

Application of Utility Maximization by Using the Cobb-Douglas Function

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Abstract. This paper is going to research about the application of utility maximization by using the Cobb-Douglas function. This paper will change the factors into the variables and use the Cobb-Douglas function to find which factor has the greatest effect to the utility maximization. The application of utility maximization in this paper is the growth of economic in different places. This paper includes four parts which are Introduction, methods and theory, as well as result and conclusion. First, the introduction will introduce the background of this paper and make some conclusion of some essay of utility maximization. Then, the method and theory will introduce the method of this paper and introduce what is the Cobb-Douglas function. Next, the result part will use the Cobb-Douglas function to find the result of the application of utility maximization in different places. In the end, this paper will make a conclusion by using the data in different places. This paper finds some factors which is the biggest effect to the utility maximization.

Keywords: Utility Maximization, Cobb-Douglas function, Growth of economic.

1. Introduction

Nowadays, there are more and more companies that make a lot of business. They all face the same question: how to earn the greatest value of money as they can. So, achieving the greatest value of money is the companies' utility. People need to find some method to achieve the utility maximization, so the utility function came out. These functions consider the factors in the business, such as the labor input and the capital. So, the growth of the economic is closely relative to the application of utility maximization. From the perspective of the supply of the public service, the optimal solution for evaluation is that marginal technology substitution rate of tangible and intangible should be equal to the marginal utility of intangible. At this time, the scale of expenditure on public sports services has reached the optimum, and the structure of the supply of government procurement services is the best [1].

Utility maximization is one of the most frequent problems in financial mathematics and has been considered by numerous authors. Here are some of the milestones viewed from people's perspective of maximization under constraints using the tools of BSDEs. For a complete market, utility maximization has been considered in Ref. [2]. Fiscal policy and monetary policy should pay attention to flexibility, mutual matching, resonance at the same frequency, and close coordination, so as to avoid a vacuum period in which policies are out of sync. Only in this way can the fiscal and monetary policies better press the "fast forward button" for this year's economic growth, and the economy can "make steady progress" [3]. In individual decision-making and market outcomes, uncertainty and consumer risk attitudes are important factors affecting consumer behavior and market operation. Uncertainly and consumer risk attitudes have an important impact on consumer decision-making in the health food market. For risk-averse consumers, they are more inclined to buy healthy foods that have been fully verified and certified to reduce uncertainty in terms of product quality and safety. This means that risk-averse consumers may avoid buying unproven health foods or products that do not have enough information. In contrast, risk-averse consumers are more willing to accept a certain level of uncertainty and risk. Consumers with a risk appetite may be more willing to try new health foods or products that are not widely recognized, and make their own judgements and evaluations, which provides opportunities for the promotion and innovation of new products in the market [4].

Shandong Province is an important industrial base in China and a strategic fulcrum for the economic development of the northern region, making important contributions to the national economic development, but innovative talents have not yet given full play to their due role [5]. In the according essay, it is clear that utility is everywhere in people's daily life, such as in the food market, the supply of public sport, the industrial and so on. So, achieving utility maximization is an important problem.

2. Methods and Theory

2.1. Methodology

Utility is a common thing in people's daily life. Utility can react the degree to which a consumer owns or consumes a good or service to satisfy a desire. Utility is greatest when consumers are most satisfied. The author uses utility function to achieve utility maximization. This paper will use the Cobb-Douglas function to solve some utility problems. It will first find some data online and then analysis them and make some tables of them, then use the Cobb-Douglas function to find the result. In the end, it will use the results to make some conclusions. It can be written to: $Y = AL^\alpha K^\beta e^{ut}$. Y is the output, L is the labor input and K is the capital input. A is the total factor productivity, is a composite indicator that covers all other factors that affect economic growth except labor and capital. ut is the random perturbation term.

This paper will first collect the data of various of places, includes the growth of local economic in Sanming city, the growth of economic of agriculture in the Heilongjiang Province, the data asset value evaluation of communication service enterprises and the estimation of Economic Growth in the USA. Then, use the Cobb-Douglas function to analysis the data to find the result. In the end, make a conclusion of them. The Cobb-Douglas function can be written to many ways in different places and different variables in this essay, but their nature is the same. Adding these factors can make the result closer to the true value.

