Research on Production Decision-Making Model Based on Monte Carlo Simulation Algorithm

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Abstract. Reasonably determining the minimum sample size of electronic device sampling and testing has an important cost-effective value for enterprises, which is conducive to improving their economic efficiency and market competitiveness. In this paper, hypothesis testing and normal distribution approximation are used to determine the minimum sample size, then the sequential probability ratio (SPRT) model test is used to derive the sampling and testing scheme, and the stability of the decision is analyzed through the operating characteristic (OC) curve and the average sample size (ASN) curve, so as to further establish the cost-benefit analysis model to assess the impact of the testing cost, defective rate, market selling price and other factors on the profit, and finally the Monte Carlo simulation algorithm is used to predict the profit under different scenarios and strategies. Finally, Monte Carlo simulation algorithms are used to predict the cost-effectiveness under different situations and strategies, and to derive the optimal strategic plan, decision-making basis and corresponding index results for each production situation.This optimal production decision allows the enterprise to effectively manage inspection costs while maintaining product quality. It also contributes to a reduction in the rate of defective products, enhances market pricing strategies, and ultimately facilitates the efficient allocation of resources.

Keywords: Production decisions, SPRT model, Cost-effectiveness, Monte Carlo simulation.

1. Introduction

The quality of the finished product depends on the quality of the spare parts: if any spare part is unqualified, the finished product will be unqualified; even if both parts are qualified, the finished product may be unqualified for other reasons [1]. Unqualified products can be scrapped or dismantled, dismantling does not damage the parts but will incur costs. Therefore, quality control and reasonable sampling and testing quantities are essential for efficient production and cost-optimized production decisions [2].

Otunno, Giuliana et al [3] proposed a decision support strategy combining Discrete Event Simulation (DES) and Cost-Benefit Analysis (CBA) by describing the target system using UML and using the output of DES in CBA. However, this approach may run the risk of simplifying real-world complexity. On the other hand, de Amorim, FR et al [4] proposed a strategy for fast automated decision making in dynamic plant environments centred on the use of Monte Carlo Tree Search (MCTS) algorithm[5] Using Adaptive Tuning. This approach achieves fast automated decision making in dynamic environments through the use of the MCTS algorithm, which is very beneficial for production environments that require immediate response to changes.

Therefore, in this paper, we determine the minimum sample size through hypothesis testing with normal distribution approximation [6] and validate the sampling and testing scheme by using the SPRT model [7], carry out further cost-benefit analysis by using the operating characteristic (OC) curve and the average sample size (ASN) curve, and predict the cost-benefit under different scenarios and strategies with the help of Monte Carlo simulation algorithms [8], thus proving the high accuracy and effectiveness of the model in production decision making.

2. Minimum Sample Size Sampling and Detection Modelling and Analysis

2.1. Model Preparation

1. Hypothesis testing: binomial distribution models

Suppose a company needs to test whether a batch of spare parts has a defective rate that exceeds its nominal value $\mathrm{p_0}$. This can be achieved by hypothesis testing [9], assuming that the defective rate p does not exceed $\mathrm{p_0}$ when the test results should conform to binomial distribution. Construct the following hypothesis:

Null hypothesis H_0: defective rate $\mathrm{p} \leq \mathrm{p_0}$ The defective rate does not exceed the nominal value;

Alternative scenario H_1 : defective rate $\mathrm{p} > \mathrm{p_0}$ The defective rate exceeds the nominal value.

Use the binomial distribution to describe the process of detecting nonconforming products. Let n be the sample size, and X be the number of defective products in the sample, then the X The obedience parameters are n and p of the binomial distribution:

$$X \sim Bin(n,p) \tag{1}$$

Under the assumption H_0 , i.e. $p \leq p_0$, the probability of the number of rejects is:

$$P(X=k) = nkp_0{}^k(1-p_0)^{nk} \tag{2}$$

Where nk is the combination number, denoting the number of pieces drawn from n pieces are drawn out of the k The probability that one piece of non-conforming product will be drawn.

