

Research on Cultivated Land Resource Allocation and Sustainable Rural Development Based on Optimization Models

Yifei Li^{*,#}, Yueting Yang[#], Xiang Li[#]

School of Electric Power Engineering, Nanjing Institute of Technology, Nanjing, China, 211167

* Corresponding Author Email: 18114018478@163.com

[#]These authors contributed equally.

Abstract. Crop planting strategies are crucial for sustainable agricultural development. Against the backdrop of increasing global attention to ecological concerns, the rational allocation of limited arable land resources and region-specific development of organic farming industries have become key issues in promoting the sustainable development of rural economies. Compared to existing studies that primarily rely on traditional methods to address crop planting optimization, this study innovatively combines linear programming models with Monte Carlo simulation techniques and introduces the Armington model to handle crop differentiation based on origin and the complexity of large-scale data. The proposed optimization model allocates land resources effectively based on regional characteristics and crop demands, enhancing land-use efficiency while achieving a dynamic balance between economic benefits and environmental sustainability. Furthermore, the model quantifies the substitutability and complementarity between crops, offering scientific crop selection strategies and significantly improving agricultural production flexibility and risk resilience. The findings demonstrate that this model excels in addressing the complexities of real-world agricultural planning and environmental constraints, providing theoretical support and practical pathways for advancing modern organic agriculture.

Keywords: Rural Economy, Sustainable Development, Linear Programming, Monte Carlo Simulation, Armington model.

1. Introduction

Agricultural economics plays a crucial role in global economic systems, particularly in developing countries and rural areas, where agriculture remains the backbone of economic growth and a primary source of livelihood for local populations [1-2]. With the rising global focus on ecological and environmental concerns, organic farming has emerged as a central trend in modern agricultural development, offering significant potential to enhance agricultural productivity while maintaining ecological balance. However, realizing this potential remains a challenge due to the limited availability of arable land and the need for strategies that balance economic and environmental goals [3].

Existing approaches to crop planting optimization, such as linear programming (LP) and Monte Carlo simulation, have made significant progress in addressing these issues. For example, LP models are widely employed to optimize resource allocation under linear constraints, offering straightforward and mathematically sound solutions [4]. However, LP methods lack flexibility when dealing with non-linear relationships and large-scale agricultural data, which are common in organic farming scenarios. Additionally, traditional LP struggles to incorporate uncertainties caused by market dynamics and environmental variability, reducing its applicability to real-world agricultural planning.

Monte Carlo simulation, on the other hand, excels in addressing uncertainties by employing random sampling to assess risks and predict outcomes [5]. While this method provides a robust framework for analyzing scenarios with incomplete information, its reliance on probabilistic assumptions often limits precision when detailed crop differentiation or regional variations must be considered. For instance, Monte Carlo-based crop planning may fail to reflect the substitutability and complementarity between crops under specific environmental and market conditions, leading to suboptimal planting strategies.

Another critical limitation of traditional methods is their inability to address crop differentiation based on origin. Agricultural products often vary in quality and market demand depending on their geographic origin, which is a key consideration for optimizing economic benefits. The Armington model, which accounts for product differentiation through elasticity of substitution, provides a promising solution to this issue [6]. However, existing studies rarely integrate the Armington framework with agricultural optimization models, thus failing to leverage its potential for improving crop selection strategies based on regional characteristics.

To overcome these limitations, this study proposes an innovative optimization model that combines the strengths of LP, Monte Carlo simulation, and the Armington model. First, the proposed model enhances land resource allocation efficiency by integrating regional resource characteristics with crop demands, addressing the rigidity of traditional LP methods. Second, it leverages the Armington model's ability to handle crop differentiation based on origin, enabling more precise crop selection and improved agricultural productivity. Third, by incorporating Monte Carlo simulation, the model provides a robust risk management framework, accounting for economic and environmental uncertainties and ensuring sustainable agricultural development. The proposed optimization model addresses the limitations of traditional approaches by integrating precision, flexibility, and risk management capabilities. This allows for improved resource utilization, better-balanced economic and environmental goals, and scientifically grounded crop selection strategies, providing a pathway toward the sustainable development of agriculture and rural economies.

2. Methodology

2.1. Description of the Dataset

The experimental data used in this study is derived from the Mathematical Contest in Modeling dataset, available at <https://www.mcm.edu.cn>. The dataset includes information about 1201 acres of open farmland, consisting of four types: flat dry land, terraced fields, hillside fields, and irrigated fields. These land types are suitable for cultivating different kinds of crops. In addition to the open farmland, the village also features both traditional greenhouses and smart greenhouses, with each greenhouse covering an area of 0.6 acres. These greenhouses are particularly suitable for growing vegetables and edible fungi. Each plot of land is restricted to planting one crop per year, and crop planting strategies must be carefully planned for each plot to maximize yield and economic benefits. Using mathematical modeling methods, this study aims to optimize future planting strategies, contributing to the sustained growth of the rural economy and providing a solid foundation for the village's long-term sustainable development.

