transmission of risk across global financial markets. This study reviews historical advancements and recent achievements in portfolio theory and practice. Furthermore, it constructs a theoretical research framework to guide the future development of quantitative investment strategies, focusing on active portfolio management. This framework also delineates several prospective research avenues. The advent of artificial intelligence and big data has transformed portfolio theory into a multidisciplinary nexus involving mathematical statistics, machine learning, and behavioral finance. This integration is crucial for refining asset portfolio optimization and understanding the dynamics of risk spillover and its determinants. The findings and theoretical advancements discussed herein are vitally important for fostering the sustainable development of capital markets and ensuring economic and financial stability. This study not only revisits foundational theories but also sets the stage for pioneering future research in financial analysis and portfolio management.

Keywords: Portfolio Theory, Risk Management, Quantitative Investment Strategies, Capital Market Efficiency, Financial Stability.

1. Introduction

This paper delves into the evolution of portfolio theory and practice, tracing its progression from the seminal introduction of Modern Portfolio Theory (MPT) by Harry Markowitz in 1952 [1]. The significance of MPT and its derivatives in contemporary finance cannot be overstated, particularly in an era characterized by rapid technological advancements, increasing market volatility, and the globalization of financial markets. These dynamics have rendered traditional investment strategies insufficient, necessitating more sophisticated analytical techniques to navigate the complexities of diversified assets and dynamic market conditions.

Portfolio optimization, a cornerstone of modern financial theory, strives to strike an optimal balance between risk and return, thereby maximizing investment returns while minimizing risks. This optimization is increasingly driven by advanced data analytics, which enhances decision-making by uncovering hidden trends, patterns, and correlations within vast datasets [2]. The integration of robust statistical methods and machine learning techniques has propelled the capabilities of portfolio management, enabling investors to achieve a more precise estimation of risk and return profiles [3].

The burgeoning field of data analytics not only equips investors with enriched information but also provides deep insights into market dynamics. This allows for more accurate identification of investment opportunities and more informed asset allocation [4]. As financial markets continue to evolve, the role of sophisticated portfolio theory grows, underpinning the development of quantitative investment strategies and the pursuit of financial stability. This paper reviews historical advancements and latest achievements in portfolio theory, setting the stage for future innovations in financial analysis and investment practice [5].

The remaining of this paper is organized as follows: section II discusses the foundational principles and key figures of early investment theory, highlighting their lasting influence on modern investment strategies. Section III explores Behavioral Investment Theory, examining how psychological factors and investor behavior impact financial decision-making and market outcomes. Section IV delves into Quantitative Investment and Algorithmic Trading, detailing the integration of advanced mathematical models, data analysis, and automated systems in portfolio management. Finally, the paper concludes by summarizing the significant insights gained from the review and discussing future research directions in portfolio theory, emphasizing the need for further exploration of interdisciplinary approaches and technological innovations in finance.

2. Modern Portfolio Management Theory

2.1. Investment Theory

The foundational framework of modern investment theory was significantly shaped by several key figures, whose pioneering contributions have enduring relevance in the field. Benjamin Graham, often heralded as the "father of modern investment," advocated for value investing, a strategy emphasizing the importance of assessing the intrinsic value of enterprises. His seminal works, "The Intelligent Investor" and "Security Analysis," remain quintessential readings in investment literature [6]. Graham and Dodd co-developed the principles of value investing, underscoring the importance of long-term investment horizons and the acquisition of stocks priced below their intrinsic value [7].

Fisher introduced a complementary perspective with his development of "growth investing," which focuses on a company's long-term growth potential and market position. His influential book, "Common Stocks and Uncommon Profits," has profoundly shaped the growth investing paradigm [8]. Additionally, Williams extended the discourse on value investing, what is commonly referred to as "Williams' Formula." This valuation model provided a robust theoretical underpinning for the concept of value investing [9].

Collectively, these theorists laid the groundwork for contemporary investment strategies. Their emphasis on the intrinsic value of companies, coupled with a long-term investment focus and rigorous risk management, continues to influence modern financial practices and investor behavior profoundly. Their enduring legacy is reflected in the ongoing relevance of their methodologies to contemporary market analyses and investment decisions.