2.2. Cobb-Douglas Function

The Cobb-Douglas function is a production function that discovered by American mathematician C. W. Cobb and economist Paul. H. Douglas. It is a production function that includes technical resources. It is an economic mathematical model use to predict the industry system of a country or an area. It plays an important part in the research and application of mathematical economics and econometrics. The Cobb-Douglas function find that, the main factors to develop industry is the labor input, fixed asset and polytechnics level (includes operating and management level, labor quality and advanced technology).

3. Results and Application

3.1. The Growth of Local Economic in Sanming City

In the following 10 years, Sanming city have completed the education reformation of Preschool Education, Compulsory Education, High School Education and Career Education and so on, and it made the population culture quantity improves a lot. At the same time, Sanming city also made a great effort on improving and adjusting the structure of various of industries, it made the level of economic in Sanming city improves gradually.

The Cobb-Douglas function can show the impact of population culture quantity to the growth of local economic. So, let the population culture quantity be C , so that this equation is come out:

$$Y = AL^\alpha K^\beta C^\gamma e^u \quad (1)$$

To make this model easier and reduce the heteroscedasticity of variable, by taking the logarithm of both sides of the equation, one can get a new model

$$\ln Y = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln C + u \quad (2)$$

This equation can react the impact of population culture quantity to the growth of local economic in Sanming city. This transforms the exponential model into a log-linear model, and the variable coefficient reflects the estimate between it and the dependent variable γ to judge the impact of the population's cultural quality on the level of economic development [6]. Y is the GDP of Sanming city, L is the average salary and K is the unchanged invest. C is the cost of education in Sanming city. The data of the cost of education and GDP in the Sanming city is shown in Fig. 1. The x axis of the Fig. 1 and 2 is the year and the y axis of the Fig. 1 and 2 is the cost of education per 100 million and GDP per billion in the Sanming city.

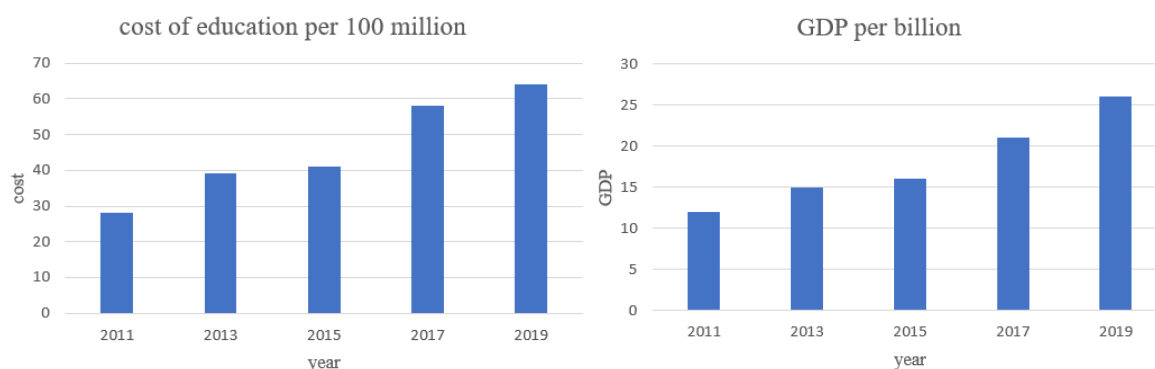


Fig. 1 the cost of education per 100 million and the GDP per billion

According to the data, the equation can be founded: $\ln Y = 3.1013 + 0.4217 \ln K + 0.2947 \ln C$. This means that the logarithmic coefficient of investment in fixed assets is 0.4217, which is significant at the level of 1%, indicating that every 1% increase in fixed asset input will bring about a 0.4217% increase in GDP output, and the coefficient of education expenditure is 0.2947 which is significant at the level of 10%, and the constant term is 3.1013, which is also significant at the level of 10%.

