Capital Asset Pricing Model and its Application on Investment Risk

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Abstract. This study explores the utilization of the Capital Asset Pricing Model (CAPM), serving as a key instrument in the realm of asset management, with the objective of maximizing financial gains while minimizing potential hazards. The introduction highlights the significance of CAPM in guiding investment decisions and portfolio management by providing a theoretical framework to assess the potential earnings of an investment in relation to the inherent risks associated with it. The body of the paper outlines the fundamental concepts of CAPM, including its underlying assumptions and the mathematical formulation that underpins the model. It also critically examines the limitations inherent in CAPM, such as its assumptions about market efficiency and investor rationality, that is not invariably applicable in practical situations. In this empirical section, the paper investigates the applicability of CAPM to the Chinese stock market, specifically the Shenzhen and Shanghai stock exchanges, using the Breusch-Pagan and Fama-MacBeth regression methods. The analysis seeks to determine whether CAPM's predictions align with the observed market behaviors and to what extent it can be relied upon for asset optimization in the Chinese context. This research's outcomes enrich the existing conversation about the real-world application of the Capital Asset Pricing Model within asset management, providing valuable perspectives for those aiming to enhance their investment strategies, particularly in the context of the distinctive features of China's equity markets.

Keywords: Capital Asset Pricing Model; Asset Optimization; Empirical Analysis; Chinese Stock Markets; Investment Decisions.

1. Introduction

The concept of capital asset pricing is of significant consequence in the field of financial engineering, serving as a fundamental component of investment practices. In the 50s of the 20th centuries, Markowitz published the paper "Asset Selection: Effective Diversification of Investments", where he pioneered the concept that the level of return volatility is quantified by return variance, and established a milestone mean-variance portfolio model. During the 1960s, William Sharpe, Jone Lintner, and Mossin introduced the renowned Capital Asset Pricing Model (CAPM), which was formulated on the principles of diversified investment strategies. combined with this principle of market securities portfolio, the separation theorem of two funds and the principle of market equilibrium [1].

There are some foundational presuppositions form the basis of the CAPM framework. Regarding market efficiency, its asset prices reflect all available information, and investors cannot achieve excess returns through information analysis or specific strategies. For the investor rationality, Investors exhibit a preference for avoiding risk and aim to achieve the highest possible profits concurrently with the reduction of potential hazards. They make decisions based on available information and reasonable expectations of future asset performance [2]. For the perfect competition, no monopoly forces, and it is presumed that investors passively adopt the prices set by the market dynamics. The actions of purchasing and divesting by them exert no impact on the valuation of assets. The zero transaction costs and taxes refer that transaction costs (commissions, slippage, etc.) and taxes are neglected in the CAPM model. For the free borrowing and lending, it is possible for investors to engage in unencumbered lending or borrowing at the prevailing risk-free rate, free from constraints [3]. For the uniform investment horizons, the investment horizons of all investors are considered the same, allowing comparisons and decisions within the same time frame. For the asset divisibility, each asset is infinitely divisible, enabling people allowing investors the flexibility to trade in any desired amount, be the entirety or a fraction of an investment collection. For the homogeneous expectations, Investors share identical expectations regarding expected returns, standard deviations, covariances, and other metrics for various assets. They employ the same methodologies and standards when evaluating assets. Systemic risk stands as the sole determinant impacting stock returns [4].

The CAPM model sees widespread usage across capital markets in various economically advanced western nations, serving as a vital instrument in the toolkit of both retail and institutional entities for managing security investments and valuing financial holdings [5]. In recent years, amidst the context of China's economic liberalization and reform initiatives, the economy has developed rapidly, The Shanghai and Shenzhen stock have continued to expand, and the capital market system has been continuously standardized. For example, the trading system in China's Shanghai stock market is "T+1," but the stock market in western developed countries generally practices "T+0." After years of rapid development, the Shanghai and Shenzhen stock exchanges have a relatively complete market structure. Moreover, they offer a diverse range of financial instruments, including bonds, mutual funds, equities, and futures. The principles of the CAPM framework enhance the ability of investors to evaluate stock values and supply a measurable index of risk for discrete investments. These elements are crucial for guiding the pricing of financial assets and shaping investment strategies, underscoring the importance of examining the CAPM framework's relevance within the contexts of the Shanghai and Shenzhen exchanges in China.