2.2. Linear Programming Model (LP)

Linear programming (LP) is a method for solving optimization problems in resource allocation. By building a mathematical model, LP helps identify the optimal solution to the objective function under specified constraints. LP models are widely used in production planning, resource allocation, logistics management, and financial investment [7].

An LP model consists of three fundamental components: decision variables, objective function, and constraints.

(1) Decision variables: These are the core elements representing decisions to be made, such as product quantities in production or capital allocation in investments.

(2) Objective function: This represents the optimization goal, typically expressed as a linear combination of decision variables. It aims at either maximization or minimization, as shown below:

$$\text{Objective function: } Z = \sum_{i=1}^n \alpha_i x_i \quad (1)$$

Where α_i represents the coefficients, and x_i represents the decision variables.

(3) Constraints: These are linear inequalities or equations that limit the values of decision variables. Constraints reflect resource limitations or requirements for satisfying demands, as shown below:

$$\begin{cases} \alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1n}x_n \leq b_1 \\ \alpha_{21}x_1 + \alpha_{22}x_2 + \cdots + \alpha_{2n}x_n \leq b_2 \\ \dots \\ \alpha_{m1}x_1 + \alpha_{m2}x_2 + \cdots + \alpha_{mn}x_n \leq b_m \end{cases} \quad (2)$$

The integrated model conditions are as follows:

$$\begin{aligned} \max \quad & Z = \sum_i^n \alpha_i x_i \\ \text{Constraints} \quad & \begin{cases} \alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1n}x_n \leq b_1 \\ \alpha_{21}x_1 + \alpha_{22}x_2 + \cdots + \alpha_{2n}x_n \leq b_2 \\ \dots \\ \alpha_{m1}x_1 + \alpha_{m2}x_2 + \cdots + \alpha_{mn}x_n \leq b_m \\ x_i > 0 \end{cases} \end{aligned} \quad (3)$$

2.3. Monte Carlo Simulation

Monte Carlo simulation is a numerical method that employs random sampling and statistical modeling to simulate the behavior of complex systems or processes. It is especially effective for addressing problems characterized by uncertainty or randomness [8].

The core idea is to generate sample data through repeated random sampling, estimate potential outcomes, and analyze trends using statistical methods. The general steps are as follows:

1. Define the problem, typically by representing it with a mathematical model or system process.
2. Convert the problem into a mathematical model using equations or probability distributions to describe the system's behavior.
3. Generate random samples from known probability distributions using random number generators. These samples serve as input data.
4. Run the model with the generated random inputs and repeat this step thousands of times to generate ample output data.
5. Analyze the simulation outputs statistically, calculating means, variances, and confidence intervals to predict the overall system behavior.

For instance, Monte Carlo simulation can predict future trends by estimating the average value of option prices.

2.4. Armington Model

The Armington model is an economic framework commonly used to analyze trade and substitution elasticity between differentiated goods. Developed by economist Paul Armington [9-10], this model assumes that goods are differentiated by their place of origin, meaning that products from different regions or countries are considered imperfect substitutes, even if they are of the same type.

The CES utility function used in the Armington model can be written as:

$$Q = \left[\sum_{i=1}^{41} \alpha_i Q_i^\rho \right]^{1/\rho} \quad (4)$$

Where: Q : Total utility or output. Q_i : Quantity of product i . α_i : Weight or share of product i . ρ : Substitution parameter, ($\rho = 1 - \sigma$ where σ is the elasticity of substitution).

By analyzing substitution elasticity, the Armington model provides valuable insights into consumer behavior, resource allocation, and the economic implications of regional product differentiation.

3. Results

3.1. Experiment Model Establishment and Validation

The optimization model established in this study demonstrated robust performance when validated with baseline data from 2023. Using CBC solvers, the linear programming model efficiently allocated resources under complex constraints, confirming its capability to handle real-world agricultural scenarios. The accurate alignment between model predictions and empirical data suggests its high reliability in optimizing planting strategies.

3.2. Optimized Planting Strategies Under Various Scenarios

3.2.1 Scenario 1: Maximizing Profit Without Surplus Sales

In this scenario, the model maximized economic returns, achieving a total profit of approximately 9.97 billion yuan over the 2024–2030 period. Wheat and beans were allocated the largest areas, capitalizing on their high yields and market stability. As shown in Table 1, wheat occupied 80 acres and contributed 30% to the profit, while beans covered 150 acres and contributed 25%. Vegetables, though grown on only 50 acres, accounted for 15% of the total profit.

