A Portfolio Study Based on the Markowitz Model - An Example of the Bitcoin Market

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Abstract. Nowadays, financial markets are becoming more and more complex, and new portfolios need to be built to cope with them. This paper aims to build a Markowitz model for portfolio research based on new calibrations for nine different industries. Firstly, the weights and minimum variance combinations are calculated by using valid information such as mean, standard deviation, variance, and covariance. Second, this paper aims to maximize the return of the portfolio, diversify the investment risk of the selected portfolio, and finally determine the optimal portfolio. The portfolio can be adjusted to reduce risk or increase return by adjusting the percentage of Bitcoin. This paper further explores the portfolio using Bitcoin as a variable. This paper derives the volatility and return of the least risky portfolio to be 11.04% and -0.46%, respectively, when the portfolio is calibrated without Bitcoin, and the volatility and return of its Sharpe optimal portfolio are 14.61% and 7.11%, respectively. When the portfolio contains Bitcoin, the volatility and return of its risk-minimal portfolio are 9.45% and 0.6%, respectively, and the volatility and return of its Sharpe-optimal portfolio are 16.31% and 37.35%, respectively. Ultimately, it is concluded that Bitcoin has some risk-reducing and return-enhancing effects.

Keywords: Markowitz model; portfolio; bitcoin.

1. Introduction

With the development of financial globalization and information technology, the correlation of international stock markets is increasing, the global financial market has become more complex, and market volatility has increased significantly [1]. The ability of traditional investment portfolios to diversify risk has gradually declined, making risk management a critical need in the investment arena. For this reason, a sound investment portfolio needs to be used to minimize investment risk and maintain financial health.

The modern study of portfolios centers on how to rationally allocate an investor's money. Investors are most concerned about the balance between risk and return and minimize risk and maximize return by appropriately allocating funds to different assets. Thus determining how rewarding and risky an asset investment is and achieving optimal asset allocation based on balancing the two metrics [2]. Markowitz [3] was the first to analyze the problem quantitatively by proposing a mean-variance portfolio model, the Markowitz model [4]. The model provides investors with a way to optimize their asset allocation based on a mean-variance framework by calculating the expected return and risk (expressed as the variance of the return) of a portfolio, helping them to make more rational and informed investment decisions. On this basis, several different assets can be selected to construct the portfolio and optimize it.

Bitcoin, as a peer-to-peer electronic cash system, has emerged as a significant aspect of modern commerce. Factors such as the attractiveness of digital markets, advancements in technology, and increased venture capital investments have driven the growing demand for cryptocurrencies [5, 6]. Although Bitcoin can generate substantial returns and diversify the downside risk of stocks, its high volatility, speculative nature, and herding nature make the Bitcoin market risky [7]. Despite the risks associated with Bitcoin, introducing Bitcoin into an investment portfolio may lead to greater returns.

In this paper, to explore the impact of Bitcoin in investment portfolios, A-shares, futures, U.S. stocks, and ETFs are introduced here to construct new portfolios with Bitcoin. This calibration was analyzed separately from the calibration when Bitcoin was not added using the Markowitz model [4].

After calculating their returns and volatilities, they are compared to analyze the impact of Bitcoin in the portfolio, while optimizing the new calibration to arrive at a more appropriate asset weight.

2. Data and methods

2.1. Data selection

In this paper, nine calibrations are selected through Investing.com, and the weekly closing data from September 24, 2023, to September 15, 2024, are selected for the study, and the closing price of the previous day is used as the closing price of the current day if a company suspension is encountered resulting in missing data. The nine calibrations are CAC (301215), China Southern Airlines (600029), Bitcoin (BTC), CICC Carbon Futures ETF (3060), ChinaAMC CSI 300 Index ETF (83188), Polypropylene Futures (DCCPc), Metallurgical Coke Futures (DCJc), Apple (APPL), and Tesla (TSLA), all replaced with the corresponding codes in the following are replaced with the corresponding codes for each calibration name. Selected data are shown in Table 1.

The table records the weekly closing prices of the nine selected calibrations over one year, with BTC experiencing greater price volatility compared to the rest of the calibrations, and 83118 experiencing less price volatility.