3.2. The Growth of Economic of Agriculture in Heilongjiang Province

The development of economic in China is always has a closely connection with the development of agriculture and the benefit of farmers. China always pays attention to the problems of agriculture, rural areas, and farmers, and it refers to “three agriculture problems”. China also published a series of policies and measures to develop the agriculture. The farming mechanization has a closely connection with the economics of agriculture. The effect to the development of agriculture of farming economic is composed of a lot of factors, the total power of farming machine is the biggest factor of all of the factors, the efficiency of the farming machine can improve the area of the crops and the load per unit area to a certain degree. The Cobb-Douglas function can also find the impact of the level of farming mechanization in Heilongjiang Province to agriculture economic. The equation is:

$$Z = C + \beta_0 B + \beta_1 D + \beta_2 J + \mu \quad (3)$$

Because having some changes (natural logarithmic transformation) to the data can improve the correlation and reliability of the data, and it can also avoid the effect of other not necessary factors in the model (doing the logarithmic transformation will not change the original relation), so doing the logarithmic transformation to the equation, and the result of its transformation is:

$$\ln Z = C + \beta_0 \ln B + \beta_1 \ln D + \beta_2 \ln J + \mu \quad (4)$$

Here, C is the constant term, Z is the total agriculture output, B is the area of agriculture, J is the total power of machine and μ error term [7].

Table 1 shows the agriculture in the Heilongjiang Province. The Table 1 shows the coefficient, standard deviation and statistical quantity p of C , $\ln J$, $\ln B$ and $\ln D$ in the Heilongjiang Province. From table 1, it is given that, $\ln Z = 6.940 + 0.484 \ln B - 0.254 \ln D + 0.549 \ln J$. From this model,

it is clear that when other variable is unchanged, the total agriculture power increase 1%, the total agriculture output increase 0.549%; The area of agriculture increases 1%, the total agriculture output increases 0.484%. The total power of machine and the area of agriculture are both has positive effects to the growth of agriculture economic in Heilongjiang Province. Also, the coefficient of the total power of machine is bigger than that of the area of agriculture, which means its effect is bigger.

Table 1 the coefficient, standard deviation and statistical quantity p of the variable

Variable	Coefficient	Standard Deviation	Statistical Quantity p
C	6.940311	1.393608	4.98
Ln J	0.5488712	0.1215716	4.51
Ln B	0.4842883	0.1155381	4.19
Ln D	-0.2541107	0.0945832	-2.69

3.3. Data Asset Value Evaluation of Communication Service Enterprises

Communication service is a service which can let the customers meet their needs by using the telephone, Internet or some other network technology to communicate. It can provide many services, for example, sound, data, graph and multimedia service. Communication service includes telephone service, data service, graph service and multimedia service. Communication service enterprise means having the production service such as communication network, protection of the network, optimization of the network and the operation of the network and so on.

The Cobb-Douglas function can find the data asset value evaluation of communication service enterprises. This equation can be used:

$$\ln Y = \ln A + a_1 \ln HRV + a_2 \ln DAV + a_3 \ln FAV + a_4 \ln AC + a_5 \ln MBC \quad (5)$$

Here, Y is the income, HRV is the labor value, DAV is the fund value, FAV is the unchanged fund value, AC is the management cost and MBC is the main business value [8]. Table 2 shows the data asset value evaluation of communication service enterprises.

Table 2. Different value of the communication service enterprises.

Variable	Outlier By Subject	Mean	Standard Deviation	Min	Max
ln AC	130	19.00022	1.531	16.716	23.972
ln DAV	130	18.79584	1.646	15.703	22.645
ln FAV	130	20.09344	1.949	15.974	26.559
ln HAV	130	19.6676	1.628	17.042	24.801
ln Y	130	22.00354	1.727	18.872	26.489

The Table 2 shows the outlier by subject, mean, standard deviation, minimum and maximum of Ln AC, Ln DAV, Ln FAV, Ln HAV and Ln Y of communication service enterprises. From table 2, it is clear that the mean of lnY is 22.003, the standard deviation is 1.727, the maximum value is 26.489, the minimum value is 18.872. It means there exist a certain gap in the income of the whole business. By using Cobb-Douglas function, the mean of lnAC is 19.00022, the standard deviation is 1.531, and this means that there is little difference in management costs between different companies in the industry. The mean of lnDAV is 18.795, the standard deviation is 1.646, and this also means that there is no big difference in the pointing out of R&D expenses, and they all attach great importance to the investment of R&D expenses. However, the standard deviation of lnFAV is the greatest of all, it is 20.093. This means that because of the unchanged fund in different communication service enterprises is not the same, there are also big differences in labor input and operating income among companies of different sizes.