2. Normal approximate binomial distribution.

By calculating the Standardized test statistic Z , the decision to reject the null hypothesis can be made by comparing it to the critical values in the standard normal distribution table. For larger sample sizes, the normal distribution can be used to approximate the binomial distribution:

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right) \tag{3}$$

Where: $\hat{p}$ is the estimate of the substandard rate in the sample.

3. Calculation of confidence intervals

Enterprises can judge whether the defective rate of spare parts is within the acceptable range by calculating the confidence interval obtained [10], the confidence interval of the defective rate $\hat{p}$ is:

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \tag{4}$$

For 95 per cent confidence level, $Z_{\alpha/2} \approx 1.96$; for 90 per cent confidence level, $Z_{\alpha/2} \approx 1.645$.

Where: $\hat{p}$ is the defective rate in the sample; $Z_{\alpha/2}$ is the critical value of the standard normal distribution corresponding to the confidence level.

2.2. Calculating the Minimum Sample Size

When the purpose of the study is to estimate the overall proportion, the following formula can be used to calculate the minimum sample size:

$$Min(n) = \frac{Z^2 \times p \cdot (1-p)}{E^2} \tag{5}$$

Where: p is the assumed defective rate and E is the allowed error.

Sample Size Example Calculation: Assume that the error allowed in this paper E is 0.05 and the defective rate $p = 0.1$, from which the sample size is calculated.

1. Sample size at 95 per cent confidence level

$n_1 = \left(\frac{1.96}{0.05}\right)^2 \cdot 0.1(1-0.1) = 138.298$ The result is $n_1 = 138$ when rounded to the nearest whole number.

2. Sample size at 90 per cent confidence level

$n_2 = \left(\frac{1.645}{0.05}\right)^2 \cdot 0.1(1-0.1) = 97.4169$ The result is $n_2 = 97$ when rounded to the nearest whole number.

Using equation (5) the program is designed to complete the calculation of the minimum sample size as shown in Figure. 1:

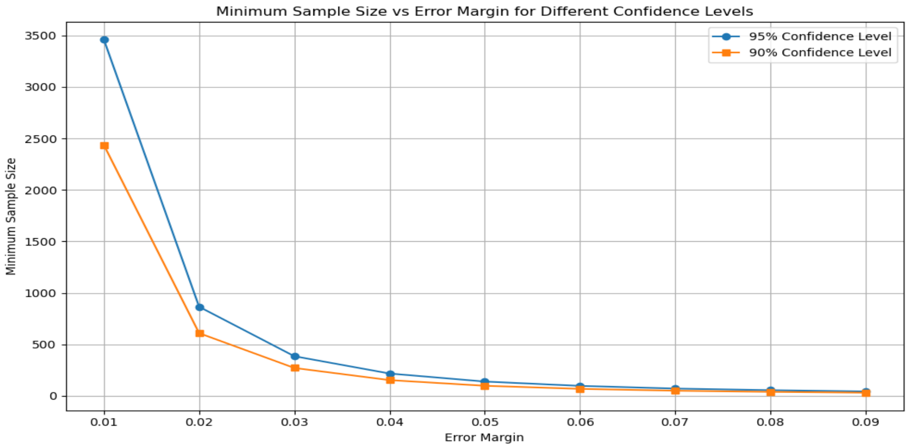


Figure 1. Minimum Sample Size Margin of Error

It is derived from equation (5) that the minimum sample size required for both confidence levels tends to decrease as the error range increases under different error ranges, while the difference in the minimum sample size between the two confidence levels decreases.

Table 1 below shows the minimum sample sizes for different confidence levels corresponding to specific error values.

Table 1. Minimum Sample Size for Different Confidence Intervals

Minimum Sample Size		
Margin of Error	95% Confidence Level	90% Confidence Level
0.02	865	609
0.03	385	271
0.04	217	153
0.05	139	98
0.06	97	68
0.07	71	50
0.08	55	39

2.3. Sequential Probability Ratio Test (SPRT) modelling

2.3.1 Sequential Probability Ratio Test (SPRT) model Principle

Based on the above idea, the core idea of the SPRT model is to calculate the likelihood ratio after each sampling test and compare it with the pre-set upper and lower limits to decide whether it is necessary to continue sampling or to make a decision of acceptance/rejection [11].