Table 1 Weekly closing data

Date	301215	600029	BTC	3060	831188	DCCPc	DCJc	Appl	TSLA
2023-09-24	6.02	6.12	26962.70	70.06	38.5	7816.00	2,605.50	171.21	250.22
2023-10-01	6.02	6.12	27961.10	69.02	37.72	7816.00	2,605.50	177.49	260.53
2023-10-08	6.17	5.91	26852.80	72.36	37.52	7518.00	2,605.50	178.85	251.12
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2024-09-01	5.08	5.61	54156.50	55.74	33.56	7429.00	1,640.00	220.82	210.73
2024-09-08	5.03	5.41	59995.40	54.6	32.68	7513.00	1,778.50	222.5	230.29
2024-09-15	5.05	5.39	63348.10	52.66	33.02	7417.00	1,803.00	228.2	238.25

2.2. Methods of analysis

In this paper, the Markowitz model is used to analyze the constructed calibration. The Markowitz model allows the calculation of the expected return and volatility of the portfolio and the ability to adjust the weights of the calibration in the portfolio to find the right portfolio according to the demand.

2.2.1 Markowitz modeling and solution

The process of building and solving the Markowitz model used in this paper is shown below [8]:

Assuming that the portfolio that minimizes risk is constructed, the objective function is:

$$min\sigma_\rho^2 = \sum_{i=1}^{n}\sum_{i=0}^{n}\omega_i\omega_j\sigma_{ij} \tag{1}$$

In equation (1), ω_i and ω_j are the weights (weights) assigned to security i and security j, respectively; $\sigma_{ij} = \sigma_i\sigma_j\sigma_{ij}$, ρ_{ij} is the correlation coefficient of security i and security j. The constraints are:

$$\sum_{i=1}^{n}\omega_i = 1 \tag{2}$$

$$\sum_{i=1}^{n}\omega_i r_i = E(r_\rho) \tag{3}$$

$$\omega_i \geq 0(\text{Allowed short selling}) \tag{4}$$

In the constraint, r_i is the expected return of security i and $E(r_\rho)$ is the expected return of the portfolio.

$$\sigma_\rho^2 = \sum_{i=1}^{n}\omega_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{i=1}^{n}\omega_i\omega_j\sigma_{ij} \tag{5}$$

In equation (5), σ_ρ^2 is the variance of the portfolio, σ_i^2 is the variance of the ith asset, ω_i is the proportion of the investment accounted for by the ith asset, and σ_{ij} is the variance of asset i and asset

2.2.2 Analytical approach to the Markowitz model

In this paper, Monte Carlo simulation of data using Python is used to analyze the Markowitz model [9]. Monte Carlo simulation is a method of modeling statistical systems based on “random numbers”, which has the advantage of expressing mathematically some complex interactions in economics without the need to solve the system analytically or perform exact enumeration.

3. Analysis of results

3.1. The process of empirical analysis of the Markowitz model

To further analyze the Markowitz model for this portfolio, the returns for each calibration over the selected time period were first calculated.

3.1.1 Calculation of the rate of return

In the Markowitz model, it is necessary to determine the optimal allocation of the portfolio through the method of data analysis, and the rate of return is a key indicator of the performance of the portfolio, which reflects the profit and loss of the portfolio in a certain period of time, and the risk and return of the portfolio is expressed by the expected rate of return and variance. In this paper, Equation (6) will be used to calculate the daily return and analyze its fluctuation trend in different time periods, which provides the data basis for the subsequent Markowitz model construction.
The formula for calculating the rate of return is:

$$yield\ rate = \frac{Yield\ for\ the\ day - Yield\ before\ day\ K}{Yield\ before\ day\ K} \tag{6}$$

The closing price data is calculated according to equation (6) to produce the yields for the nine calibrations over the selected time period. The yield data are shown in Fig. 1 and Fig. 2.

In Fig. 1, the maximum return of 83188 reaches 0.1868. 3060 and 600029 are relatively stable, maintaining between -0.05 and 0.05. In the case of BTC, for example, its maximum yield reached 0.1841, but the overall volatility is higher, showing a higher risk.

In Fig. 2, TSLA is volatile but its maximum return reaches 0.2399. DCCPc and Appl are relatively stable. While DCJc is more volatile and its minimum return reaches -0.0677. This indicates that DCJc is more risky in the selected time period.