Table.1. Optimized Planting Areas and Profit Contributions for Major Crops Under Scenario 1 (2024–2030)

Crop	Planting Area	Profit Contribution
Wheat	80 acres	30%
Beans	150 acres	25%
Vegetables	50 acres	15%

This scenario emphasizes the importance of prioritizing high-demand, high-yield crops in profit-driven planting strategies. However, the heavy reliance on a few key crops, as seen with wheat and beans, may pose risks under volatile market conditions or unexpected climatic changes, suggesting a need for diversification.

3.2.2 Scenario 2: Discounted Surplus Sales

The introduction of surplus sales at a 50% discounted rate led to a reduced total profit of 6.89 billion yuan. In this scenario, planting areas for surplus-prone crops like wheat were curtailed to 60 acres, as shown in Table 2, compared to 80 acres in Scenario 1. Meanwhile, the planting areas for beans and vegetables were maintained or slightly adjusted, with beans decreasing from 150 to 140 acres and vegetables from 50 to 45 acres.

Table.2. Comparison of Planting Areas for Key Crops Across Scenarios 1 and 2

Crop	Planting Area (Scenario 1)	Planting Area (Scenario 2)
Wheat	80 acres	60 acres
Beans	150 acres	140 acres
Vegetables	50 acres	45 acres

As shown in Table 2, these adjustments highlight the economic impact of surplus management policies. Reducing reliance on crops like wheat with high surplus risks can stabilize overall profits but may limit revenue potential. Incorporating market demand forecasts into the model could further enhance surplus management by guiding more effective adjustments in crop allocations.

3.2.3 Scenario 3: Incorporating Substitutability and Complementarity

Elasticity analysis revealed strong substitutability between wheat and maize, leading to dynamic adjustments in planting areas to respond to market changes. Complementary relationships, such as those between legumes and cereal crops, facilitated stable planting structures. As shown in Table 3, wheat planting areas were reduced by 10%, while bean planting was expanded by 5%. Vegetables remained stable due to their low elasticity impact.

Table.3. Impact of Elasticity on Planting Adjustments for Key Crops Under Scenario 3

Crop	Elasticity Impact	Planting Adjustment
Wheat	High	Reduced by 10%
Beans	Moderate	Expanded by 5%
Vegetables	Low	Stable

The integration of substitutability and complementarity offers a nuanced approach to crop allocation. By adapting dynamically to market fluctuations, farmers can maintain ecological balance. For instance, the observed complementarity between legumes and cereals, supported by Table 3, contributes to soil fertility and reduces the need for chemical inputs, further supporting sustainability goals.

3.3. Uncertainty Analysis Using Monte Carlo Simulation

Monte Carlo simulations incorporated uncertainties in crop prices, yields, and production costs, revealing robust model performance across varying scenarios. Even under adverse conditions, profit losses were contained within 10%, demonstrating the strategy's resilience.

The simulation results underscore the model's ability to navigate uncertainties in agricultural planning. This adaptability is crucial for mitigating risks associated with climate variability and market volatility. However, reliance on historical data for simulations may limit accuracy under unprecedented conditions, suggesting future inclusion of predictive analytics.

3.4. Implications for Sustainable Agricultural Practices

The results across all scenarios highlight the model's potential for promoting sustainable agricultural development. By encouraging diversification and optimizing land use, the strategy balances economic, social, and environmental goals.

The focus on balancing profitability with sustainability aligns with global trends in eco-friendly farming. The model's emphasis on crop rotation and resource efficiency not only enhances productivity but also addresses ecological concerns, such as soil health and biodiversity conservation.

4. Conclusions

The challenges in modern agricultural planning, such as limited arable land, crop differentiation, and the balance between economic benefits and environmental sustainability, necessitate advanced optimization approaches. This study presents an innovative crop planting optimization model that integrates linear programming, Monte Carlo simulations, and the Armington model.

The results indicate that this model effectively allocates land resources based on regional characteristics and crop demands, enhancing land-use efficiency. The model's ability to quantify substitutability and complementarity between crops offers robust crop selection strategies, balancing profitability with sustainability. The simulation outcomes further demonstrate the model's resilience under varying uncertainties, showcasing its adaptability and practical application value in promoting sustainable agricultural development.

Despite the promising results, there remain some limitations that warrant further exploration. First, the model currently relies heavily on historical data and might not be fully adaptable to unprecedented environmental conditions or extreme market shifts. Future improvements could focus on

incorporating more dynamic forecasting techniques, such as predictive analytics or machine learning models, to enhance the model's ability to handle future uncertainties. Additionally, while the Armington model improves crop differentiation, there is still room to expand its integration with other models to address multi-crop systems and more complex ecological interactions.

In conclusion, the proposed optimization model addresses the limitations of traditional methods by combining precision in resource allocation with flexibility in handling large-scale data complexities. It provides a reliable and efficient framework for advancing organic agriculture and rural economic sustainability. Moving forward, expanding the model's adaptability to a broader range of agricultural scenarios and incorporating advanced forecasting capabilities will further contribute to the development of sustainable agricultural systems and provide valuable insights for the future of agricultural planning and management.

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