Application and Empirical Analysis of Random Volatility Model in Financial Markets

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Abstract. The Stochastic Volatility Model (SVM), as a key innovation in modern financial engineering, has profoundly changed our understanding of the volatility characteristics of financial markets. This model not only considers the volatility of asset prices as a time-varying random variable, but also simulates this uncertainty by introducing stochastic processes such as Brownian motion or more complex stochastic differential equations, thereby achieving a precise characterization of the dynamic characteristics of volatility. Compared to traditional fixed or historical average volatility assumptions, SVM can capture sudden changes in market sentiment, the impact of news events, and the influence of macroeconomic factors on volatility, providing investors with a more realistic perspective on the market. This article not only deeply analyzes the theoretical basis and construction methods of SVM, but also verifies its effectiveness and applicability in different financial market environments through empirical analysis and actual market data, providing valuable references and insights for financial practitioners, scholars, and policy makers.

Keywords: Random volatility model; financial markets; applications and empirical analysis.

1. Introduction

In today's complex and ever-changing global economic system, the volatility of financial markets is not only a barometer of market efficiency, but also a core consideration factor for risk management and investment decisions [1]. With the increasing globalization and informatization of financial markets, the speed and scale of capital flows are unprecedented, which not only promotes the optimal allocation of resources, but also exacerbates the interdependence and risk contagion between markets [2]. In this context, how to accurately capture and predict the volatility of financial assets has become a major issue faced by both the theoretical and practical fields of finance [3]. Volatility, as a direct reflection of financial market uncertainty, reflects the degree to which asset prices fluctuate over time [4]. In the fields of economics and finance, volatility is considered one of the important sources of risk, which directly affects investors' risk preferences, asset portfolio choices, and overall market stability [5].

Specifically, high volatility often comes with higher investment risks, which may lead to decreased investor confidence, restricted capital flows, and even trigger market panic and systemic risks [6]. Therefore, a deep understanding of the dynamic characteristics of volatility and the construction of effective volatility prediction models are of great significance for maintaining financial stability and promoting effective resource allocation [7]. Traditional methods of measuring volatility, such as historical volatility or implied volatility, can to some extent reflect market volatility, but often struggle to capture the temporal variability and agglomeration effects of volatility [8]. The emergence of SVMs provides a more refined solution to this problem. This type of model assumes that volatility itself is a stochastic process that can capture both the horizontal changes and dynamic characteristics of volatility, such as volatility clustering, long-term memory, etc., in order to more accurately depict the true volatility of financial markets. The theoretical basis of SVM is rooted in the stochastic process theory in modern financial theory, especially Brownian motion and its variants.

Common SVM models include GARCH family models (such as GARCH, EGARCH, GJR-GARCH, etc.), SVM and its extended forms (such as thick tailed SV model, leveraged SV model, etc.). These models use specific conditional variance or conditional volatility equations, combined with market data, and statistical methods to estimate model parameters, thereby achieving predictions of future volatility. When constructing SVM, the key is to reasonably set the dynamic equation of volatility and consider the characteristics of market data (such as non-normality, leverage effect, etc.). Meanwhile, the estimation and validation of the model are also indispensable, and commonly used estimation methods include maximum likelihood estimation, Bayesian estimation, etc. The diagnosis of the model relies on tools such as residual analysis and information criteria. In summary, SVM, as an important tool for studying financial market volatility, has a solid theoretical foundation, diverse construction methods, and effective empirical analysis. It has broad application prospects and important theoretical value.

2. Application of SVM in Financial Markets

2.1. Risk Management

The application of SVM in financial market risk management is crucial. Through deep mining and analysis of massive historical data, it can reveal the inherent dynamic laws of volatility and predict the future trend of volatility changes based on this [9]. This predictive ability provides investors with powerful risk management tools, enabling them to proactively assess market uncertainty and develop more robust investment strategies [10]. In the core area of risk management, Value at Risk (VaR) is a key indicator for measuring the maximum loss that an investment portfolio may suffer within a specific period of time.

Traditional VaR calculation methods are often based on fixed volatility assumptions, making it difficult to capture the time-varying and complex nature of market volatility. SVM, on the other hand, can more accurately calculate VaR values by dynamically simulating the changes in volatility and combining it with the latest market information and historical data. This not only improves the accuracy of risk measurement, but also provides more reliable decision-making basis for financial institutions. By using SVM for VaR calculation, financial institutions can more comprehensively assess the potential risks faced by their investment portfolios and take corresponding risk management measures, such as adjusting asset allocation, setting stop loss points, purchasing insurance, etc., to effectively reduce investment risks.

