

Using Simplex Method to Maximize Profit for McDonald's

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Abstract. Generally, the main purpose of a corporation or businessman is to maximize profit. Mathematically, a function of revenue, which consists of the prices and quantities of goods, is needed to achieve their goal. However, in reality, there are always some constraints or limitations that restricts production. For instance, the time needed to produce the goods, the number of raw materials prepared the space occupied by products and so on. In this article, an approach called the simplex method is introduced to solve this kind of problem. This method is straight and simple to understand and usually works when dealing with questions with inequalities and constraints. There are some drawbacks, though, such as the linearity and curvature of given functions and inequalities and the time required in a real situation. This essay will focus on problems that consist of only two variables and four constraints. The detailed explanations of introduction, detailed information of simplex method, and an application where an example of McDonald's will be given. The limitations and conclusion are followed.

Keywords: Simplex method, Optimization, McDonald's, Constraints.

1. Introduction

As a Chinese idiom says, “Merchants won’t operate without gain”, which shows that making profit is the main purpose of traders throughout history. Therefore, particular methods are needed to figure out a solution which brings out the most reward for merchants, although the method differs from time to time. In 2021, linear programming model was used in maximizing profit for a mixture of small and medium enterprises [1]. In the same year, another method called dynamic programming was used in profit maximization of energy storage merchants [2]. In addition, graphical method can also be used like Cobb B. did in 2011 [3]. However, there are some limitations for each of the methods mentioned. Firstly, linear programming model cannot apply when there is more than one objective function or when the constraints are non-linear. Secondly, dynamic programming cannot be used in complicated non-linear problems and it often requires huge number of calculations. Thirdly, graphical model can only apply in linear programming problems that only consist of two decision variables. It is not convenient when there are many constraints either. All in all, these three methods are a bit complex in some ways and may not apply in some situations. In contrast, simplex method, as its name suggests, is quite simple in both understanding and applying. Therefore, this essay will mainly base on this method and how it helps in profit maximization problems using examples.

Simplex method was first applied in the second world war. During the period, the problem of allocating supporting resources was a big concern of the US, especially one of its air forces. In 1947, a member of the air force called G. B. Dantzig figured out an approach to solve this kind of problem which shortly developed to what is so called simplex method. This method turned out effective in allocating limited resources properly. Furthermore, simplex method was also used in production planning, such as minimizing production cost of a company [4]. From the middle of twentieth century, simplex method could be applied in supply chain management. Recently, in 2005, it was used in a manufacturing supply chain plants with distilling process [5]. To summarize, simplex method was applied in many territories before it was used in profit maximization problems.

The detailed procedures will be discussed in the application section using an example of McDonald's. The author is interested in how McDonald's balances the amount produced of beef and chicken burgers. In this example shown below, the author tries to find the revenue function of McDonald's that consists of only these two burgers, as well as the inequalities of the constraints using data preciously found online.

2. Simplex method

2.1. History

Simplex method was first mentioned by G. B. Danzig in his paper titled Linear programming in problems for the numerical analysis of the future in 1948. In 1990, he published another paper called Origins of the simplex method [6], where he introduced the development and historical context of the simplex method as well as its geometry basis. The whole essay is based on his two papers, because the methodology comes from there. Moreover, simplex method was first been improved by Nelder, John A., and Roger Mead in 1965 [7]. Indeed, the method, so called the Nelder-Mead or downhill simplex method, is different from the original one by Danzig. It is particularly effective when dealing with multi-dimensional problems where derivatives are not available. In addition, in 2021, this method was applied in economics in titled State budget revenue optimization using the simplex method. In this paper, the authors focused on optimizing Ukraine's state budget revenues using the method, analyzing data from 2007-2019 to find the optimized revenue structure by comparing the data with 25 countries in the Europe Union [8]. That is a recent application of simplex method on optimization problems.

The original simplex method mentioned in the earliest paper was quite complex. Some simplifications, since only four constraints will be discussed in this essay, are quite necessary. As mentioned, this essay will focus on profit maximization problems when there are only two variables. To solve this kind of problem, the purpose to represent one variable using the other to get a new revenue function that contains only one variable [9]. However, before doing this, slag variables are needed to process the constraints. In particular, the limitations are inequalities, and the slag variables are used to cover the gap between two sides of equality. In other words, they convert inequalities to equations. By doing this one can approach to our purpose using the new equations that each contains two variables and one slag variable. More precisely, the new revenue function contains one variable and slag variable. In this way, the maximum value of the revenue will be clear. The basic steps are shown below in a flow chart of Fig. 1.