2.2. Portfolio Theory

2.2.1. Markowitz Portfolio Theory

The evolution of modern portfolio management theory has yielded significant theoretical and practical advancements. A pivotal moment in this development was in 1952, when Markowitz introduced the mean-variance (MV) model, positioning risk and return at the core of investment decision-making. This model is widely regarded as the inception of contemporary investment theory [10]. Markowitz's hypothesis was premised on the notion that investors diversify their holdings across a range of securities to form a portfolio, rather than investing in a single asset.

Markowitz articulated that the primary concern for investors should be the construction of an optimal asset portfolio that minimizes risk for a given level of expected return, or maximizes return for a given level of risk. This conceptual framework uses the mean-variance approach to effectively capture the essential elements of return and risk [11]. The introduction of Markowitz’s portfolio theory addressed three fundamental challenges that had long vexed securities investment:

The Rationale for Portfolio Investment: Modern portfolio theory posits that diversifying investments across a portfolio of securities can substantially reduce investment risk.

Determining the Efficient Frontier: Markowitz described how, given a fixed set of securities, variations in the proportion of investments can yield an infinite number of portfolios, each with different risk-return profiles—collectively known as the feasible set. The efficient frontier is then defined as the set of portfolios offering the highest expected return for a given level of risk or the lowest risk for a given level of expected return, forming a convex curve on the risk-return graph, often referred to as the Markowitz Boundary [12].

Selection of the Optimal Investment Portfolio: The selection of an optimal portfolio along the efficient frontier is influenced by an investor's risk preference, which is represented by a set of indifference curves. These curves, typically parallel to one another, form a family indicating various levels of investor satisfaction. The optimal portfolio is identified at the point where an indifference curve is tangent to the efficient frontier [13].

Markowitz's Portfolio Theory laid the groundwork for further developments in investment theory, including the Capital Market Line (CML) and the Capital Asset Pricing Model (CAPM), which elaborated on the relationship between risk and return. These models have profoundly influenced both academic research and practical investment strategies, continuing to be fundamental to the understanding of financial markets and portfolio management [14].

2.2.2. The Application of Markowitz Portfolio Theory

MV model integrates risk and return into the core of investment decision-making, has been the foundation for numerous subsequent innovations in portfolio management. Over the years, many scholars have extended Markowitz's theory to include various transaction cost constraints and other real-world considerations.

For instance, Pogue was one of the first to address the MV portfolio problem under the transaction cost constraint in 1970 [15]. Subsequently, Konno introduced an alternative risk measure using absolute deviation, which he effectively solved using an equivalent linear programming model [16]. Furthermore, Zhou and Russell explored the dimensions of liquidity preferences and investment behaviors, incorporating these into risky portfolio choices. They developed a multi-objective portfolio model aimed at maximizing utility while minimizing risky assets, which they termed the Target Portfolio Model [17]. However, these models typically rest on the assumption of a "completely rational man," primarily focusing on asset returns and transaction costs to analyze portfolio issues.

The adoption of portfolio optimization techniques has seen rapid expansion since its inception. A survey by Fabozzi et al. indicated that 92% of asset management firms utilize some form of portfolio optimization [18]. Among these, the mean-variance optimization technique remains predominant, employed by 83% of asset management firms to enhance portfolio performance. Additionally, 42% of firms utilize Utility Function Optimization (UFO) for similar purposes. Meanwhile, 25% of surveyed firms have adopted Robust Optimization, with a growing number beginning to explore Stochastic Optimization.

These developments highlight the enduring impact and continuous evolution of Markowitz's mean-variance optimization framework within the field of financial portfolio management. The widespread adoption and adaptation of this model underscore its foundational role in shaping modern investment strategies and risk management practices.

2.2.3. The Constraints Of Portfolio Theory

Barberis conducted an empirical analysis of the dynamic portfolio choices of long-term investors, particularly considering the uncertainty inherent in these decisions [19]. The findings suggest that with increased predictability of long-term investment returns, investors are inclined to allocate a higher proportion of their portfolio to equities. Moreover, the propensity to invest in equities intensifies with an extended investment horizon. However, Barberis cautioned that long-term investors must carefully consider risk estimation factors to prevent excessive equity allocation, particularly when future returns are uncertain [19].