2.2. Option Pricing

Option pricing is one of the core issues in financial engineering, which directly affects the decision-making and risk management of market participants. Traditional option pricing models, such as the famous Black Scholes model, are based on a key assumption at the beginning of construction: the volatility of the underlying asset price is constant. However, this assumption is often difficult to hold in real market environments, as market volatility is actually influenced by multiple factors such as macroeconomic conditions, market sentiment, policy changes, etc., exhibiting significant temporal variability and uncertainty.

To overcome this limitation, SVM emerged. These models no longer consider volatility as a static parameter, but rather as a stochastic process that can dynamically adjust with changes in time and other market factors. This approach is more closely aligned with market reality, thereby improving the accuracy and flexibility of option pricing. Among numerous SVM models, Heston model, SABR (Stochastic Alpha, Beta, and Rho) model, and SVI (Surface Volatility Interpolation) model are particularly prominent. They each model the dynamic changes in volatility through different mathematical methods and assumptions, and derive more accurate option pricing formulas based on this. The application of these models not only enriches the theoretical system of option pricing, but also provides market participants with more scientific and reasonable pricing tools, promoting the healthy development of the option market.

2.3. Portfolio Optimization

SVM plays a crucial role in portfolio optimization. Faced with complex and ever-changing financial markets, investors pursue the best balance between risk and return. SVM provides investors with a window to gain a deeper understanding of market uncertainty by capturing the dynamic changes in volatility. When building and optimizing investment portfolios, investors need to flexibly adjust their asset allocation according to changes in the market environment. SVM can analyze the volatility characteristics of different assets and their interrelationships, helping investors identify which assets have higher risks or potential returns under current market conditions. Based on this, investors can adjust the allocation ratio of various assets (such as stocks, bonds, commodities, etc.) in a timely manner to achieve risk diversification and maximize returns.

In addition, risk budgeting, as an important part of portfolio management, also relies on the support of SVM. Risk budgeting aims to allocate appropriate risk levels to different assets or asset portfolios based on investors' risk tolerance and investment objectives. SVM can accurately evaluate the risk contribution of each asset through quantitative analysis, providing strong support for investors to develop scientific and reasonable risk budget plans. In this way, investors can better control and manage risks while pursuing returns, ensuring the long-term stable operation of investment portfolios.

3. Empirical Analysis

3.1. Model Building

The standard SVM is as follows:

$$y_t = \exp(h_t / 2)v_t \tag{1}$$

$$h_{t+1} = \mu + \varphi(h_t - \mu) + \sigma\eta_t, \eta_t \sim N(0,1) \tag{2}$$

Here, y_t represents the yield of assets at time t; h_t appears as an implicit variable, which specifically characterizes the logarithmic volatility of asset returns at time t.

On the basis of standard SVM, we innovatively incorporated threshold effects and non-parametric distributions to construct a semi parametric threshold stochastic fluctuation model (TSV-DPM). The purpose of this model is to model the return volatility of financial assets in a more refined and flexible way, so as to capture the complex and volatile dynamic characteristics of the market. Through TSV-DPM, we can simultaneously consider the nonlinear volatility characteristics under extreme market conditions and the potential non normal distribution of data, providing investors with more accurate risk assessment and decision support.

$$\begin{aligned} r_t &= h_t + \varepsilon_t, \varepsilon_t \sim F \\ h_{t+1} &= \varphi_{s_t} h_t + \sigma_{s_t}\eta_t, \eta_t \sim N(0,1) \end{aligned} \tag{3}$$

Among them, ε_t follows an unknown distribution F.

3.2. Comparison of Model Effects

In the financial field, predictive ability is undoubtedly an important criterion for measuring model performance and practical value. Accurate predictions can not only help investors avoid potential risks, but also guide them to capture market opportunities and achieve effective asset allocation and appreciation. Figure 1 shows the significant difference in prediction mean square error (MSE) between the TSV-DPM constructed in this paper and the traditional process mixed stochastic fluctuation model (SV-DPM) through intuitive comparison. It can be clearly seen from the graph that the TSV-DPM model has a significantly lower MSE than the SV-DPM model when predicting the volatility of financial asset returns. This result fully demonstrates the superiority of the TSV-DPM model in prediction accuracy. This is mainly due to the clever integration of threshold effects and non-parametric distribution concepts in the design of the model, which enables the model to adapt more flexibly to the complex changes in market volatility, especially under extreme market conditions. The TSV-DPM model demonstrates stronger capture and prediction capabilities. Therefore, for financial practitioners who pursue high-precision prediction, the TSV-DPM model is undoubtedly a more ideal choice.