2. Methods and Theories

2.1. Basic Concepts

The Market Portfolio encompasses every financial instrument available for trade, with each asset's weight determined by its market valuation relative to the overall market valuation, thus reflecting the composite financial landscape. Efficient Set When the risk level (standard deviation) is the same, investors, acting rationally, prioritize portfolios offering superior returns; given identical expected returns, they opt for those exhibiting lower variability, as indicated by standard deviation. When portfolios satisfy these dual criteria, they form what's known as the efficient set. Assets devoid of risk, such as certain government securities, exhibit a return volatility of zero and are commonly exemplified by treasury bonds, fixed-term savings, and distinct sovereign debt instruments [6].

The portfolio with an element of risk involves the exclusion of low-risk securities typically found in a comprehensive market portfolio. Investment ratio: The allocation of individual assets within the portfolio. The composite market ensemble validity: When each investor tends to optimize the investment portfolio, the investment portfolio of everyone in the end will be the same. Each asset's allocation within the portfolio corresponds to its share of the total market asset base, with each security's investment proportion aligning with its valuation in the market. Because: such as a stock is not in the combination, it means that everyone doesn't like it. The stock price will decline, and finally attract investors to the bottom, and the stock will return to the portfolio. Ultimately, the market portfolio encompasses every stock, ensuring that none are excluded from consideration. That is, when all investors hold the same portfolio, this portfolio is a market portfolio [7]. And this portfolio is effective. According to the Separation Principle, an investor's risk-return inclinations do not affect the ideal makeup of their risky asset portfolio. An exemplary risk-adjusted portfolio achieves the highest Sharpe ratio, signifying its optimal balance between risk and return. Investment Diversification In equilibrium, each security has a non-zero percentage in the portfolio at the equilibrium point. The Mutual Fund Theory posits that an investor's ideal risky asset portfolio, often referred to as the critical portfolio, mirrors the market portfolio, with each security's weightage reflecting its market valuation.

2.2. Formula Derivation

The portfolio's expected return, denoted as $E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$, represents a weighted sum of the individual expected returns of its assets. Here, w_i signifies the proportionate share of the i--th asset within the overall investment mix. The variance of a portfolio σ_p^2 is determined by the variances and covariances of its assets [8]

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(R_i, R_j). \tag{1}$$

For a simplified model with only two assets, it is: $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1, R_2)$. The beta value β_i for a specific asset i indicates how its returns fluctuate in comparison to the overall market portfolio(M):

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)} = \frac{Cov(R_i, R_M)}{\sigma_M^2} \tag{2}$$

For the market portfolio(M) the beta coefficient is unity, signifying it embodies the market's aggregate risk profile $\beta_M = 1$. The composite beta (β_p) for a portfolio (p) is calculated as the weighted mean of the individual betas of the assets it comprises $\beta_p = \sum_{i=1}^{n} w_i\beta_i$. The premium for market risk represents the additional anticipated return of the market portfolio above the risk-free rate $\mathrm{R}P_M = \mathrm{E}(R_M) - R_f$.

As per the CAPM, an asset's prospective return is calculated by adding its beta-adjusted market risk premium to the risk-free rate $E(R_i) = R_f + \beta_i RP_M$. Substituting$(RP_M)$ with $(E(R_M) - R_f)$ yields [9]

$$E(R_i) = R_f + \beta_i\big(E(R_M) - R_f\big). \tag{3}$$

A portfolio's forecasted return is the sum of the risk-free rate and the portfolio beta multiplied by the market's expected excess return [9]

$$E(R_p) = R_f + \beta_p\big(E(R_M) - R_f\big). \tag{4}$$

Investors assemble portfolios by strategically choosing assets and their proportions, aiming to optimize the anticipated returns for a set level of risk $\max E(R_p)$ subject to $\sigma_p^2 \leq$ given level of risk.

An investor's contour maps of satisfaction delineate various portfolio blends that yield equivalent gratification levels. Under the CAPM framework, investors will choose portfolios on the CML because they provide the highest anticipated earnings at a specified degree of risk exposure. At market equilibrium, all investors' optimal portfolios will lie on the CML, and the prices of all assets will reflect their risk and expected returns.