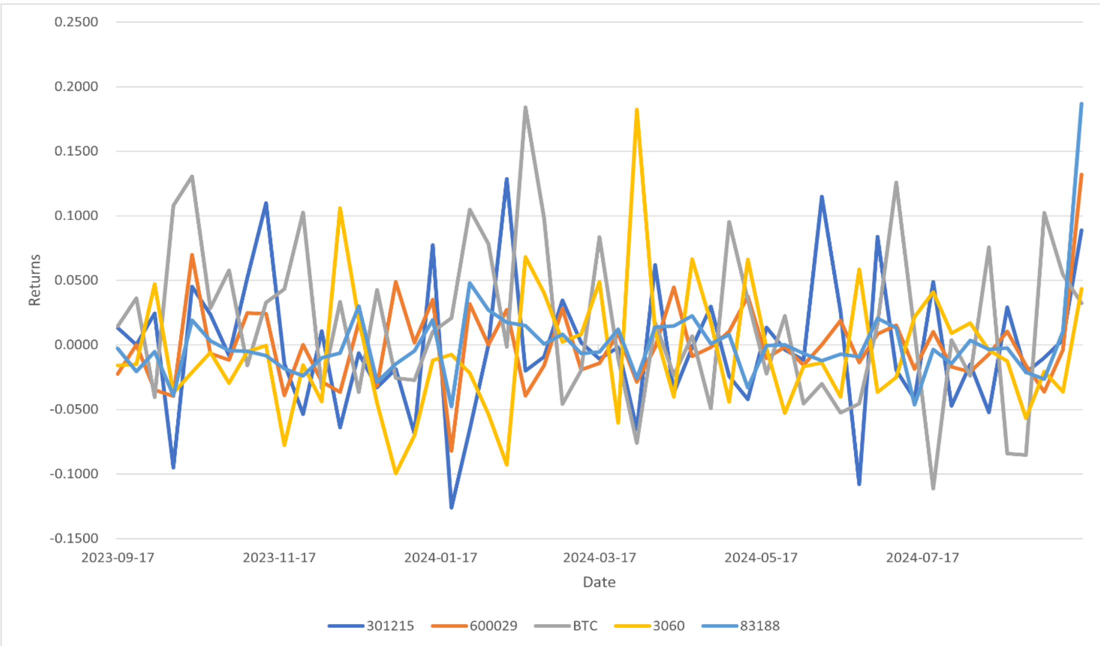

Fig.1 301215, 600029, BTC, 3060, 83188 Yield

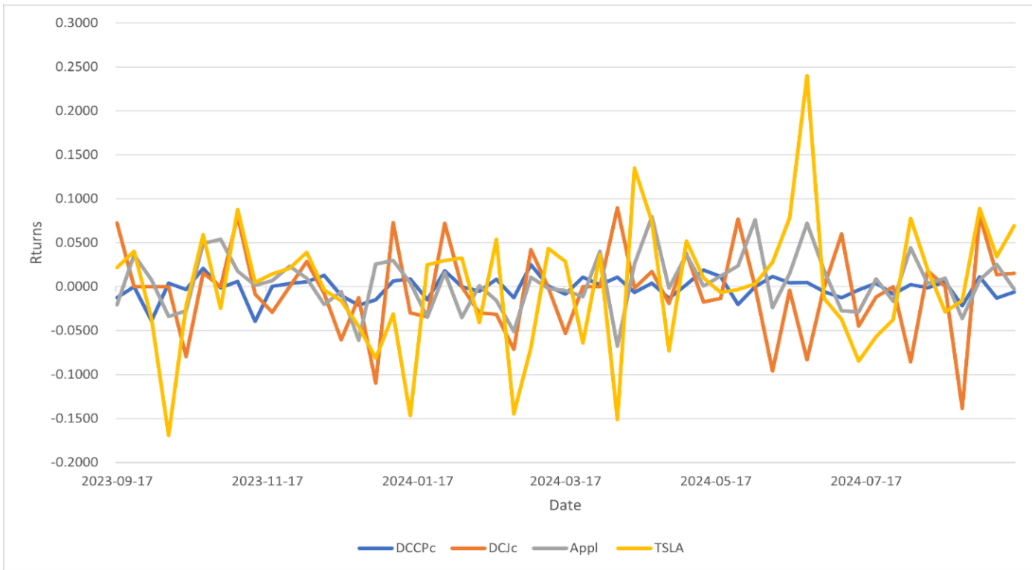

Fig.2 DCCPc, DCJc, Appl, and TSLA yields

3.1.2 correlation analysis

The correlation coefficient is a measure of the degree of correlation between two variables and is a measure of the linear relationship between two variables. The closer the value of the correlation coefficient is to 1, the stronger the correlation between the two variables; the closer the value of the correlation coefficient is to 0, the weaker the correlation between the two variables. When the correlation coefficient of two stocks is greater than 0, the two stocks are positively correlated, and when the correlation coefficient of two stocks is less than 0, the two stocks are negatively correlated. The size of the correlation coefficients between the stocks affects the magnitude of the variance of the portfolio model, and it also affects the degree of risk diversification of the stock portfolio. The smaller the correlation coefficient between two stocks, the smaller the standard deviation and variance of the stock portfolio, so the better the stock portfolio diversification, and the larger the correlation coefficient between two stocks, the larger the standard deviation and variance of the stock portfolio, so the worse the stock portfolio diversification.

After calculating the variance and standard deviation between the 9 calibrations, the correlation coefficients between the 9 calibrations were calculated using Python and the correlation coefficients heat map is shown in Fig. 3. As shown in Fig. 3, the correlation coefficients between BTC and 301215, 3060, DCJc, and TSLA are -0.63, -0.57, -0.68, and -0.57, whose absolute values are