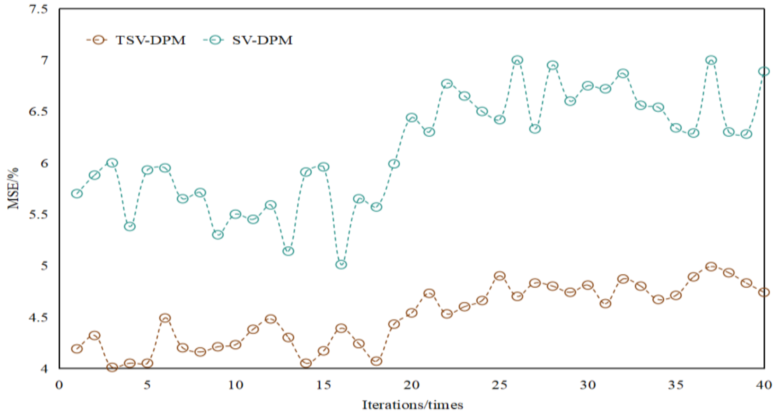

Figure 1. Prediction MSE comparison.

Since the global financial crisis ravaged in 2008, stock markets and their derivatives markets in various countries have suffered heavy losses, and the global economy has entered a period of deep adjustment. In order to revive the economy, the United States and the European Union frequently adjusted their monetary policies, especially the fluctuation of interest rates, which directly exacerbated the volatility of the exchange rate between the US dollar and the euro. Especially after the outbreak of the European debt crisis in 2011, the euro continued to depreciate, which not only deeply affected the stability of the foreign exchange market, but also prompted unprecedented attention to risk management and avoidance strategies for the US dollar/euro exchange rate worldwide. In this context, in-depth research on the volatility characteristics of the US dollar/euro exchange rate has become a focus of attention for both the financial academic and practical communities.

This article selects daily data of the euro/dollar (EUR/USD) exchange rate as the research foundation, aiming to explore its volatility patterns through innovative models. Figure 2 visually presents the logarithmic volatility time series calculated using the semi parametric threshold stochastic fluctuation model (TSV-DPM) proposed in this paper. In the figure, the logarithmic volatility trend captured by the TSV-DPM model is highly consistent with the volatility characteristics of the actual return of the euro/dollar exchange rate. This not only verifies the effectiveness of the model in characterizing the time-varying volatility of complex financial time series, but also demonstrates its excellent ability to capture volatility characteristics under extreme market conditions. Therefore, the TSV-DPM model provides more accurate risk assessment tools for foreign exchange market participants, helping them make more robust investment decisions in the volatile market environment.

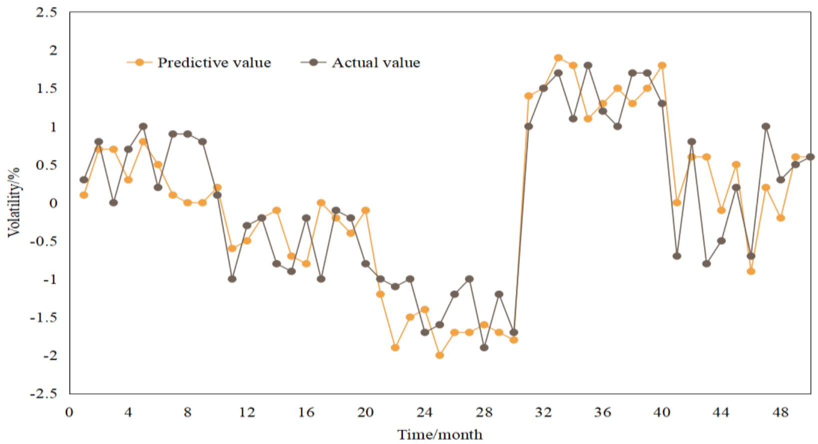

Figure 2. Estimation of logarithmic volatility.

4. Conclusions

Volatility, as one of the core indicators in financial market research, has immeasurable value in accurately predicting and guiding investors to formulate reasonable strategies, avoid potential risks, and capture market opportunities. Against the backdrop of rapid economic and social development in China, our environment is undergoing unprecedented profound and complex changes. The rise of anti-globalization sentiment globally not only challenges the existing international economic order, but also exacerbates market uncertainty. Therefore, actively promoting the reconstruction of globalization and promoting the deep integration and coordinated development of economies of various countries has become an important mission entrusted to us by the times. In this process, it is particularly crucial to have a deep understanding and precise grasp of the information transmission mechanism between various markets. As a barometer of the economy, the volatility of financial markets often contains rich economic information, which can reflect the market's expectations and judgments on future economic trends. By constructing efficient and accurate volatility prediction models, such as the TSV-DPM model discussed in this article, we can better understand market dynamics, predict market trends, provide scientific basis for policy makers, and provide decision support for investors. This not only helps to enhance the international competitiveness of China's financial market, but also promotes the stable and healthy development of the global economy. Therefore, strengthening research on volatility and enhancing predictive capabilities have important practical and far-reaching historical significance for addressing the current complex and ever-changing domestic and international environment, promoting globalization reconstruction.

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