2.3. Data Select

The study employs a conventional two-tiered regression analysis, integrating both the BJS and FM methodologies, mainly using R for operational test. The BJS method is proposed by Black, Jensen and Scholes mainly for CAPM model test with time series method; the FM method is the cross-section method for CAPM model test proposed by Fama and MacBeth in 1973. CSI 300 Index covers the Shanghai and Shenzhen A-share markets, objectively and truly reflecting the operating conditions of the A-share market. It also excludes inferior stocks. The sample stocks are all large-scale and liquid high-quality stocks, which can comprehensively reflect the price fluctuation trend and market investment changes of the Chinese stock market. The compilation method of this index (Pass the Weighted Composite Price Index Formula) is like the market portfolio required by the CAPM model. Excluding stocks with discontinuous time intervals, 193 constituent stocks of the Shanghai and

Shenzhen 300 Index were selected as the research objects, accounting for approximately 2/3 of the total. The sample interval consists of 129 months of data from January 9, 2009 to September 30, 2019. The sample stocks cover more than 20 industries including finance, utilities, basic chemicals, metals, catering and tourism, pharmaceuticals and biology, electrical equipment, etc., with a weight proportion of 76 55%, with distinct representativeness. According to the chronological order, the sample stock data is divided into three groups. The first group has a time interval from January 9, 2009 to July 27, 2012, the second group has a time interval from August 3, 2012 to February 26, 2016, and the third group has a time interval from March 4, 2016 to September 30, 2019. Each group has weekly returns spanning 43 months. One year treasury bond yield represents risk-free return rate. Calculating weekly return rate

$$R_t = (P_t - P_{t-1})/P_{t-1} \tag{5}$$

Here, R_t is closing price of the market portfolio at week i, and P_{t-1} is closing price of the market portfolio at week $i-1$.

Using CSI group 1 data, perform time series testing to estimate the risk of a single stock β. The least squares regression equation [10]

$$R_{it} - R_f = \alpha_i + \beta_i \times (R_{mt} - R_f) + \varepsilon_i \tag{6}$$

Here, R_{it} is the Stock return rate at week t, and R_{mt} is the Return rate of the market index. R_f is risk-free return rate, ε_i is Random error term, and α_i, β_i are parameters to be estimated. The relevant data are shown in Table 1.

Table 1. Partial results of least square regression equation for single stock. Note: ***, ** and * are significant at 0. 01, 0.05, 0.1 respectively; standard deviation in parentheses.

Stock Name	Ping’an	WanKeA	ZTE	OCTA	TCL	ZOOMLION
α	0.003.6	0.0010	-0.0007	0.0036	0.0021	0.0061
	(0.0026)	(0.0032)	(0.0037)	(0.0032)	(0.0034)	(0.0024)
β	0.9504*	1.0711	0.6994	1.1302*	0.9407*	1.1711
	(0.0742)	(0.0938)	(0.1079)	(0.0938)	(0.0970)	(0.0688)
σ	0.0324	0.0350	0.0441	0.0508	0.0441	0.0453
R^2	0.51	0.53	0.23	0.37	0.35	0.45
F	190.9	208.1	55.62	109.8	100.6	145.7

The stock data is sorted from small to large according to the risk coefficient value estimated during the period of the first set of samples, and then grouped in size order, forming 10 securities portfolios for every 19 stocks. The first stock, the ticket portfolio has 22 stocks, and the last 9 stock portfolio has 19 stocks in each group. Use group2 data calculate portfolio weekly return rate (arithmetic average)

$$R_{pit} = (1/N) \sum R_{it} \tag{7}$$

R_{pit}: return rate of ith securities portfolio at week $t.R_{it}$:Return rate on the first i stock in this portfolio.N: number of stocks contained in this portfolio. Weekly excess return rate of CSI 300 as the independent variable. The surplus return from a stock portfolio on a weekly basis serves as the variable being predicted. Do regression analysis, calculate each group β_p

$$R_{pt} - R_f = \alpha_p + \beta_p \times (R_{mt} - R_f) + \varepsilon_{pt}. \tag{8}$$

According to weekly return rate of group3 data to calculate average weekly return of 10 stock portfolios, then test linear relationship

$$R_p = \gamma_0 + \gamma_1 \beta_p + \varepsilon_p \tag{9}$$

Here, R_p is the average weekly return rate of each stock portfolio in the third set of time intervals, β_p is the Risk coefficient values estimated by using the second set data, and γ_0, γ_1 are the parameter to be estimated. These results are shown in Table 2.

