Research on Optimal Production Decision Based on Dynamic Programming Model

Chujun Wang *

Department of automation, Hebei University of Technology, Tianjin, China, 300000

* Corresponding Author Email: 18332588637@163.com

Abstract. In the fiercely competitive machine parts manufacturing industry in China, companies must enhance production efficiency and reduce costs to maintain a competitive advantage. To achieve this, enterprises need to make optimal decisions during the production process to maximize profits. This paper proposes a decision-making framework aimed at addressing complex production and cost structures to ensure profit maximization under different conditions. A dynamic programming model is constructed to manage multi-stage decision-making, enabling enterprises to adjust strategies dynamically. To further improve decision outcomes, the simulated annealing algorithm is applied, which helps avoid local optima and identify the global optimal solution. The key innovation of this research is the integration of dynamic programming with the simulated annealing algorithm, offering significant flexibility and adaptability in optimizing complex production processes. This approach not only helps businesses improve efficiency, reduce costs, and strengthen their market competitiveness but also provides valuable insights for optimizing decision-making across the manufacturing industry.

Keywords: Dynamic Programming, Simulated Annealing Algorithm, Production Decision model.

1. Introduction

The competition in China's machinery parts manufacturing industry is intense. To succeed, companies must adopt advanced methods to improve production efficiency and reduce costs [1]. The industry generates significant material waste and energy consumption, so understanding the production process, identifying key issues, and continuously improving are essential for maintaining a competitive edge. This requires aligning theory with practice and making adjustments based on the actual situation of the enterprise [2]. This paper establishes the objective function and constraints, using dynamic programming combined with simulated annealing for optimization [3]. This approach overcomes the limitations of traditional methods, which often rely on using only one algorithm, such as dynamic programming, decision trees, or simulated annealing, and are only suitable for simpler problems with fewer parts. By addressing these challenges, this combination improves solution efficiency and quality.

2. Establishment of comprehensive decision model

This paper assumes that disassembly or inspection must be performed uniformly on the same batch of identical objects, without partial operations. If two qualified parts produce an unqualified finished product, reassembly after disassembly ensures qualification. The disassembly and reassembly process are allowed only once.

2.1. Single operation

Taking the single process of two parts synthesizing a finished product as an example, since the key to enterprise decision-making is to obtain the maximum profit, a comprehensive decision-making model is established on the basis of profit to make decisions in the production process. Different production decisions will lead to different costs, obtain different benefits, and finally determine the final profit of the enterprise. Therefore, the core problem of making the optimal decision is profit maximization, which can be optimized for each link by analyzing profit. The specific formula is as follows:

$$W_{total}=\sum_{i=1}^{5}W_i \tag{1}$$

Where W_{total} represents the total cost, and W_i (i=1,2,3,4,5) respectively represents the replacement cost of parts inspection, finished product inspection, assembly cost, disassembly cost, and customer refund.

$$L_{total}=Q_{former}+Q_{dismantle}-W_{total} \tag{2}$$

L_{total} represents the total profit, and Q_{former} represents the profit of qualified finished products sold on the first sale (excluding the part sold after the second assembly). $Q_{dismantle}$ represents the income obtained from the sale of the returned and tested nonconforming products after disassembly and reassembly. To calculate the maximum profit, the following article will investigate and optimize the value of each part of the profit calculation formula.

Income from the first sale of qualified finished products (Q_{former}):

$$Q_{former}=n_{Primary}\times(1-P_B)\times C \tag{3}$$

Where P_B represents the defective rate of finished products and C is the market price. $n_{Primary}$ represents the total number of finished products produced for the first time (excluding the finished products reassembled after disassembly). Since the finished product is composed of component 1 and component 2 one to one, its quantity is equal to the minimum value of component 1 and component 2 entering the assembly link, and its calculation formula is as follows:

$$n_{Primary}=\min\left(n_1(1-p_1x_1),n_2(1-p_2x_2)\right) \tag{4}$$

The formula for calculating the number of finished products produced by secondary assembly is as follows:

$$n_{Secondary}=\min\left(n_{Primary}\,p_{finished}\left[1-(1-x_1)\,p_1\right],n_{Primary}\,p_{finished}\left[1-(1-x_2)\,p_2\right]\right) \tag{5}$$

The probability of defective products entering the assembly process is:

$$P_A=(1-x_1)\,p_1+(1-x_2)\,p_2-(1-x_1)\,(1-x_2)\,p_1p_2 \tag{6}$$

Where x_1 and x_2 are the state variables of component 1 and component 2. When the value is 1, it means that it has been tested; when the value is 0, it means that it has not been tested. p_1 and p_2 are the defective rates of component 1 and component 2 respectively. Revised defective rate of finished products:

$$p_{finished}=P_A+P_B \tag{7}$$

Where P_B is the defective rate of finished products after the assembly of genuine parts.