Recent research has also focused on the implications of securities transaction costs, liquidity, and various constraints on portfolio theory. Transaction costs encompass both direct expenses such as fees, taxes, and bid-ask spreads, and indirect costs like slippage. Liquidity, often measured by bid-ask spreads, trading volume, or the cost of trading, reflects the ease with which securities can be bought or sold at reasonable prices in the market. Additionally, investment strategies are frequently subject to a range of limitations, including regulatory constraints and market trading conditions. Barberis's analysis extended to exploring how such factors as market trading limitations and risk requirements influence the decision-making of long-term investors. Barberis argued that ignoring these elements in portfolio optimization models generally leads to suboptimal outcomes in real-world portfolio management [19].

Longstaff demonstrated that even short periods of illiquidity could significantly lower the discount rate of securities [20]. And further expanded this research by examining asset portfolio optimization under liquidity constraints, such as borrowing limits and short-selling restrictions. These constraints tend to lead investors to allocate greater weights to stocks, particularly over longer investment horizons [21].

These studies underscore a broader trend in portfolio management research, which involves considering more dynamic and realistic constraints that reflect actual market conditions. This shift acknowledges that long-term investment behavior often diverges notably from theoretical models due to practical constraints like short-selling restrictions or the regulatory environment.

The evolution of modern portfolio theory is likely to embrace inter-period investment under continuous time conditions, asset pricing with dynamic features, and models that incorporate global influencing factors and non-linear relationships. Additionally, quantitative research on behavioral finance, non-normal distribution phenomena under extreme market conditions, and advanced quantitative modeling leveraging big data and complex computation are expected to be pivotal in refining and advancing modern portfolio theory. These developments will not only enhance theoretical understanding but also improve the pragmatic application of portfolio management strategies in increasingly complex financial markets.

2.2.4. Optimization of Portfolio Theory

Prospect theory and its enhanced variant, cumulative prospect theory (CPT), have significantly influenced the fields of project analysis, risk control, and investment management. In the context of complete markets, these theoretical frameworks often reduce the portfolio selection problem to a static optimization issue, with the decision variable being the optimal terminal wealth. This approach has been extensively explored in continuous-time market settings [22].

However, the scenario in incomplete markets, particularly those analyzed in discrete-time frameworks, necessitates specific assumptions for model development. For instance, Barberis and Huang explored the CPT optimization model assuming a normal distribution, while Pirvu and Schulze extended this to elliptical distributions [23]. Further, Barberis and Xiong conducted studies on bifurcation distributions, including a bifurcated tree distribution [24]. Additionally, Wu et al. examined the interplay between irrational expectations and the pronounced disposition effect among financial market investors within a discrete-time portfolio decision model, framed by prospect theory [25].

Moreover, there has been significant scholarly interest in multi-period CPT modeling. Researchers such as De Giorgi and Legg, Shi et al. have delved into the dynamics between various periods [26]. Yan Shuli et al. [27] proposed a decision-making framework based on cumulative prospect theory and the gray-target approach, tailored for dynamic risky decisions with unknown indicator weights.

Despite these advancements, the field still faces substantial challenges, particularly in the development of high-performance algorithms for solving these complex models. Currently, heuristic algorithms and general nonlinear optimization solvers are predominantly used [28]. The optimization problem under incomplete market conditions, as addressed by CPT, remains ripe for further theoretical and algorithmic exploration. This gap underscores the need for more sophisticated analytical methods to enhance the robustness and efficacy of portfolio optimization models in these settings.

2.3. Investment Models Proposed Based on Markowitz Portfolio Theory

2.3.1. CAPM

In 1964, William F. Sharpe introduced a groundbreaking approach to evaluating investment risk and return through the development of CAPM [29]. This model utilizes the capital market line, derived under the assumption of market equilibrium, as a benchmark. Specifically, CAPM employs the total risk of a portfolio, quantified by its standard deviation, to normalize the risk premium of the portfolio. This normalization provides a measure of the return per unit of total risk, thereby facilitating a quantitative analysis of portfolio performance. The elegance of Sharpe's CAPM lies in its simplicity and the minimal number of parameters it requires. CAPM not only enhances the theoretical understanding of the risk-return tradeoff but also boasts considerable practical application value, offering a robust tool for financial analysts and portfolio managers in diverse market scenarios.