3.4. Estimation of Economic Growth in the USA

The growth of economic is changing in the USA during these years. Many factors can affect the growth of economic, for example, labor, capital, technical progress and so on. Various of factors often intertwine with the other factors. People can use some factors to estimate the economic growth in the USA by collecting the data, analysis the data and make a conclusion. The Cobb-Douglas function can describe the economic growth in the USA by using this model:

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L$$

Q is the output (GDP-billion US dollar), L is the labor (10,000 people), K is the capital (fixed asset investment-billion US dollar), A is the productivity of existing technology (total factor productivity). Fig. 2 shows the trends in Labor, Capital and GDP from 1951 to 2006:

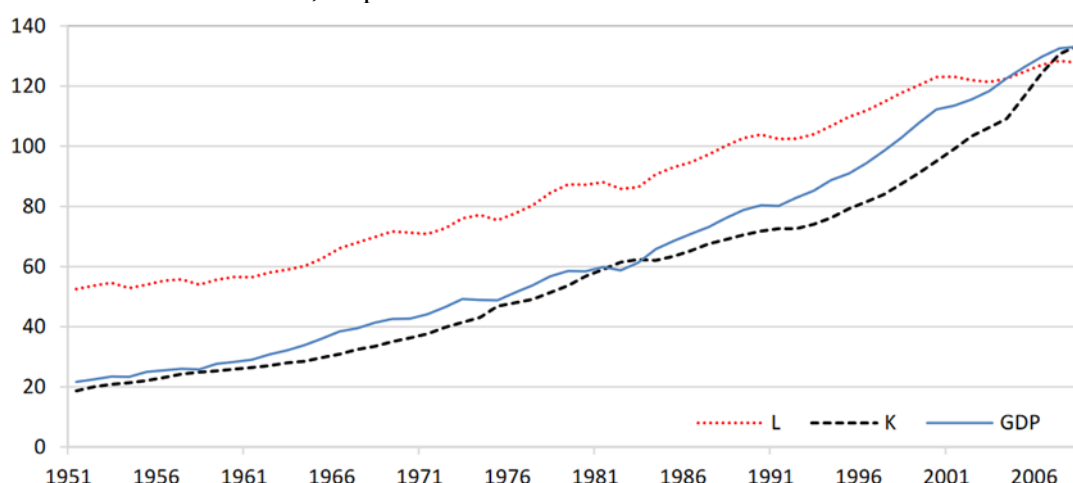


Fig. 2 the value of labor, capital and GDP from 1951 to 2006 in the USA [9]

Using the model that is given, the result is founded in Table 3. The table 3 shows the coefficient, standard error and t start of intercept, Ln K and Ln L of the data in the Fig. 2 [9]. From the Fig. 2 and table 3, it is clear that the figure for labor, capital and GDP all increased gradually from 1951 to 2006. If the figure for the labor increased, the figure for GDP increased; If the figure for capital increased, the figure for GDP increased. It is clear that, when the other variable is unchanged, the capital increase 1%, the total output increase 0.403%; The labor increase 1%, the total output increase 1.094%. Both the capital and the labor have positive effects to the growth of GDP in the USA from 1951 to 2006. The coefficient of the labor is bigger than the coefficient of the capital, so the effect of the labor is bigger than the effect of the capital.

Table 3. the coefficient, standard error and t start of variable.

	Coefficient	Standard Error	t Start
Intercept	-5.7761	1.1151	-5.1799
Ln K	0.4030	0.0677	5.9545
Ln L	1.0936	0.1281	8.5377

4. Conclusion

It is important for people to find the factor which has the biggest effect to the utility maximization. The cost of education higher and the government pay more attention to the education, the growth of GDP in Sanming city increase faster. The level of farming mechanization can also affect the agriculture economic in the Heilongjiang Province. If people increase the total power of farming machine, and the total area of the crops, the economic growth will be more significant. In the communication service enterprises, people should increase the labor value, the fund value, and the management cost to make the economics of that grows faster. Many factors can affect the growth of economic in the USA. From this paper, the effect of the labor is the biggest which means the labor

input is the biggest factor of the growth of economic in the USA. So, people should pay more attention in the labor input so that the economic in the USA will grow faster than before. According to these studies, to achieve utility maximization, people should first find out the main factors of that. The application of utility maximization is everywhere, and people can use the utility function to achieve utility maximization.

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