Parts inspection cost (W_1):

If the enterprise carries out spare parts testing, the qualified products are assembled, and the unqualified products are directly discarded. Therefore, it is possible to compare the testing cost and the loss caused by unqualified products after assembly to determine whether to carry out parts testing. The calculation formula of inspection cost is as follows:

$$W_1=n_1\times x_1\times W_{1e}+n_2\times x_2\times W_{2e} \tag{8}$$

n_1 and n_2 are the total number of parts 1 and 2 respectively. W_{e1} and W_{e2} are the single inspection costs of component 1 and component 2 respectively.

When all parts are not inspected, there may be a certain inspection cost, and if unqualified parts enter the assembly, it may lead to defective assembly, resulting in potential assembly cost losses and user return and replacement cost losses. The formula of expected loss caused by assembly of unqualified products is:

$$E[C_1] = W_3{}' + W_5{}' + W_1 \tag{9}$$

State variables x_1 and x_2 are 0 when different. $W_3{}'$ is the assembly loss from defects, and $W_5{}'$ is the refund and replacement cost. Formula:

$$W_5{}' = (1 - x_3) n_{Primary} P_A W_{exchange} \tag{10}$$

Where x_3 represents the inspection status of the finished product, $W_{assemble}$ represents the cost of assembling a finished product, and $W_{exchange}$ represents the cost of returning defective finished products. If a value comparison is required, the cost of both parts must be compared with the expected loss cost, that is, the auxiliary variable $W_1{}'$ is introduced, when x_1=x_2=1, $W_1{}' = W_1$.

$$W_1{}' = n_1 \times W_{1e} + n_2 \times W_{2e} \tag{11}$$

Therefore, there are parts testing cost decisions: $If\ W_1{}' < E[C_1], select\ detection$

Finished product inspection cost (W_2):

Inspection cost:

$$W_2 = (n_{Primary} + n_{Secondary}) \times x_3 \times W_d \tag{12}$$

Where W_d is the single inspection cost of each finished product and x_3 is the inspection status of the finished product. When the test is not carried out, if the unqualified product enters the market, it will bring potential loss of user return and exchange cost. Assuming that the defective rate of finished products is $p_{finished}$, the expected loss is:

$$E[C_2] = W_5 \tag{13}$$

Where W_5 is the replacement cost of customer refund, the calculation formula is as follows:

$$W_5 = (1 - x_3) n_{Primary} p_{finished} W_{exchange} \tag{14}$$

If a value comparison is required, the cost of the finished product inspection must be compared with the expectation of the loss cost, that is, the auxiliary variable $W_2{}'$ needs to be introduced, when x_3=1, $W_2{}' = W_2$:

$$W_2{}' = (n_{Primary} + n_{Secondary}) \times W_d \tag{15}$$

Therefore, there are product testing cost decision principles: $If\ W_2{}' < E[C_2], select\ detection$

Where $W_3{}'$ is the assembly loss cost caused by unqualified parts. $W_{assemble}$ is the cost of a single assembly. $n_{Primary}$ is the number of finished products obtained by initial assembly.

Dismantling cost and dismantling benefit (W_4 and $Q_{dismantle}$):

The unqualified finished products detected by the enterprise can be discarded or recycled after disassembly, so it is necessary to consider the disassembly cost and disassembly income to analyze whether the decision is worthwhile.

Dismantling cost:

$$W_4 = W_s n_{Primary} p_{finished} x_4 \tag{16}$$

Where $n_{Primary}$ is the finished product produced by the initial assembly, W_s is the single disassembly cost of each finished product. x_4 is the state variable indicating whether the finished product is disassembled (disassembled when equal to 1, and vice versa when equal to 0).