2.3.2. APT

In 1976, building upon the foundational work of Markowitz and Sharpe, Ross introduced a novel evaluative framework for investment portfolios—APT [30]. The APT provides a flexible framework, positing that returns are linearly dependent on various macroeconomic and firm-specific factors, thereby allowing for the analysis of multiple sources of risk. This multivariate approach is contrasted with the single-factor model of CAPM, thus offering a more nuanced understanding of the factors driving returns and their sensitivities.

Despite its significant contributions and broad applicative value, the APT is not without limitations. One key critique is its ambiguity concerning the exact number and nature of the factors that should be included. While market influence is undoubtedly a crucial factor, the APT does not specify which additional factors should be considered to encapsulate this influence fully. Moreover, the theory does not address potential substitutes for the market factor when it is absent from the model [31]. This lack of specificity can complicate the practical implementation of the APT, as the selection of relevant factors becomes subjective, potentially varying between different implementations and market conditions.

3. Behavioral Investment Theory

3.1. Behavioral Finance

Early research in portfolio theory predominantly focused on static portfolio problems under discrete-time settings, often overlooking critical practical factors such as liquidity, transaction costs, and taxes. However, recent advancements in mathematical modeling, operations research, and computational technology have catalyzed the development of sophisticated methods in portfolio analysis. These include the incorporation of Behavioral Finance, Value-at-Risk (VaR) methodologies, dynamic portfolio management under continuous-time frameworks, and models that integrate various real-world constraints such as liquidity and transaction costs.

Behavioral finance, an interdisciplinary field combining finance and psychology, has emerged as a potent framework to address these gaps. This approach suggests that market prices are not solely determined by the intrinsic values of securities and macroeconomic or microeconomic factors, but are also profoundly influenced by the psychological and behavioral patterns of investors. Kahneman and Tversky have been pivotal in this field, introducing Prospect Theory, which integrates psychological insights into economic models. They described decision-making features such as reference dependence, diminishing sensitivity, and loss aversion, portraying a more nuanced view of investor rationality [32].

Prospect Theory posits that investors are not always fully rational. Their decision-making often reflects bounded rationality, influenced by their psychological biases and expectations. In financial markets, this is manifested in a "judgment-regulation-feedback" cycle, where investors continually adjust their perceptions and investment strategies based on new market information. This cycle leads to the evolution of investment behaviors as investors' psychological states and cognitive assessments evolve, offering a dynamic and realistic depiction of financial decision-making in fluctuating markets.

3.2. The Integration of Modern Portfolio Theory with Behavioral Finance

The integration of modern portfolio theory with behavioral finance represents a significant contemporary trend in financial research. Numerous studies have highlighted irrational behaviors in the stock market, providing empirical evidence and theoretical frameworks for understanding these phenomena. DeBondt and Thaler demonstrated that markets tend to overreact, leading to a long-term reversal trend of three to five years, which underpins the strategy of contrarian investment [33]. Similarly, Jegadeesh and Titman identified that a strategy of buying stocks that are performing well and selling those in a downtrend over a six to twelve-month period can yield returns that exceed average market performance, thereby supporting the viability of momentum investment strategies [34].

Several studies have sought to explain these irrational behaviors from the perspective of behavioral finance. For instance, Daniel et al., attributed such market anomalies to investor overconfidence and self-attribution, suggesting that these psychological biases lead to market overreactions and price anomalies such as the price-to-book effect and long-term reversals [35]. Barberis et al. focused on the roles of human extrapolation and conservatism in psychological processes, arguing that investors often erroneously interpret market patterns from limited data, a behavior that propels stock market overreactions and subsequent corrections [36]. They also posited that a conservative mindset can cause investors to underestimate market signals, contributing to momentum effects in stock prices. Stein introduced another dimension by suggesting that the momentum effect in stock markets can be attributed to the gradual dissemination of information, where investors' tendencies to rely on past performance can exacerbate market overreactions [37].

In the realm of portfolio management, behavioral finance continues to be a fertile area of exploration. Shefrin and Statman developed the Behavioral Portfolio Theory (BPT), which integrates the concept of mental accounting within the framework of investment decision-making [38]. This theory distinguishes between 'single-investment mental accounts' and 'compound investment mental accounts'. The former adheres to a basic investment level aimed at "preventing poverty," while the latter seeks "pursuing affluence," reflecting different risk and return objectives and investment strategies. This multilayered approach to portfolio construction provides a nuanced understanding of how different investment goals influence investor behavior and decision-making processes. This theoretical framework, therefore, offers a rich insight into the complexity of investment strategies shaped by varied investor psychologies and objectives.