close to 0.7 but do not exceed 0.7, indicating that the correlation between these calibrations is relatively high but can still play a certain role in diversifying risk. Therefore the selected calibrations can be used in the subsequent analysis of portfolio optimization.

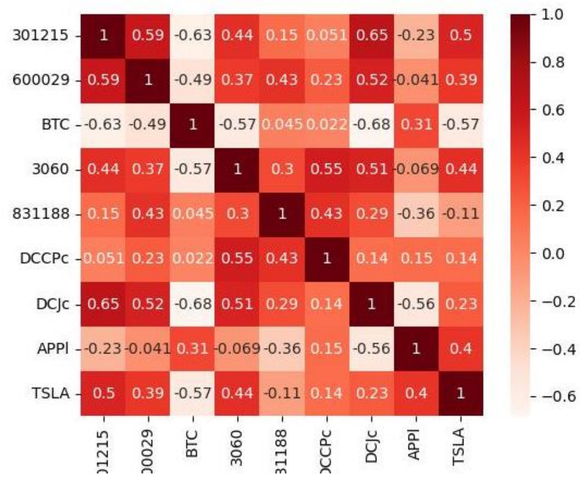

Fig. 3 Heat map of correlation coefficient

3.2. Simulating the Markowitz model using Monte Carlo simulation

This paper uses Monte Carlo simulation[6] in Python to simulate and analyze the selected newly calibrated Markowitz model. The Markowitz model is simulated by randomly generating a set of weights, calculating the return and standard deviation under that portfolio, repeating the process several times, and plotting the results of the calculations as a scatterplot. Fig. 4 illustrates the efficient frontier curve and market line when BTC is included in the portfolio. Fig. 5 illustrates the efficient frontier curve and market line when BTC is not included. Fig. 4 and Fig. 5 show the effective frontier curves, market values (green marked points), and minimum volatility points (red marked points) for the Markowitz model for both cases. The coordinates of the red marker point in Fig. 4 are (0.0945,0.0006) and the coordinates of the green marker point are (0.1886,0.7256), while the coordinates of the red marker point in Fig. 5 are (0.1145, -0.0046) and the coordinates of the green marker point are (0.2315,0.2944). It can be seen that the market value of the portfolio containing BTC is significantly higher than the market value of the portfolio not containing BTC. However, due to the limitations of the model, it cannot completely simulate the actual market situation and therefore has some discrepancies with the actual market.