Obtained stock portfolio β is taken as the systematic risk. The residual standard deviation signifies the level of unsystematic risk. Perform secondary regression test

$$R_p = \gamma_0 + \gamma_1\beta_p + \gamma_2\beta_p^2 + \gamma_3\sigma_p + \varepsilon_p \tag{10}$$

Here, R_P is the average weekly return rate for a stock portfolio, β_p is value of the risk factor for the stock portfolio, σ_p is the standard deviation of the estimated residuals in the least squares regression equation. As a non-systemic risk, $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ are parameters to be estimated. This parabolic regression model further investigates if systemic risk is the sole determinant of stock pricing and confirms whether there is a direct proportionality between the stock's profitability and its associated risk.

Table 2. The results of least square regression equation of stock portfolio. Note: ***, ** and * are significant at 0. 01, 0.05, 0.1 respectively; standard deviation in parentheses.

Stock portfolio	Group1	Group2	Group3	Group4	Group5
α	0.0026	0.0026*	0.0031*	0.0051*	0.0040*
	(0.0012)	(0.0010)	(0.0012)	(0.0015)	(0.0010)
β	0.75°*	0.82*	0.90*	0.88*	0.94*
	(0.0318)	(0.0260)	(0.0336)	(0.0403)	(0.0260)
Residual standard error	0.0158	0.0130	0.0168	0.0201	0.0130
R^2	0.75	0.85	0.80	0.72	0.88
F	550.6	1003.0	717.8	476.3	1323.0
Stock portfolio	Group6	Group7	Group8	Group9	Group10
α	0.0040*	0.0028*	0.0029	0.0023	0.0014
	(0.0012)	(0.0011)	(0.0015)	(0.0014)	(0.0011)
β	0.93*	1.00*	1.02	1.10*	1.18
	(0.0327)	(0.0297)	(0.0413)	(0.0379)	(0.0293)
Residual standard error	0.0163	0.0148	0.0206	0.0189	0.0146
R^2	0.82	0.86	0.77	0.82	0.90
F	805.7	1123.0	609.0	837.6	1638.0

3. Results of Data Analysis and Modeling

As can be seen from the data shown in Table 1 and Table 2, the proportion of β less than 1 increases and the extreme value decreases. It shows that the way of investing in stock portfolio can effectively disperse risks, Moreover, there persists the influence of non-systematic risks that can alter stock returns [11].

Examining the direct correlation between risk exposure and investment profitability, the author designates the stock portfolio's beta (β_p) as the predictor variable, while the portfolio's profitability rate takes on the role of the response variable. A scatter plot is then constructed to visualize this relationship. There is a notably apparent linear association between a stock portfolio's beta coefficient and its rate of return, Despite the existence of some stock portfolios with high returns accompanied by low risks, the overall profitability of these portfolios is surprisingly modest. This discrepancy suggests an underdeveloped market structure, where the influence of speculative trading is still palpable. By applying the least squares regression technique, the study explored the link between the weekly financial outcomes of stock portfolios and the systemic risks they are subjected to. For more details, see the following Table 3.

Table 3. Least squares regression results of stock portfolio. Note: ***, ** and * are significant at 0. 01, 0.05, 0.1 respectively; standard deviation in parentheses.

regression coefficient	coefficient value	T-test
Constant term	0.007 516	5.431
	(0.001 384)***	
β_P	-0.005 271	−3.656
	(0.001 442)***	
R^2	0.625 6	
Residual standard error	0.000 557 3	
F-test	13.37	

On the one hand, when $\gamma_0 = 0.007516 > 0$, the risk-free interest rate is greater than 0 indicating that there are risk-free assets in the capital market, which is consistent with the reality. On the other hand, when $\gamma_1 = -0.005271 < 0$, it is counter to the expected scenario that a stock portfolio's profitability inversely correlates with the risks it assumes. The results of quadratic regression equation are shown in Table 4.