Dismantling income:

Disassembly income refers to the sales revenue obtained after disassembling the nonconforming finished product, recycling the parts, and reconstituting the new finished product. During the dismantling process, the utilization rate of the parts depends on whether the parts have been inspected. If tested, the utilization rate is 1; The utilization rate of untested parts is 0.9.

$$Q_{dismantle} = n_{Secondary} \times C \tag{17}$$

$Q_{dismantle}$ is disassembly revenue, $n_{Secondary}$ is the number of finished products after reassembling the parts obtained after disassembly, and C is the market price. The expected formula of total profit is shown as follows:

$$E[L_{total}] = \sum_{i=1}^{n} (Q_{former}(i) + Q_{dismantle}(i) - W_{total}(i)) \tag{18}$$

The total profit expectation is calculated using the above formula, enabling decision-making at each stage.

2.2. Multiple processes

Using eight parts and three processes as an example: parts 1, 2, and 3 form semi-finished product 1; parts 4, 5, and 6 form semi-finished product 2; parts 7 and 8 form semi-finished product 3. These semi-finished products combine to create the final product. Since the production is complex, each process is treated as a series of decisions, making it a multi-stage decision problem. The process is divided into three stages, and for easier calculation, state variables T and K are used to represent the condition of parts, semi-finished, and finished products (1 for detection/disassembly, 0 for no detection/disassembly) [4]. Parts detection stage: state variable T_{1j} (j=1,2...8) indicates the detection status of the Jth part. Semi-finished product stage: state variable T_{2j} (j=1,2...3) indicates the detection status, and K_{2j} (j=1,2...3) indicates whether defective products are removed. Finished product stage: state variable T_3 indicates whether to test, and K_3 indicates whether to disassemble a defective product. The goal is to find the optimal decision for maximum profit by quantifying decision values at each stage, including the recycling value of disassembled semi-finished products V_1.

$$V_1 = \sum (W_{bj} - T_{1j} W_{dj}) \tag{19}$$

Where W_{bj} is the purchase cost of part j; W_{bj} is the inspection cost of part j; T_{1j} indicates the detection status of the JTH part in the first stage. j is the set composed of the Ordinal Numbers of disassembled parts.

Recycling value of finished products disassembled into semi-finished products V_2:

$$V_2 = \sum (W_{hbj} - T_{2j} W_{hdj}) \tag{20}$$

Where W_{hbj} is the cost of semi-finished products, which can be expressed as:

$$W_{hbj} = \begin{cases} W_{hb1} = \sum_{i=1}^{3} \frac{V_1}{1-p_i}, j=1 \\ W_{hb2} = \sum_{i=4}^{6} \frac{V_1}{1-p_i}, j=2 \\ W_{hb3} = \sum_{i=7}^{8} \frac{V_1}{1-p_i}, j=3 \end{cases} \tag{21}$$

T_{2j} represents the inspection status of the JTH semi-finished product in the second stage, W_{hdj} represents the inspection cost of the JTH semi-finished product, and p_j represents the defective rate of the JTH part.

Decisions based on value quantification affect the defect rates of semi-finished products through part inspections and finished products through semi-finished testing [5].

Revised defective rate of semi-finished products:

$$p_{hfj} = p_{hj} + P(D_j) \tag{22}$$

Event D_j indicates that there is at least one defect in the parts that make up the semi-finished product j:

$$D_j = \begin{cases} (1-T_j)A_j \cup (1-T_{1(j+3)})B_j \cup (1-T_{1(j+6)})C_j, j=1,2 \\ (1-T_j)A_j \cup (1-T_{1(j+3)})B_j, j=3 \end{cases} \tag{23}$$

The formula for calculating $P(D_j)$ is as follows, refer to the finite set union formula:

When j is equal to 1,2:

$$\begin{gathered} P(D_j) = (1-T_{1j})P(A_j) + (1-T_{1(j+3)})P(B_j) + (1-T_{1(j+6)})P(C_j) \\ -(1-T_{1j})P(A_j)(1-T_{1(j+3)})P(B_j) - (1-T_{1j})P(A_j)(1-T_{1(j+6)})P(C_j) \\ -(1-T_{1(j+3)})P(B_j)(1-T_{1(j+6)})P(C_j) + \\ (1-T_{1j})P(A_j)(1-T_{1(j+3)})P(B_j)(1-T_{1(j+6)})P(C_j) \end{gathered} \tag{24}$$

When j is equal to 3:

$$P(D_j) = (1-T_{1j})P(A_j) + (1-T_{1(j+3)})P(B_j) - (1-T_{1j})P(A_j)(1-T_{1(j+3)})P(B_j) \tag{25}$$

Where p_{hfj} is the defective rate of the JTH semi-finished product after modification, p_{hfj} is the defective rate of the JTH semi-finished product, p_{hj} is the defective rate of the JTH part, and T_{1j} represents the detection status of the JTH part.