4. Quantitative Investment and Algorithmic Trading

The domain of Quantitative Investment and Algorithmic Trading represents a significant evolution in the field of portfolio management, characterized by the integration of sophisticated mathematical models, comprehensive data analysis, and automated trading systems. This section elaborates on how quantitative strategies have refined the capabilities of investors and reshaped the landscape of financial markets.

4.1. Quantitative Investment Strategies

Quantitative investment strategies employ advanced mathematical models to analyze financial data and make investment decisions [39]. These strategies are primarily data-driven, relying on historical and real-time data to predict market trends and asset price movements. The core advantage of quantitative investment is its ability to process vast amounts of information, far beyond the capability of human analysts, thereby uncovering profitable opportunities that might be invisible to traditional analysis.

One of the seminal models includes the APT, offerring a more flexible alternative to the CAPM by considering multiple factors that might influence the returns of a portfolio, such as macroeconomic variables or firm-specific attributes.

4.2. Algorithmic Trading

Algorithmic Trading is another pillar of modern financial strategies. The algorithms are designed to identify slight inefficiencies in the market or to implement complex trading strategies across multiple markets simultaneously. For instance, High-Frequency Trading (HFT) strategies, which involve buying and selling securities in fractions of seconds, leverage advanced algorithms to exploit small price gaps between markets [40].

The deployment of these algorithms has not only increased the efficiency of markets but also raised several regulatory and ethical concerns due to their potential to create significant market impacts within milliseconds. The Flash Crash of 2010, where the Dow Jones Industrial Average plummeted over 1000 points only to recover those losses within minutes, serves as a stark reminder of the potent influence algorithmic trading can wield over financial markets [41].

4.3. Integration of Machine Learning and Artificial Intelligence

Recent advancements in Artificial Intelligence (AI) and Machine Learning (ML) have further propelled the capabilities of quantitative investment strategies. Machine learning models, such as neural networks and decision trees, are now commonly used to predict stock prices and market movements based on complex patterns and relationships that traditional statistical methods might not easily capture.

The integration of AI into algorithmic trading has also led to the development of autonomous trading systems that can adapt their strategies in real-time based on market conditions. These systems are capable of learning from past trades and adjusting their parameters for future trades to optimize performance.

4.4. Ethical and Regulatory Considerations

As the field of Quantitative Investment and Algorithmic Trading matures, it brings with it a host of ethical and regulatory challenges. The opaque nature of algorithmic decisions, combined with their potential to affect market stability, necessitates transparent and robust regulatory frameworks. Regulators and market participants continue to debate the extent of oversight required, balancing innovation with the need to prevent market abuse and ensure investor protection.

4.5. Future Directions

The field of Quantitative Investment and Algorithmic Trading is poised for further growth, driven by continuous advancements in technology and data analytics. The development of quantum computing, for instance, could revolutionize the speed and efficiency of algorithmic calculations, potentially opening new frontiers in financial strategy formulation.

Moreover, as data sources grow both in volume and variety, there is a burgeoning opportunity to refine investment strategies further and enhance market prediction models. The integration of unstructured data sources like social media sentiment, news articles, and economic reports into quantitative models could provide even deeper insights into market dynamics and investor behavior.

In summary, the evolution of Quantitative Investment and Algorithmic Trading continues to play a pivotal role in the development of modern portfolio theory and practice. As these strategies become increasingly sophisticated and integral to market operations, the need for advanced analytical tools and robust regulatory frameworks becomes ever more critical. This dynamic sector promises to remain at the forefront of financial innovation, continually challenging and reshaping the boundaries of investment strategy and market theory.ith the development of computer technology and big data, quantitative investing and algorithmic trading have gradually become a hot topic in the investing field. By constructing mathematical models and computer programs, investors can analyze a large amount of data in a short period of time and achieve automated trading. The application of these techniques improves investment efficiency and reduces transaction costs.

