Game Theory and Nash Equilibrium in Rock Paper Scissors

Hongbo Chen

Shanghai Wenlai International School, Shanghai, 201101, China

Abstract. Game theory is a theory that has been widely used and analyzed in real life circumstances such as trading or investing. It is the study of strategic decision making. This theory can be modeled and applied to mathematics or economy to help people, societies, and governments make better decisions. This paper will discuss the basic concepts of game theory, such as what is game theory, the basic elements of game theory and the practical application of game theory. This paper will elaborate the principles of game theory through the example of rock paper scissors. This paper will also explain the concept of Nash equilibrium, which is a special equilibrium in game theory. This paper will use another example to show under which circumstances Nash equilibrium will be achieved. Through Nash equilibrium theory and case analysis, it is found that rational people will maximize their own interests by looking for a Nash equilibrium in their daily behavior decisions. This indicates that when making some decisions, society and the government need to consider and look for Nash equilibrium as much as possible, so that the whole country can obtain the maximum benefit.

Keywords: Game Theory; Nash Equilibrium; Rock Paper Scissors; Decision Making.

1. Introduction

From a certain point of view, one has to make many decisions through one's life. As small as what to eat today or as big as choosing which work or project to undertake, does a person need to make their own decisions. This kind of decisions may affect only one person, or they may affect the whole society or even the country, such as political or economic decisions. To maximize benefits, one can model an event using game theory and analyze it to figure out what choice is the best decision. In fact, game theory has been widely used in real life, and the research on game theory has become more and more in-depth in recent years. As mentioned earlier, game theory is often applied to economic or political decision making. As more and more people realize the importance of game theory, this paper will provide a simple analysis of game theory from a simple real-world case, namely rock-paper-scissors, to help people understand the concept of game theory and enable people to apply game theory to other life scenarios, especially political and economic decision-making, so as to maximize national interests.

Basically, game theory is a theory that can be applied to games in some particular situation. In such games, two or more players, unable to share information, attempt to make one of many decisions to win the game. Once all players have made their decisions, a settlement is made according to the rules of the game, and the winner is determined. Sometimes the game goes on for several rounds until a winner is determined. In this case, these players are considered rational, meaning that they will use the best decision at hand to maximize their interests. Since players cannot share information with each other and they all want to maximize their own interests, we can analyze each player's decision to obtain their optimal decision, or dominant strategy. This is consistent with most of the situations for people to make decisions in real life, because people have little way to predict the decisions of others, which means that in most cases, information is not available. Therefore, people can only choose their own strategies based on their subjective experience. But once people model this real-world event and use game theory to solve it, they are more likely to make the decision that maximizes their benefit or the decision that minimizes their risk. In other words, game theory can help people analyze or solve some complex or intractable real problems to a certain extent.

It is worth noting that there are situations in which a game is not fair. In this unfair game, one player may have only a few relatively poor decisions and obtain less benefit, while other players may have several relatively good decisions, each of which enables them to obtain more benefit. The difference in the quality of decisions leads all players to choose the decision of the highest quality

among the relative all decisions when considering their final decision. When the best decisions of all players are determined, this unequal game may give rise to a special kind of equilibrium, called Nash equilibrium. In this equilibrium, none of the players will try to change their dominant strategies, so that the entire game will result in only a few combinations of decisions resulting from the dominant strategies. Simply put, in order to maximize profits in an unfair game, people will only choose the dominant strategy that gives them the greatest payoff. But these dominant strategies may combine to form an equilibrium, which is the Nash equilibrium. In effect, since Nash equilibrium arise precisely because each player makes his or her best decision, then these decisions leading to Nash equilibrium are considered the best strategies one needs to look for in game theory. This is also the biggest significance of Nash equilibrium, that is, to help people find their best decision under the model of game theory so as to maximize their own interests. Therefore, in real life, when people make decisions, Nash equilibrium can help people find the optimal plan and help people make profits in reality.

Game theory is a modern theory. Since the combination of game theory and real life examples is less, this paper aims to combine game theory with reality, so as to make people have a basic understanding and concept of game theory, and help people to apply game theory in reality so as to obtain more benefits. At the same time, this paper also enlightens people to seek Nash equilibrium as much as possible when encountering unfair games, so as to make more accurate decisions and maximize their own interests.

2. Case Study

To further elaborate on game theory, let's first look at a real-life example of rock-paper-scissors. In rock paper scissors, rock can win scissor, scissor can win paper, and paper can win rock, as shown in Figure 1.

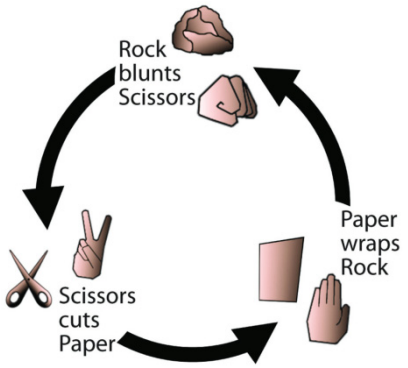

Figure 1. Game rule of rock paper scissors

If write this rule in a simple notation, it would be R wins S, S wins P, and P wins R. Now suppose that two players, A and B, are playing this game. For statistical purposes, if player A wins, 1 point is added to player A and 1 point is deducted from player B, and vice versa; If it is a tie, neither player A nor B get point. Thus, we can make a table of all the decisions made by players A and B. The results are shown in Table 1.