A_j event indicates that the j part is defective; Event B_j indicates that the j+3 part is defective; Event C_j indicates that part j+6 is defective.

Revised defective rate of finished products:

$$p_{cfj} = p_{cj} + P(E) \tag{26}$$

Where event E indicates that there is at least one defective product among the semi-finished products that make up the finished product:

$$E = (1-T_{21})E_1 \cup (1-T_{22})E_2 \cup (1-T_{23})E_3 \tag{27}$$

Event E_j indicates that the JTH semi-finished product is defective, and the calculation of reference $P(D_j)$ about $P(E)$ is formula (26). Where p_{cj} is the defective rate of finished products, and p_{cfj} is the corrected defective rate of finished products.

3. Establishment of dynamic programming model

3.1. Single operation

The process studied in this paper involves multi-stage decision [6], and the decision has a continuous impact on the subsequent stage, so dynamic programming method can be used to optimize the decision. The recursive formula is

$$E(s)=\max\left[L(s,a)+\sum E(s_0)\right] \tag{28}$$

Where $E(s)$ is the maximum expected profit when starting from state s and is the value function of state s. $L(s,a)$ is the immediate profit from acting a in state s.

3.2. Multiple processes

After quantifying the value and the related defective rate and establishing the production decision model, the dynamic programming model [7] is established to optimize the decision process.

Stage three:

$$F(3,T_3,K_3)=\begin{cases} -T_3W_{cdj}-p_{cdj}T_3K_3(W_4-V_2)+C(1-p_{cdj}),T_3=1 \\ -p_{cdj}W_5+C(1-p_{cfj}),T_3=0 \end{cases} \tag{29}$$

Where W_{cdj} is the testing cost of the finished product, W_4 is the dismantling cost of the finished product, p_{cdj} is the corrected defective rate of the finished product, V_2 is the recycling value of the finished product disassembled into semi-finished products, W_5 is the replacement loss of the finished product refund, and C is the market price of the finished product.

Stage two:

$$\begin{aligned} F(2,T_{21},K_{21},T_{22},K_{22},T_{23},K_{23})= & \sum_{j=1}^{3}(T_{2j}W_{hdj}+p_{hdj}T_{2j}K_{2j}(W_{hbj}-V_1)) \\ & +\sum_{j=1}^{3}(1-p_{hfj})F(3,T_3,K_3) \end{aligned} \tag{30}$$

Where W_{hdj} is the testing cost of semi-finished products, W_{hdj} is the dismantling cost of semi-finished products, p_{hdj} is the corrected defective rate of semi-finished products, and V_1 is the recycling value of disassembled semi-finished products.

Stage 1:

$$F(1,T_{11},T_{12},T_{13},...,T_{18})=-\sum_{j=1}^{8}(T_{1j}W_{dj}+W_{bj})+F(2,T_{21},K_{21},T_{22},K_{22},T_{23},K_{23}) \tag{31}$$

W_{dj} is the inspection cost of part j, and W_{bj} is the procurement cost of part j. $F(2,T_{21},K_{21},T_{22},K_{22},T_{23},K_{23})$ is the optimal value of the second stage.

4. Solve the model

4.1. Single operation

Each part has two states: detected or undetected, resulting in 2×2 combinations. Finished products can be detected or not, and unqualified products can be disassembled or not, yielding 2×2 combinations. Thus, there are 16 possible strategies, with six cases solved by traversal, as shown in the figure 1[8].