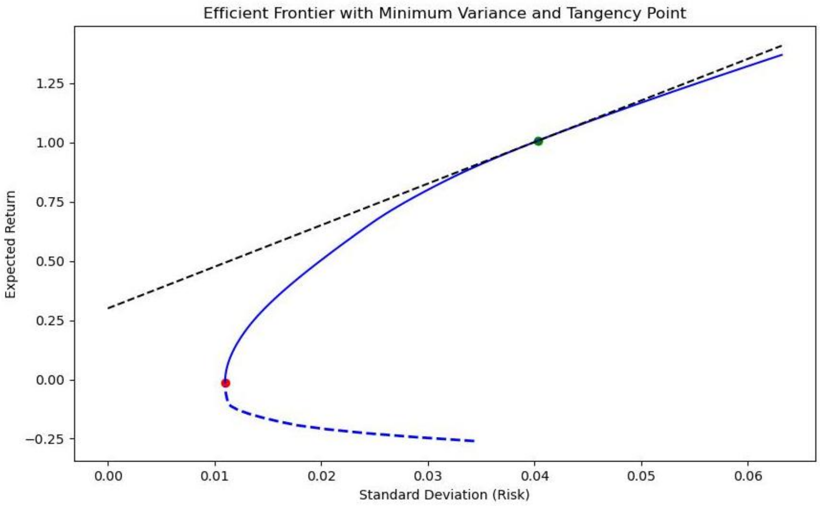

Fig. 4 Effective frontier curve when BTC is included

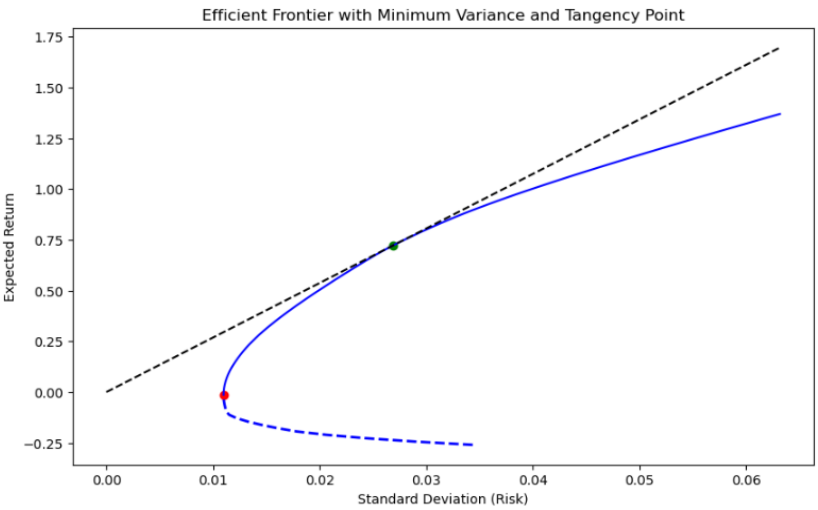

Fig. 5 Effective Frontier Curve without BTC

3.3. Minimal investment risk portfolio

To find the portfolio that minimizes the risk of the investment, the volatility and return for the volatility-minimum case are calculated using Python, which is labeled in a scatterplot, and after that, the calibrated weights under the corresponding volatility and return, i.e., the volatility-minimum portfolio, are listed using python. When BTC is included in the calibration, the results calculated using the Markowitz model are demonstrated by a scatterplot, as shown in Fig. 6. The red-marked point in Fig. 6 is the minimum volatility point (0.0945,0.0006), which shows the return when the volatility of this portfolio is at its minimum value. Further, this paper uses Python to find out the corresponding weights of each calibration under this portfolio (Fig. 7). As shown in Fig. 6, the volatility of this calibration is 9.45% and the return is 0.06% when volatility is minimum.

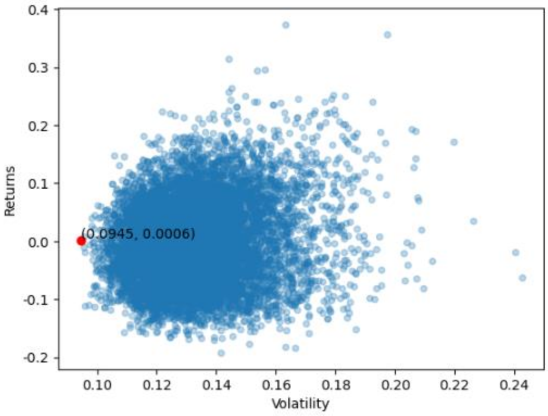

Fig.6 Risk-minimizing Markowitz model (with BTC)

As shown in Fig. 7, in this portfolio, 831188 has the largest weight of 0.2345, followed by DCCPc with 0.1883, while TSLA has the smallest weight of 0.0097.