Likelihood ratio function:

$$L(x) = \frac{p_1{}^{x}(1-p_1)^{n-x}}{p_0{}^{x}(1-p_0)^{n-x}} \tag{6}$$

Where: n is the current sample size and x is the number of failed samples observed.

To simplify the calculations, the logarithm of the likelihood ratio function is obtained:

$$\ln L(x) = x\ln\left(\frac{p_1}{p_0}\right) + (n-x)\ln\left(\frac{1-p_1}{1-p_0}\right) \tag{7}$$

The decision boundary becomes:

$$\ln A = \ln\left(\frac{1-\beta}{\alpha}\right),\ \ln B = \ln\left(\frac{\beta}{1-\alpha}\right) \tag{8}$$

2.3.2 SPRT Model Solution and Result Analysis

1. Parameter setting for sequential testing

Scenario 1: Rejection of programme at 95 per cent confidence level

Significance level $\alpha = 0.05$ (probability of wrongly accepting substandard spare parts)

Test efficacy $\beta = 0.10$ (probability of incorrectly rejecting a qualified spare part)

Upper Threshold: $\ln A_1 = \ln\left(\frac{1-\beta}{\alpha}\right) = \ln\left(\frac{1-0.10}{0.05}\right) = 2.89037$

Lower threshold: $\ln B_1 = \ln\left(\frac{\beta}{1-\alpha}\right) = \ln\left(\frac{0.10}{1-0.05}\right) = -2.25129$

Scenario 2: Receiving programme at 90 per cent confidence level

Significance level $\alpha = 0.10$ (probability of incorrectly accepting substandard spare parts)

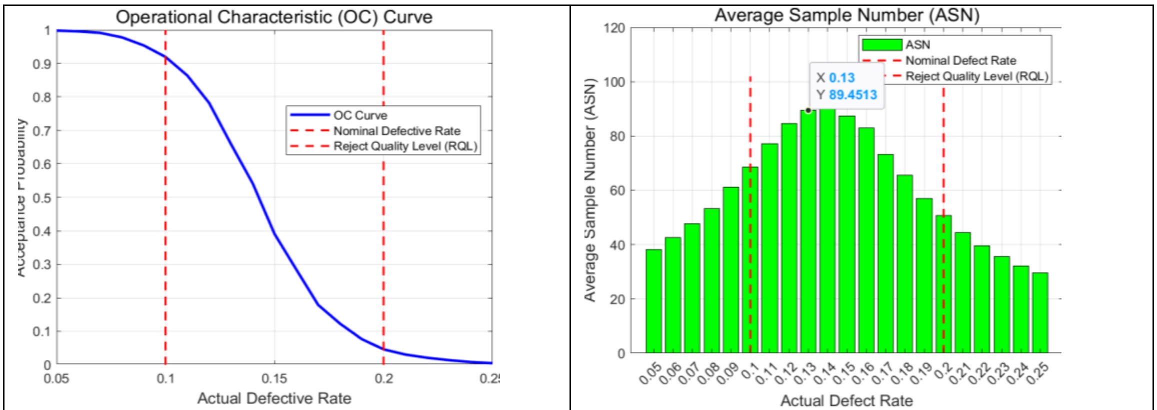
Test efficacy $\beta = 0.05$ (probability of incorrect rejection of eligible spare parts)

Based on the set parameters, the two decision boundaries for the sequential test are calculated as above.