Table 1. Results of each player's decision in rock paper scissors

A \ B	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

By analyzing Table 1, we can draw the following conclusions:

Firstly, for each player, there is no dominant strategy given that all players are rational, because all decisions lead to the same outcome, plus one point, minus one point, and get no point. Therefore, as long as both players play with the probability that each decision is one in three, they are guaranteed to maximize their profits

Secondly, if one player does not consider each decision equally and instead makes one decision with a higher probability, then the game is no longer fair because the other player has a dominant strategy. For example, player A is much more likely to play rock than scissors. So for player B, player B should try to avoid the scissors, because now player B has a greater chance of being beaten by the rock. Instead, player B should play more paper, so that player B has a better chance of beating player A. In this example, player B's playing more paper and less scissors becomes his dominant strategy, because Player A's strategy is changed, resulting in an unfair game.

In fact, in unfair games, Nash equilibrium is sometimes achievable. Now, let's analyze Nash equilibrium with another example. Assume that players C and D are playing an unfair game, and their respective decisions result in scores shown in Table 2 below.

Table 2. Results of each player's decision in an unfair game

D / C	M	N
X	4,3	2,2
Y	3,1	1,0

By analyzing Table 2, we can draw the following conclusions:

In this unfair game, for player C, there are only two decisions, decision X and decision Y. Of these two decisions, decision X is clearly superior to decision Y, because player C always gains more benefit from decision X than from decision Y, regardless of which decision D chooses. For example, when player C chooses decision X, if player D chooses decision M, then player C will earn 4 points. However, if player C chooses decision Y, then player C can only earn 3 points if player D also chooses decision M. Similarly, when player D chooses decision N, player C can get more points by choosing decision X rather than decision Y. Therefore, the dominant strategy for Player C is decision X, which means that Player C only needs to choose decision X to maximize profit.

For player D, there are also only two decisions, decision M and decision N. Similar to player C, player D's dominant strategy is decision M. When player D chooses decision M, the benefit is significantly higher than N, regardless of which decision player C chooses. For example, when player C chooses strategy X, player D chooses strategy M or strategy N to get 3 points or 2 points respectively. However, when player C chooses strategy Y, player D chooses strategy M or strategy N to earn 1 point or 0 point respectively. Therefore, for player D, the payoff of decision M is significantly higher than that of decision N, so player D only needs to choose decision M to maximize his profit.

As a result, provided that both players C and D are rational players, player C will always choose decision X and player D will always choose decision M to maximize their own profits. So we can say that player C and player D have reached Nash equilibrium, because they each have their own dominant decision, whatever the other's decision is. And in this Nash equilibrium, the combination of decisions is decision X for player C and decision M for player D. At this Nash equilibrium, both players receive the maximum benefit of their decisions.

3. Conclusion

This paper explains the basic concepts of game theory and Nash equilibrium, and shows how game theory and Nash equilibrium can be used reasonably through two examples. Rock-paper-scissors is an example of how people use game theory and Nash equilibria to make accurate decisions in real

life. Through game theory and Nash equilibrium, people can make efficient decisions to help themselves get more benefits.

However, there are still many problems and controversies in the practical application of game theory. At the most basic level, for example, the game theory assumption of rational players is almost impossible to achieve in life, since there are few purely rational people in reality. Similarly, for countries and governments, if they blindly use game theory on one issue without considering other factors such as environmental damage and long-term impact, and just make the best decision for their own interests, then this decision is likely to have certain defects and cause certain negative impacts on the society. In other words, because of the complexity of real life, game theory needs to be applied and analyzed more carefully, especially when making decisions with a wide range of impacts.

References

[1] Eyler, Derek, et al. "Winning at Rock-Paper-Scissors." The College Mathematics Journal, vol. 40, no. 2, 2009, pp. 125–28. JSTOR, http://www.jstor.org/stable/25653686. Accessed 18 Sep. 2022.

[2] CASON, TIMOTHY N., et al. "Cycles and Instability in a Rock-Paper-Scissors Population Game: A Continuous Time Experiment." The Review of Economic Studies, vol. 81, no. 1 (286), 2014, pp. 112–36. JSTOR, http://www.jstor.org/stable/43551668. Accessed 18 Sep. 2022.

[3] Batzilis, Dimitris et al. "Behavior in Strategic Settings: Evidence from a Million Rock-Paper-Scissors Games." Games 10 (2019): 18.

[4] Benz, A., Jäger, G., van Rooij, R. (2006). An Introduction to Game Theory for Linguists. In: Benz, A., Jäger, G., van Rooij, R. (eds) Game Theory and Pragmatics. Palgrave Studies in Pragmatics, Language and Cognition. Palgrave Macmillan, London. https://doi.org/10.1057/9780230285897_1.