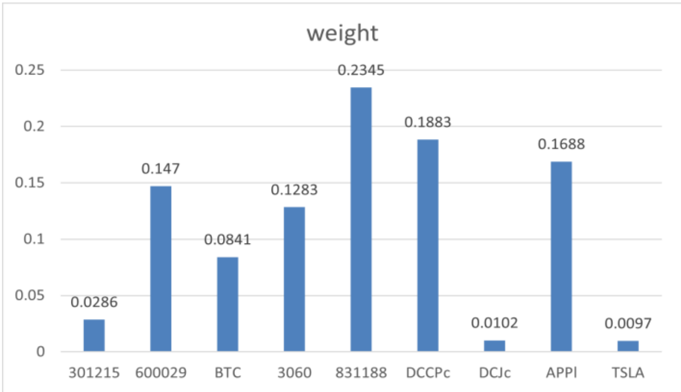

Fig.7 Risk Minimum Portfolio Weights (BTC included)

When the calibration does not contain BTC, its Markowitz model is shown in Fig. 8, and the volatility in this case is 11.04% and the return is -0.46%. The weights of the least risky portfolio at this point are shown in Fig. 9, with DCCPc having the largest weight at 0.4234, followed by 600029 at 0.2414, and 3060 having the smallest weight at 0.0026.

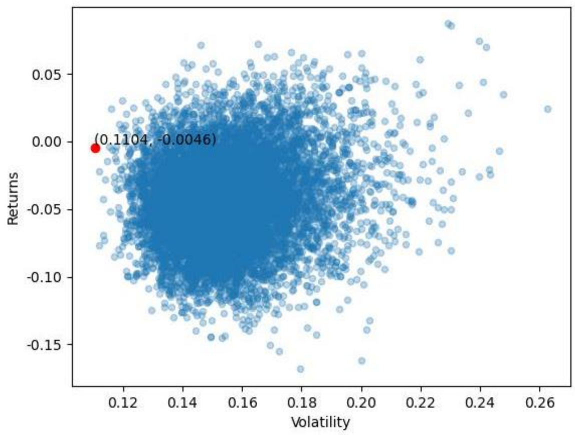

Fig.8 Risk minimization Markowitz model (without BTC)

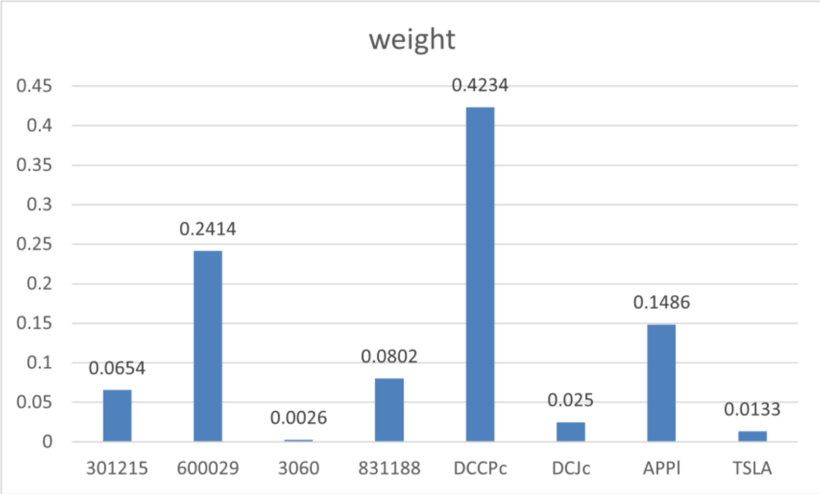

Fig.9 Risk minimum portfolio weights (excluding BTC)

Comparing Fig. 6 and Fig. 8, a portfolio containing BTC leads to less volatility and higher yields. Comparing Fig. 7 and Fig. 9, the portfolios containing BTC have more diversified weights across calibrations. In contrast, the weight of DCCPc in the portfolio without BTC is much higher than the other calibrated weights, reaching 0.4234, a phenomenon that is not conducive to reducing the risk of the portfolio. This shows that BTC plays a role in reducing the risk in this portfolio.