2. Analysis of results

Scenario 1: Rejection of programme at 95 per cent confidence level

The procedure was designed according to the principles of SPRT model to complete the test of the rejection scheme at 95% confidence level, and running the procedure yielded the results as shown in Figure. 2:



(a) OC Curve for Different True Defect Rates (b) ASN Curve for Different True Defect Rates

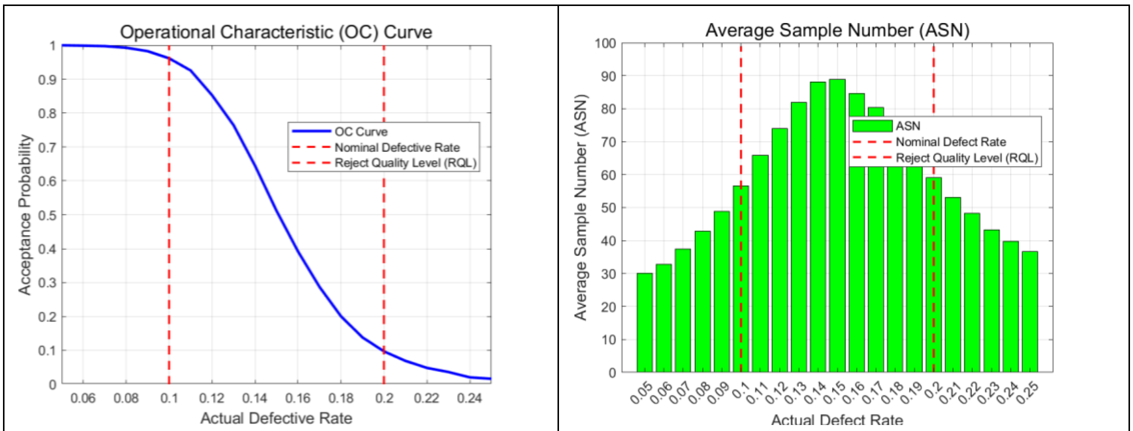
Figure 2. Rejecting Scheme at a 95% Confidence Level

The OC curves show the probability of accepting batches at different true defective rates. It can be seen that when the true defective rate is close to or lower than p0(0.10), the acceptance probability is very high, about 0.9609; when the true defective rate is close to or higher than p1(0.20), the acceptance probability decreases sharply. This shows that the SPRT model can effectively distinguish between "good" and "bad" batches.

The ASN curve shows the average sample size required at different true defective rates. It can be observed that when the true defective rate is close to p0 or p1, the average sample size is smaller, about 56.4403, while when the true defective rate is between p0 and p1, the average sample size is larger, about 88.9941.This is because more samples are needed to make a reliable decision when the quality level is close to the decision boundary.

Scenario II: Intake programme at 90 per cent confidence level

The procedure was designed according to the principles of the SPRT model to complete the test of the rejection scheme at 90% confidence level, and running the procedure yielded the results shown in Figure 3:



(c) OC Curve for Different True Defect Rates (d) ASN Curve for Different True Defect Rates

Figure 3. Rejecting Scheme at a 90% Confidence Level

The OC curves show the probability of accepting batches at different true defective rates. It can be seen that when the true defective rate is close to or lower than p0(0.10), the acceptance probability is very high, about 0.9208; when the true defective rate is close to or higher than p1(0.20), the acceptance probability decreases sharply. This shows that the SPRT model can effectively distinguish between "good" and "bad" batches.

The ASN curve shows the average sample size required at different true defective rates. It can be observed that when the true defective rate is close to p0 or p1, the average sample size is smaller, about 69.481, while when the true defective rate is between p0 and p1, the average sample size is larger, about 91.2353.This is because when the quality level is close to the decision boundary, more samples are needed to make a reliable decision.

3. Cost-benefit Analysis of Monte Carlo Simulation Algorithms

3.1. Model Preparation

3.1.1 Analysis of Cost-effective Decision-making

1. Cost decision-making for spare parts testing

State representation: $\{s_1, s_2\}$

$s_1 \epsilon \{0, 1\}$: Whether or not to detect spare parts 1 (0 for no detection, 1 for detection).

$s_2 \epsilon \{0, 1\}$: Whether or not spare parts 2 are detected (0 is not detected, 1 is detected).