5. Conclusion

This paper has embarked on a comprehensive exploration of the evolution and practical implications of portfolio theory, beginning with the seminal work of Harry Markowitz in 1952, which established the foundational principles of Modern Portfolio Theory (MPT). The historical and recent advancements in portfolio theory culminated in a detailed examination of its impact on current financial practices and its potential future developments. This paper revisited the classical models that have shaped investment strategies over the decades, highlighting the enduring relevance of thinkers like Graham, Fisher, and Williams. Their contributions to value and growth investing have not only enriched the theoretical landscape but have also provided practical frameworks that continue to influence investment decisions in contemporary markets. The paper also discussed the pivotal role of Markowitz's mean-variance model in modern portfolio management, emphasizing its significance in aligning risk and return objectives. Advancements in this area have been greatly accelerated by the integration of statistical methods and machine learning, which facilitate sophisticated data analysis and enhance the precision of risk-return estimations. Moreover, the study addressed the dynamics of behavioral finance and the psychological factors influencing investor decisions, underscoring the complexity of financial markets beyond mere numerical data. The exploration into quantitative investment and algorithmic trading further illustrated how technological innovations are continuously reshaping the landscape of portfolio management, pushing the boundaries of traditional investment strategies. In contemplating the future of portfolio theory, it is evident that the field is moving towards an increasingly interdisciplinary approach. The integration of artificial intelligence and big data analytics into portfolio management not only opens up new avenues for optimizing asset allocation but also presents challenges that require novel theoretical frameworks and practical solutions. This evolution calls for a sustained commitment to research that bridges the gap between theoretical finance and real-world applications.

In the future, the need for robust, adaptable strategies that can handle the complexities of global financial markets is more pressing than ever. The theories discussed herein provide a solid foundation, but they also invite further innovation and refinement. Future research should continue to explore the integration of technology in finance, the implications of global economic shifts on investment strategies, and the development of tools that can effectively manage the growing interconnectivity of markets and assets.

Although this paper has covered substantial ground in tracing the trajectory and transformations within portfolio theory, it is clear that the journey is far from complete. The rapid evolution of financial markets demands continuous theoretical and practical advancements. By building on the robust frameworks established by past and current scholars, and by embracing the potential of emerging technologies, the field of portfolio management can look forward to a future that is not only rich with challenges but also abundant with opportunities for groundbreaking research and innovative investment strategies.

References

[1] Markowitz, H. (1952) the utility of wealth. Journal of political Economy, 60, 151-158.

[2] Ibeh, C.V., Asuzu, O.F., Olorunsogo, T., Elufioye, O.A., Nduubuisi, N.L. and Daraojimba, A.I. (2024) Business analytics and decision science: A review of techniques in strategic business decision making. World Journal of Advanced Research and Reviews, 21, 1761-1769.

[3] Moreno Paredes, J.C. (2018) Developing insights related to portfolio management and individual investors by overcoming problems associated with analysing large scale financial data. Doctoral dissertation, University of Southampton.

[4] Michaud, R.O. and Michaud, R.O. (2008) efficient asset management: a practical guide to stock portfolio optimization and asset allocation. Oxford University Press.

[5] Tufano, P. (2003) financial innovation. Handbook of the Economics of Finance, 1, 307-335.

[6] Graham, B. and McGowan, B. (2003) the intelligent investor. HarperBusiness Essentials, New York.

[7] Graham, B., Dodd, D.L.F., Cottle, S. and Tatham, C. (1951) Security analysis: Principles and technique. McGraw-Hill, New York, p. 851.

[8] Fisher, P.A. (2003) Common stocks and uncommon profits and other writings. John Wiley & Sons, 40.

[9] Williams, J.B. (1938) the theory of investment value. (No Title).

[10] Markowitz, H.M. and Todd, G.P. (2000) Mean-variance analysis in portfolio choice and capital markets. John Wiley & Sons, 66.

[11] Mangram, M.E. (2013) A simplified perspective of the Markowitz portfolio theory. Global journal of business research, 7, 59-70.

[12] Markowitz, H.M., Schirripa, F. and Tecotzky, N.D. (1999) A more efficient frontier. Journal of Portfolio Management, 99.

[13] Kumar, N., Singari, R.M. and Archana, S. (2014) Portfolio optimization: Indifference curve approach. International Journal of Advance Research and Innovation, 2, 127-133.