3.4. Sharpe Optimal Combination

To find the Sharpe optimal portfolio, the volatility and return for the maximum Sharpe ratio case are calculated using Python and labeled in a scatter plot. The formula for calculating the Sharpe ratio is given below.

$$Sharpe\ ratio = \frac{\mathcal{R}_\rho - \mathcal{R}_f}{\sigma_\rho} \tag{7}$$

The weights of the calibrations under the corresponding volatilities and yields, i.e., Sharpe optimal portfolios, are listed afterward using Python. When the calibration contains BTC, its Sharpe optimal point as well as its weights are shown in Fig. 10 and Fig. 11. In Fig. 10, the portfolio has a volatility of 16.31% and a return of 37.35%. In Fig. 11, APPL has the largest weight of 0.3445, followed by BTC with 0.3217, while DCCPc has the smallest weight of 0.0055.

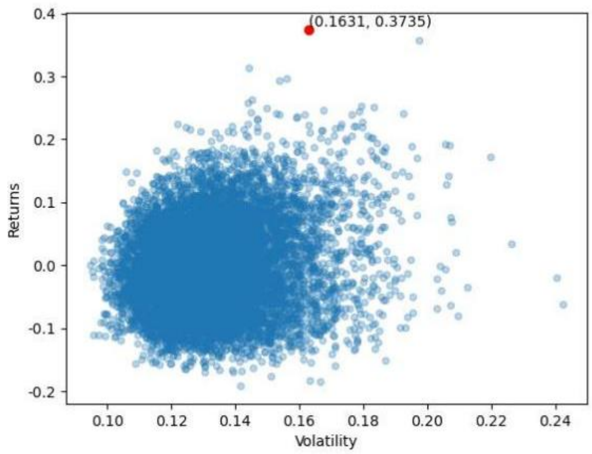

Fig.10 Sharpe optimal Markowitz model (with BTC)

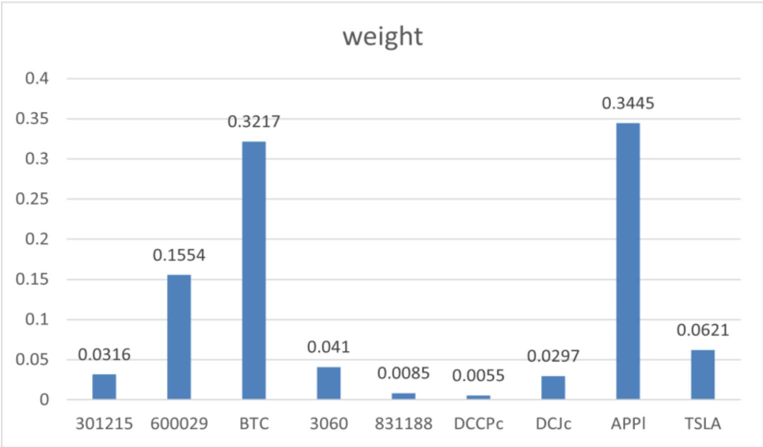

Fig.11 Sharpe's optimal portfolio weights (BTC included)

When the calibration does not contain BTC, its Sharpe optimal point as well as its weights are shown in Fig. 12 and Fig. 13. In 12, the volatility under this portfolio is 14.61% and the return is 7.11%. In Fig. 13, APPL has the largest weight of 0.4033, followed by DCCPc with 0.2199 and DCJc has the smallest weight of 0.03.

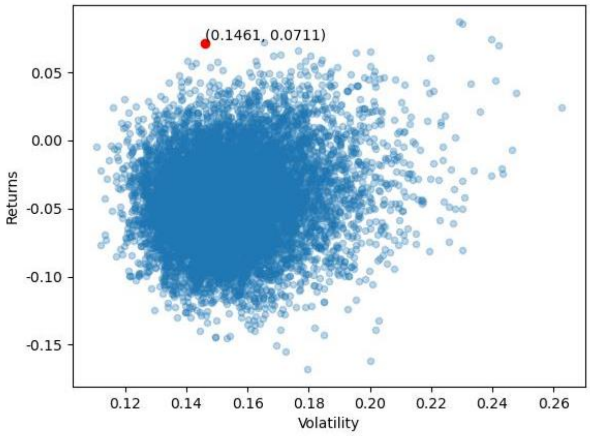

Fig.12 Sharpe optimal Markowitz model (without BTC coins)