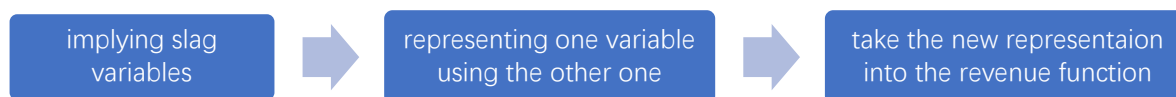


Fig. 1 Flow chart for simplex method's steps

2.2. Advantages and Disadvantages

There are, obviously, a lot of benefits of using simplex method in profit maximization problems. Among them, the method is easy to understand, due to its simple steps. However, the author will focus on the imitations of this method more than its advantages.

On the other hand, some drawbacks of simplex method exist. For instance, this method cannot apply in every situation. Firstly, the objective function must be linear. Nevertheless, simplex method cannot apply on polynomials, exponential or logarithm functions. There are multiple reasons, among them is the complexity of non-linearity. In other words, when dealing with non-linear functions using simplex method, the optimum value observed, or calculated may be a local maximum or minimum point instead of the global one which is indeed the expected answer. More precisely, the constraints together illustrate the available area if plotted on a rectangular coordinate system. Normally, when the limitations are linear, the area is actually a polygon, the optimum value is located on one of its vertices. However, when it comes to non-linear ones, the available area may be non-convex or concave, which means that the optimum value may not be located on any of its vertices. Secondly, the time required using this method is a bit long. The time needed to get the right answer using simplex method greatly depends on the number of variables and constraints. In the example above, there are only two variables and four constraints, and it is obvious that the process was simple. But if there is,

say ten variables and constraints, then 10 slag variables will be needed, which requires a large number of calculations and time. Moreover, the calculations can less likely been down by hands.

3. Application

As previously mentioned, in this essay, the author will focus on problems that contain two variables and four constraints. To clearly display the steps of using simplex method to solve profit maximization problems, an example will be used. The author looked for data about the prices and costs of a single McDonald's beef burger and chicken burger (the data was cited from website www.mcdonalds.com.cn). When it comes to a beef burger, 6 types of it sold in McDonald's, along with related chicken burgers are displayed in a form shown in Table 1.

Table 1. prices and names of burgers

Beef burgers		Chicken burgers	
Title	Price (\$)	Title	Price (\$)
Cheeseburger	2.65	McChicken	2.97
Double Cheeseburger	3.47	Spicy McChicken	3.27
Big Mac	6.29	Crispy Chicken Sandwich	5.06
Quarter Pounder with Cheese	6.42	Deluxe Crispy Chicken Sandwich	5.93
Double Quarter Pounder with Cheese	8.32	Spicy Crispy Chicken Sandwich	5.24
Bacon Quarter Pounder with Cheese	7.93	Deluxe Spicy Crispy Chicken Sandwich	6.10

To make things clearer, the authors calculated the average of these 6 prices as the price of a single beef burger, and the mean observed was approximately \$5.85.

$$(2.65 + 3.47 + 6.29 + 6.42 + 8.32 + 7.93) \div 6 = 5.85 \quad (1)$$

Similarly, the average price of a chicken burger was calculated after the prices of six types of burgers were given, the average of a chicken burger is approximately \$4.76.

$$(2.97 + 3.27 + 5.06 + 5.93 + 5.24 + 6.10) \div 6 = 4.76 \quad (2)$$

Therefore, the author can propose a revenue function for McDonald's beef and chicken burgers: $z = 5.85x + 4.76y$, where z is the daily revenue that comes from selling beef and chicken burgers. While x and y represent the amount of the two burgers sold every day.