[14] Markowitz, H.M. (1991) Foundations of portfolio theory. The journal of finance, 46, 469-477.

[15] Pogue, G.A. (1970) an extension of the Markowitz portfolio selection model to include variable transactions' costs, short sales, leverage policies and taxes. The Journal of Finance, 25, 1005-1027.

[16] Konno, H. and Koshizuka, T. (2005) Mean-absolute deviation model. Iie Transactions, 37, 893-900.

[17] Roman, D., Darby-Dowman, K. and Mitra, G. (2006) Portfolio construction based on stochastic dominance and target return distributions. Mathematical Programming, 108, 541-569.

[18] Fabozzi, F.J., Kolm, P.N., Pachamanova, D.A. and Focardi, S.M. (2007) robust portfolio optimization and management. John Wiley & Sons.

[19] Barberis, N. (2000) Investing for the long run when returns are predictable. The Journal of Finance, 55, 225-264.

[20] Longstaff, F.A. and Schwartz, E.S. (1995) a simple approach to valuing risky fixed and floating rate debt. The Journal of Finance, 50, 789-819.

[21] Longstaff, F.A. (2001) optimal portfolio choice and the valuation of illiquid securities. The Review of Financial Studies, 14, 407-431.

[22] Jin, H. and Yu Zhou, X. (2008) Behavioral portfolio selection in continuous time. Mathematical Finance: An International Journal of Mathematics, Statistics and Financial Economics, 18, 385-426.

[23] Pirvu, T.A. and Schulze, K. (2012) Multi-stock portfolio optimization under prospect theory. Mathematics and Financial Economics, 6, 337-362.

[24] Barberis, N. and Xiong, W. (2009) what drives the disposition effect? An analysis of a long-standing preference-based explanation. The Journal of Finance, 64, 751-784.

[25] Wu, H., Zeng, Y. and Yao, H. (2014) Multi-period Markowitz's mean–variance portfolio selection with state-dependent exit probability. Economic modelling, 36, 69-78.

[26] Shi, Y., Cui, X. and Li, D. (2015) Discrete-time behavioral portfolio selection under cumulative prospect theory. Journal of Economic Dynamics and Control, 61, 283-302.

[27] Yan, S. and Liu, S. (2016) Multi-stage group risk decision making with grey numbers based on grey target and prospect theory. Grey Systems: Theory and Application, 6, 64-79.

[28] Hens, T. and Mayer, J. (2014) Cumulative prospect theory and mean variance analysis: A rigorous comparison. Swiss finance institute research paper, 14-23.

[29] Sharpe, W.F. (1964) Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 19, 425-442.

[30] Ross, S. (1976) the arbitrage pricing theory. Journal of Economic Theory, 13, 341-360.

[31] Ross, S.A. (2013) the arbitrage theory of capital asset pricing. In Handbook of the fundamentals of financial decision making: Part I, pp. 11-30.

[32] Kahneman, D. and Tversky, A. (1984) Choices, values, and frames. American psychologist, 39, 341.

[33] De Bondt, W.F. and Thaler, R. (1985) Does the stock market overreact?. The Journal of finance, 40, 793-805.

[34] Jegadeesh, N. and Titman, S. (2002) Cross-sectional and time-series determinants of momentum returns. The Review of Financial Studies, 15, 143-157.

[35] Daniel, K., Hirshleifer, D. and Subrahmanyam, A. (1998) Investor psychology and security market under- and overreactions. The Journal of Finance, 53, 1839-1885.

[36] Barberis, N., Shleifer, A. and Vishny, R. (1998) A model of investor sentiment. Journal of financial economics, 49, 307-343.

[37] Stein, R.S. (1999) the role of stress transfer in earthquake occurrence. Nature, 402, 605-609.

[38] Shefrin, H. and Statman, M. (2000) Behavioral portfolio theory. Journal of financial and quantitative analysis, 35, 127-151.

[39] Focardi, S.M. and Fabozzi, F.J. (2004) the mathematics of financial modeling and investment management. John Wiley & Sons, 138.

[40] Nocera, A. (2020) High Frequency Trading and Financial Stability.

[41] Gu, P. (2024) The Flash Crash: The Impact of High-Frequency Trading on the Stability of Financial Market. Highlights in Business, Economics and Management, 24, 1158-1163.