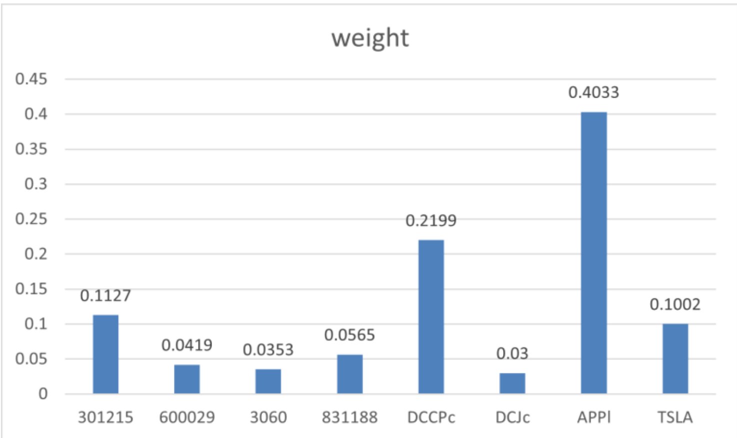

Fig.13 Sharpe optimal portfolio weights (excluding BTC)

Comparing Fig. 10 and Fig. 12, it can be seen that the volatility of the portfolio with BTC is 16.31% and the return is 37.35%, while the volatility of the portfolio without BTC is 14.61% and the return is 7.11%. Although the volatility of the former is slightly higher than that of the latter, the return of the former is much higher than that of the latter. Comparing Fig. 11 and Fig.13 shows that the calibration weight dispersion of the portfolio containing BTC is slightly lower than that of the portfolio without BTC, which indicates that although BTC can bring higher returns, there is still a higher risk.

3.5. Discussion

Through the above analysis, this paper finds the minimum risk portfolio, and the Sharpe optimal portfolio, and determines the weight allocation of each calibration, after which the comparison establishes the existence of a certain impact of BTC on the portfolio. Bitcoin, as represented by BTC, has a high degree of volatility, but investing a certain percentage in a portfolio can help reduce the risk of the portfolio and increase the risk-adjusted return of the portfolio.

Nevertheless, the Markowitz model is based on a series of theoretical assumptions that may not fully correspond to the complexities of real markets. In the Markowitz model, investors' decision-making behavior is rational and based on historical data to calculate expected returns and risks.

However, in actual markets, investor behavior may be influenced by a variety of irrational factors, such as emotions and cognitive biases, as well as market uncertainty. In addition, the Markowitz model uses standard deviation to measure portfolio risk, but standard deviation is not a good indicator of all types of risk. Standard deviation may not accurately capture the risk characteristics of investments that have the potential for a large percentage of losses, such as options and futures.

4. Conclusion

In this paper, based on the Markowitz model, which is a mean-variance model, the portfolio is optimized in some way by selecting nine new calibrations and calculating the return, standard deviation, variance, mean, etc., and comparing it with the portfolio without adding BTC.

After optimizing the portfolio, the volatility and return of its risk-minimum portfolio are 11.04% and -0.46%, respectively, and the volatility and return of its Sharpe-optimal portfolio are 14.61% and 7.11%, respectively, when there is no BTC in the portfolio as a calibration. When the portfolio contains BTC, the volatility and return of its risk-minimum portfolio are 9.45% and 0.6%, respectively; the volatility and return of its Sharpe-optimal portfolio are 16.31% and 37.35%, respectively. When the risk-minimizing portfolio is used, the volatility of the portfolio containing BTC is lower than that of the portfolio without BTC, and the return of the portfolio containing more than the special one is higher; when the Sharpe-optimal portfolio is used, the volatility and return of the former are significantly higher than the latter. Adding Bitcoin to the portfolio can reduce risk and enhance effective returns to a certain extent.

However, the Markowitz model has some shortcomings in practice. First, the model assumes that investors are rational and can accurately predict future returns and risks. However, in reality, investors' decisions are often influenced by various factors, such as market sentiment and information asymmetry.

Secondly, the model does not consider the impact of factors such as transaction costs and liquidity on portfolio performance. In addition, for large-scale portfolios, the computational complexity of Markowitz's model is high and may be difficult to apply in practice.

Therefore, when applying the Markowitz model for stock portfolio analysis, investors need to consider various factors, such as the market environment and personal preferences. At the same time, investors can also combine other investment theories and tools to further improve and optimize their portfolio strategies.

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