Detection costs:

$$C_1 = m \times C_1' \tag{9}$$

Where: m is the number of parts tested and C_1' is the cost per part tested.

2. Cost decision-making for finished product testing

The company identifies and eliminates unqualified products to reduce the loss of returns, repairs and reputation caused by product quality problems.

State representation:$\{s_3\}$

$s_3 \epsilon \{0, 1\}$: Whether or not to test the finished product (0 is not tested, 1 is tested).

Finished product testing costs:

$$C_{11} = n \times C_{11}' \tag{10}$$

Where:C_{11}' is the cost of testing each finished product.

3. Cost decision-making for the dismantling of non-conforming finished products

For substandard finished products detected, enterprises may choose to discard them directly or dismantle the substandard finished products to recover qualified spare parts for reuse. There will be a certain cost for dismantling, so it is necessary to judge whether to dismantle or not.

State representation:$\{s_4\}$

$s_4 \epsilon \{0, 1\}$: Whether or not to dismantle substandard finished products (0 for no dismantling, 1 for dismantling).

Disassembly costs:

$$C_3 = n_d \times C_3' \tag{11}$$

Where:n_d is the number of non-conforming finished products and C_3' is the dismantling cost per finished product.

4. Integrated decision-making models

In order to comprehensively consider the decision-making of each stage in the whole production process, the following total cost expectation model can be constructed:

$$E(C) = \sum_{i=1}^{n} (C_1(i) + C_2(i) + C_3(i) + C_4(i)) \tag{12}$$

Where:$C_i (i = 1, 2, 3, 4)$ is the cost of each stage and determines whether testing or dismantling is performed at each stage, thus optimizing the cost structure of the entire production process.

5. Return of non-conforming finished goods

When a user returns a non-conforming finished product, the company has the option of directly scrapping or dismantling the returned product.

If the disassembled returned non-conforming product still has a revenue value, it can be calculated according to the following formula:

$$C_{rd} = C_t + C_3 E(DR) \tag{13}$$

Where:C_t is the logistics cost of the return process and $E(DR)$ is the expected return on eligible spare parts recovered from dismantling.

3.2. Modelling and Solving

3.2.1 Monte Carlo Simulation Decision-making Model

Due to the large uncertainty in the defective rate and production process in each stage of the production process described above, this paper uses a Monte Carlo simulation algorithm to estimate the expected cost under different strategies by randomly simulating different defective rates and production processes several times. As a result, the expected profit per piece under different strategies is:

$$E(R) = S - E(C) \tag{14}$$

Where: S is the market price.

There are a total of $2^4=16$ different combinations of strategies based on the strategies used in the 3 production stages described above. Each combination represents the inspection decision for part 1, part 2, the inspection of the finished product and the disassembly decision.

In this paper, Monte Carlo simulation is used to simulate the decision combinations of four strategies [12], 16 in total, and also need to consider the spare parts obtained from disassembling the substandard finished products to be re-tested and assembled.

3.2.2 Analysis of Results

Based on the Monte Carlo simulation algorithm the program is designed to derive the profit per piece for different scenarios and strategies as well as the optimal combination of decisions for each scenario, running the program yields the results as shown in Figure. 4:

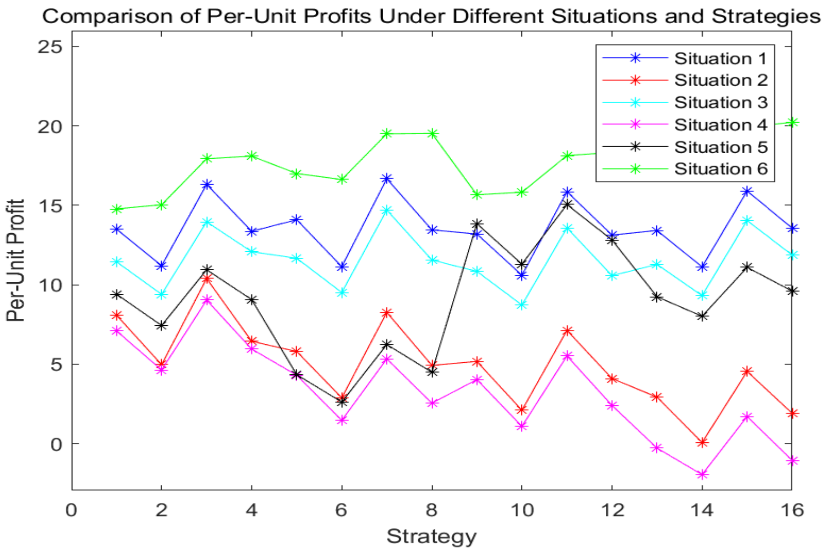


Figure 4. Comparison of Per-Unit Profits under Different Situations and Strategies

Under the 16 strategy combinations, for the six cases, respectively, the single piece of profit analysis, through the nested loop for 10,000 simulations, calculate and accumulate the net profit of each simulation, it can be concluded that the single price profit ups and downs fluctuate greatly, most of the aggregation of 10-20, this is due to the number of simulations is large, the parameter adjustment between the situation strategy is small [13], the impact on the results is not significant. Some more than 20, a small number of less than 5, case 4 there is less than 0 loss situation, which may be due to the case of 4 higher rate of defective products, up to 20%, exchange loss is larger for 30, there is a frequent detection of unqualified finished products as well as the return of exchange dismantling of the situation, which leads to a reduction in profits.

Table 2. Decision Combinations Corresponding to Specific Situations

sight	Spare part 1 testing	Spare part 2 testing	Finished product testing	Dismantling of substandard products
1	be	clogged	clogged	be
2	be	be	clogged	be
3	be	clogged	clogged	be
4	be	be	clogged	be
5	clogged	be	clogged	be
6	clogged	clogged	clogged	clogged

Table 3. Profit Table for Different Situations and Strategy Combinations

Profit unit (yuan)		
Type of situation	best combination of strategies	Maximum profit per unit
1	(1, 0, 0, 1)	16.67694
2	(1, 1, 0, 1)	10.38893
3	(1, 0, 0, 1)	14.70840
4	(1, 1, 0, 1)	9.03463
5	(0, 1, 0, 1)	15.05093
6	(0, 0, 0, 0)	20.23140

From Tables 2 and 3 comparing the maximum single profits obtained from each of the 16 strategies corresponding to the 6 cases:

Scenario 1 has a low defect rate, the cost of testing spare part 2 is higher than 1, and spare part 2 is not tested.

Scenario 2 has a high defective rate of 20 per cent, and finished product testing is costly; finished product is not tested.

Scenario 3 has a low rate of substandard products, and the potential gain in substandard finished products is greater than the cost of dismantling them, and dismantling the substandard finished products.

The profit obtained in case 4 is $9.03463, the lowest profit obtained per piece, the rate of defective products is larger, the cost of detecting the parts are lower for 1, the spare parts 1 and 2 are tested.

Case 5 detects high cost of spare parts 1 as 8 and low cost of spare parts 2.

The profit obtained in case 6 is $20.23140, which yields the largest profit per piece, a lower defective rate of 5 per cent, and a higher cost of testing accessories and finished products, which are not tested and dismantled.

4. Conclusion

This paper addresses the problem of determining cost-effective decision making for electronic devices by calculating the minimum sample size based on the estimated overall proportion formula and using the SPRT model to test the designed sampling and testing scheme. Based on the optimization scheme, a Monte Carlo simulation model is developed, and the results show that the model yields the profit per unit under different scenarios and strategies as well as the optimal decision combination in each case.

In terms of designing solutions for actual production decisions, Monte Carlo method results are stochastic in nature and require extensive simulation to obtain stable and reliable estimates. Therefore, enterprises should use Monte Carlo models to conduct cost-benefit analyses in the context of their own practices, and periodically validate and update the models to ensure that they reflect the current business environment and risk profile.

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