Table 4. Results of quadratic regression equation

regression coefficient	coefficient value	T-test
γ_0	0.0046208	0.442
	(0.010 461)	
γ_1	-0.000 317 3	-0.014
	(0.022 241 0)	
γ_2	0.002 617 4	-0.229
	(0.011 439 4)	
γ_3	0.036 016 8	0. 456
	(0.079 055 6)	
R^2	0.644 6	
Residual standard error	0.000 626 9	
F-test	3.627	

In this situation, when $\gamma_1 = -0.0003 < 0$, a direct proportionality between the risk inherent in a stock portfolio and its profitability does not exist and there are speculators in the market but $\gamma_2 = 0.0026 > 0$. The influence factor is very small and basically consistent with the CAPM theory. In addition, $\gamma_3 = 0.0360 \neq 0$. Factors affecting asset pricing behavior in the stock market include non-systemic risk, which is inconsistent with the basic assumptions in the CAPM model. Also, $\gamma_0 = 0.0462 > 0$. In the capital market, the return rate on risk-free capital is positive, which is basically consistent with the reality.

4. Conclusion

Investing in diversified stock portfolios is a strategic approach to mitigating investment risk, and by employing the CAPM, one can better comprehend the complex relationship that dictates the trade-off between risk and reward in asset valuation. However, the adaptability of CAPM within the context of the China Securities Index (CSI) has been found to be limited, suggesting a need for a more nuanced approach to asset management within this market. The systematic risk measure, as posited by CAPM, offers a robust framework for explaining the profitability of equity portfolios., yet it is the interplay between systemic and non-systemic risks that truly shapes asset pricing. This paper suggests that while CAPM provides valuable insights, its application in the CSI should be tempered with caution, acknowledging the significant impact of non-systemic risk factors on stock prices. To enhance the efficacy of financial management in the CSI, it is imperative to further refine the financial supervision system, thereby fortifying market effectiveness and preempting potential risks. Additionally, elevating the marketization level of the securities market is essential to improve the financial literacy of investors, thereby curbing speculative behaviors that can lead to a "high risk, low return" scenario. The paper concludes with a critical reflection on the limitations of the current study, highlighting the need for more robust data to strengthen the model's fit and persuasiveness. The goodness of fit in this research falls short of the desired 80% threshold, indicating a necessity for additional empirical validation through multiple experiments with more contemporary data sets.

In light of these findings, subsequent studies should concentrate on refreshing the data pool to encompass the latest market developments and financial practices. This will not only provide a more accurate assessment of CAPM's applicability in the current market environment but also offer a more compelling argument for its integration into investment strategies within the CSI. There are some suggestions for Future Research. One can conduct further empirical studies with updated and more extensive data sets to validate the findings, and explore the integration of non-systemic risk factors into asset pricing models to provide a more comprehensive understanding of stock portfolio returns. In addition, people can also investigate the role of financial literacy and investor behavior in the context of market efficiency and risk management.

References

[1] Jesen M C. Studies in the theory of capital markets. New York: Praeger Publishers Inc., 1972.

[2] Fama E F, Macbeth J D. Risk return, and equilibrium: empirical tests. Journal of Political Economy, 1973, 81(3): 607-636.

[3] Fama E F, French K R. The cross-section of expected stock returns. The Journal of Finance, 1992, 47 (2): 427-465.

[4] Shi Donghui. An empirical Study on the Risk of Shanghai Stock Market. Economic Research, 1996 (10): 44-48.

[5] Ruan Tao, Lin Shaogong. CAPM model test of Shanghai stock market. Mathematical Statistics and Management, 2000 (4): 12-17.

[6] Xu Dilong, Zhang Yu. Empirical Analysis of capital asset pricing Model and Shanghai Stock Market. Journal of Nanchang University (Science edition), 2005 (2): 142-145,150.

[7] Zhu Shunquan. Empirical test of Capital Asset Pricing Model CAPM in China's capital market. Statistics and information theory Tan, 2010, 25 (8): 95-99.

[8] Zhai Yu. Empirical test of capital asset pricing model in China's stock market. Economist, 2015 (3): 114-115.

[9] Wang Dantong. Empirical test of the applicability of CAPM model in Shanghai and Shenzhen Stock markets. China market, 2018 (29): 48-50.

[10] Min Geng. Test of the effectiveness of CAPM model in A-share market. Journal of Chifeng University (Natural Science Edition), 2014, 30 (17): 34-36.

[11] Zou Zhou, Lou Baijun. Test of the validity of the CAPM model in the Shanghai Stock Market. Enterprise Economy, 2013, 32 (1): 173-175.