Since the author now has the objective function, the next step is to find the constraints. Firstly, the time needed to make one burger is 50-80 seconds (take the average time 65 seconds for calculation). And assume there are ten employees working 8 hours a day making burgers. Then a single shop can produce maximum 4431 burgers per day. So, the first constraint will be $x + y \leq 4431$. Secondly, the grill used in McDonald's can grill about 12-24 patties simultaneously. Hourly, it can grill a maximum of 240 patties to make the burgers (the patties are promised by McDonald's to be grilled for 37 seconds). Then at most 5760 burgers can be made. As the type of burgers are displayed above, most beef burgers require two beef patties while most chicken burgers require only one. So, the second constraint will be $2x + y \leq 5760$. Now, the constraints are all observed and displayed bellow,

$$\begin{cases} z = 5.85x + 4.76y \\ x + y \leq 4431 \\ 2x + y \leq 5760 \\ x \geq 0, y \geq 0 \end{cases} \quad (3)$$

From the revenue function, it is clear that the price of beef burger is higher than the price of chicken burger, so it is expected that producing more beef burgers (x) than chicken burgers (y) will bring more profit, so in the following procedures, x is needed to be represented by y . To solve this question, the first step is to imply slag variables in order to process the constraints. Take the first constraint as

an example. If s_1 is added to its left-hand side, there will be no difference between the two sides of the inequality. Of course, s_1 , in this case, is an integer. The same way applies in the second constraint. Therefore, two new equations are observed as shown.

$$x + y + s_1 = 4431 \quad (4)$$

$$2x + y + s_2 = 5760 \quad (5)$$

After that, since x is needed to be represented by y , equation (4) is converted to make y the subject (the equation (5) can be used as well in this step, but equation (4) looks simpler). The new equation is

$$x = 4431 - y - s_1 \quad (6)$$

Next, the author took the new presentation of y , which is equation (3), into the origin revenue function to observe a new function that contains only x and s_1 , i.e., $z = 5.85(4431 - y - s_1) + 4.76y$. To simplify it, it is found that $z = 25921.35 - 5.85y - 5.85s_1 + 4.76y$, or

$$z = 25921.35 - 1.09y - 5.85s_1 \quad (7)$$

Lastly, the new revenue function is used to calculate the maximum value. The equation consists of one constant and two subtractors. If both $1.09y$ and $5.85s_1$ are equal to 0, then z is maximized and is equal to 25921.35. And the value of x can be calculated using equation (6), which is 4431.

In summary, the maximum of the revenue in this case is \$25921.35, when 4431 beef burgers and no chicken burgers are produced. The answer to the question above may not apply in real situation, as it is impossible that a McDonald's shop does not sell chicken burgers. The authors solved the question above using simplex method by applying slag variables and the detailed steps are displayed. From the result, in order for the manager to maximize his revenue, or profit, he needs to produce only Y . Back to the step, the author suggested that since the price of Y is higher than X , the manager should produce more Y than X , which turned out true. There may be concern, when dealing with such kind of problem, there must be a variable equal to zero. But this is not always the case, it depends on the ratio of the two products. And this question can be further discussed.

All in all, the method used is different from the ones found in the origin papers of simplex method. In the papers, the values of x and y keep altering, and so do the two slag variables the changes could be very small. Therefore, iterative calculation is necessary. Through a form and comparing the value of the revenue after entering and leaving variables are chosen, the maximum of x and y can be observed. Nevertheless, in this case, where x and y are definitely integers, using simple calculations can approach to the right answer as well. Moreover, it is a quick and simple way at least in this kind problem where there are only few variables. But in some more practical situations which the range and number of variables are much larger, the origin simplex method applies better.

Indeed, in most cases there are more factors that affect or limit the values of x and y , but to help beginners understand simplex method, and use it to solve simple profit maximization problems like the example taken, the steps above are more understandable and effective since there is no need to try all the values available on x and y or to draw a complicated form.

4. Conclusion

In this essay, the author explained how simplex method applies in profit maximization problems through an example of McDonald's beef and chicken burgers. Before that, the author wrote about the methods used in optimization problems in history, and an introduction to simplex method. By looking at the steps displayed in the example, the author believes that simplex method is perfectly fit in this kind of questions. In particular, optimization questions that evolve few variables and limitations (each less than 5). In conclusion, although there are some drawbacks of this method, it is still an easy approach when dealing with optimization problems with some constraints. Especially, when the accuracy of the answer is not high, just like the example. The manager could quickly come out of the

ideal collocation of the amounts of two products. Further discussion and research can focus on problems that consist of more variables and constraints in a more practical situation, as the author has mentioned in the advantages and disadvantages session. Also, just like the interior-dot method shows, such problems can be dealt with using graphical models. By plotting the limitations on a graph, find the intersections and try each of them into the objective function to calculate the value. Compare the values obtained and the largest one will be the final answer. More research could be done on combining the calculation (as the example shown) method and graphical model to deal with more complicated situations.

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