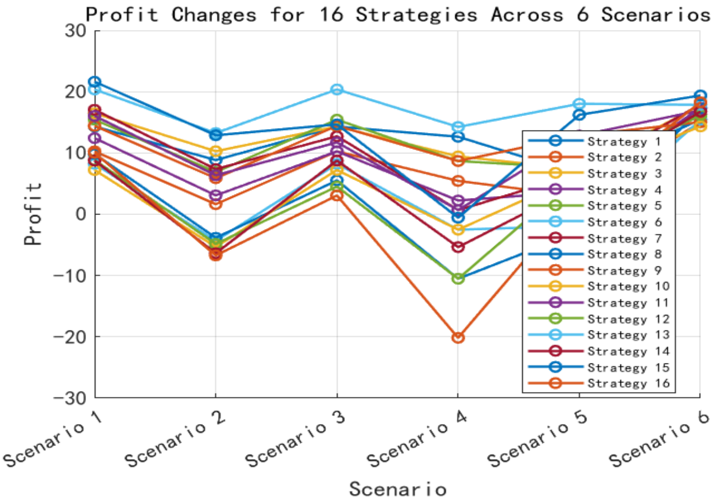

Figure 1 Profit of sixteen schemes corresponding to six scenarios

Strategies 13 and 15 are stable with high profits, making them ideal in most cases, while strategy 16 is unstable with low profit and should be avoided. In case 4, most strategies performed poorly, especially strategy 16, suggesting high costs or uncontrollable factors. In case 6, most strategies yielded high profits, indicating minimal impact of strategy choice on final profit. The remaining profit results are shown in the table 1 below.

Table 1 Optimal strategies for the six cases

scenario	optimal decision				Maximum profit per group of parts (unit: Yuan)
	x_1	x_2	x_3	x_4	
1	0	0	0	1	21.62
2	0	0	1	1	13.24
3	0	0	1	1	20.36
4	0	0	1	1	14.24
5	0	0	1	1	18.02
6	0	0	0	1	19.37

(Note:x_1, x_2, x_3 respectively represent the inspection decision of component 1, component 2, and finished product (0 is not detected, 1 is detected); x_4 represents the disassembly decision for the finished product (0 for no disassembly, 1 for disassembly)).

4.2. Multiple processes

The simulated annealing algorithm, based on physical thermodynamics, mimics the annealing process by heating a system to allow free particle movement and then gradually cooling to reach the lowest energy state, the ground state [9]. This heuristic algorithm is effective for optimization problems, offering global search capabilities, avoiding local optima, and handling high-dimensional problems [10]. When solving multi-part, multi-process dynamic programming models, the large state space complicates high-dimensional problems. Simulated annealing is used here as it avoids full state enumeration and solves the model through probabilistic search, making it suitable for high-dimensional challenges. The solution steps are as follows:

(1) Initialization: Set the initial temperature T0=1000, generate the initial solution S0 using Monte Carlo simulation, set the maximum number of iterations ML=1000 for the maximum temperature value, and set the temperature attenuation to α=0.95.

(2) For k=1,2... L does steps (3) through (6) and produces a new solution S '

(3) Take the recursive equation of the dynamic programming model in multiple processes as the evaluation function and calculate the increment: $df = f(S') - f(S_0)$. If $df > 0$, S' accepted as the new current solution, otherwise S' is accepted as the new solution with probability $e^{df/T}$.

(4) When the termination condition is met, the current solution is output as the optimal solution and the program is terminated.

(5) T gradually decreases, and T approaches 0, and then turns to step (2).

Results for [0,0,0,0,0,0,0,0,1,1,1,0,0,0,0,0], that is, the optimal strategy for parts inspection, three pieces of semi-finished products detection do not disassemble, finished product testing and dismantling. Currently, the maximum expected profit of a single finished product is 92 yuan.

5. Conclusion

This paper proposes a production decision optimization method based on a dynamic programming model and an intelligent optimization algorithm to address production process optimization in the machinery parts manufacturing industry. Dynamic programming effectively handles multi-stage decision-making, while the simulated annealing algorithm enhances performance by overcoming the limitations of traditional methods. In single-process problems, the exhaustive method finds optimal strategies, but it becomes inefficient in complex multi-process, multi-part scenarios due to state space expansion. By integrating simulated annealing, the method avoids local optima and improves efficiency through probabilistic search. Experimental results demonstrate significant advantages in profit improvement and cost reduction. Future research could explore genetic algorithms or particle swarm optimization to enhance computational efficiency and adapt to dynamic market demands and multi-objective problems.

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