A Research on the Application of Option Pricing Strategy Framework Based on BS Model

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Abstract. This paper first reviews the development history of option modeling and its application in corporate strategy, providing a comprehensive background for understanding its evolution. It introduces the European option pricing model studied by scholars in the past, particularly focusing on the Black-Scholes (BS) model, a foundational tool in financial modeling. After establishing the research significance, this paper discusses several basic properties and influencing factors of the BS model while introducing real options as a dynamic framework for strategic decision-making. The core focus of this paper lies in the modification and adaptation of the BS model at the corporate strategy level. Specifically, it develops a combined model that integrates the BS real option method to address the limitations of traditional models, enhancing their applicability to complex business scenarios. Xinhua Wenxuan Company is employed as a case study to validate the feasibility of this refined model, providing a practical evaluation of its results. Lastly, this paper highlights the future research direction of this model, underlining its adaptability and potential for broader applications in corporate strategic management, particularly in industries characterized by high uncertainty and innovation.

Keywords: Option Pricing, Black-Scholes Model, Real Option.

1. Introduction

The original modeling of financial derivative pricing can be traced back to the Black-Scholes model (BS model for short) derived by American financial economists Fischer Black and Myron Samuel Scholes in their 1973 paper "The Pricing of Options and Corporate Liabilities".

Robert Merton, one of the earliest researchers in option pricing theory, who developed the Black-Scholes-Merton model by using a simple equilibrium model and selecting an asset portfolio, believed that a crucial shift in option pricing theory in the future would be a gradual change from the company's simple strategic application of derivative securities to the company's strategic application level. In addition, Merton also predicted that option pricing theory would gradually become a standard tool for companies to implement their strategic goals, whether in relatively wealthy developed countries or in less developed developing countries [1].

Timothy A. Luehrman believes that the investment strategy related to options can be regarded as a series of learning processes and options themselves, pointing out that it "builds a bridge between the practicality of real-world capital projects and the advanced mathematics related to option pricing theory" [2, 3]. In this strategy, option pricing methods are of great value to the formation and implementation of innovative strategies, and can continuously improve the timing and sequence of companies in strategic investment portfolios.

After that, Trigeorgis expanded the formula of project value analysis of NPV = static (passive) NPV + option premium and the conceptual option framework of project investment based on this formula, namely, "whether the project can be postponed, the complexity and competition within and between projects" to study the application of option theory and methods at three levels [4]. Amram and Kulatilaka proposed four major steps: constructing an application framework; completing the option pricing model; checking the results; and redesigning the application framework when necessary [5].

In addition, domestic scholars such as Liu Ying also believe that the once popular investment decision-making methods have great shortcomings in strategic investment decisions in modern complex and diverse markets [6]. Obviously, unlike the daily operational investment of modern companies, the company's strategic investment requires a large amount of funds and has a long payback period, so the uncertainty is greater. Another domestic scholar Yan Jun believes that option pricing in the field of corporate strategic investment decision-making will not only have the function of value assessment, but also has a unique way of thinking and strategic model. Companies should pay attention to the flexibility of option pricing, which can encourage managers to invest in the direction of obtaining investment opportunities with future development [6].

The current research on option pricing theory in China alone can greatly improve the company's strategic investment analysis and technical planning capabilities, but most of the existing option pricing research on corporate strategic management still focuses on conceptual frameworks and ideal abstraction. In the fields of actual corporate governance and practical management tools, domestic research on option pricing theory rarely provides companies with effective methods. This article attempts to use real companies as actual cases to explore the strategic analysis role that option pricing theory based on the BS model can play in corporate operations.

2. Introduction to the BS Model and Real Option Theory

2.1. BS Model

A random process model whose price is determined by the probability distribution of its underlying asset can be used to measure the pricing of the underlying asset. Option pricing theory is more complicated. The BLACK-SCHOLES pricing formula is the foundation theory in the development of option pricing theory, so it has the most extensive reputation and application in the history of option pricing theory.

In the first fundamental theorem of asset pricing, there is a market model with risk-neutral probability measure without arbitrage [1]. In the more complete no-arbitrage pricing theory, there is a risk-neutral measure, which makes the asset price have no arbitrage space under the assumption of the theory.

From a statistical point of view, the purpose of option pricing is to ensure that the long and short positions of the option are in a state of no profit or loss at the same time after a series of transactions, and then the price of the option that best matches the actual market can be derived. If the current value of the stock is equal to the present value of the price during the trading period, this state can be achieved under the present value theory.

In the basic BS model that does not consider dividend payments, the formula for pricing European options needs to be based on five basic assumptions. According to the research on the BLACK-SCHOLES model, there are six factors that affect the price of European options: T (effective period), σ (volatility), K (strike price), t (time), S (stock price), r (risk-free interest rate) (taking stock options that do not pay dividends as an example, the dividend rate q is not discussed) [7].

Table 1 below shows the increase or decrease in the price changes of European call options and European put options using the six factors mentioned above as variables.

Table 1. The Impact of Six Major Factors on the Changes in European Option Prices

Factors	European call options	European put options
S (stock price)	+	-
K (strike price)	-	+
R (risk-free interest rate)	+	-
T (effective period)	?	?
T (time)	?	?
Σ (volatility)	+	+

The BS option pricing model is shown below (taking put options as an example):

$$P\,(S,t) = Ke^{-r(T-t)}N\,(-d_2) - SN\,(-d_1) \tag{1}$$

$$d_1 = \frac{ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\quad (T-t)}{\sigma\sqrt{T-t}} \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = \frac{In\left(\frac{S}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)\quad (T-t)}{\sigma\sqrt{T-t}} \tag{3}$$

The GARCH model (Autoregressive conditional heteroskedasticity model), is an improved model proposed by Bollerslev and Engle in 1986. It is a classic model for dealing with the volatility of time series data [8]. The GARCH model can effectively capture the clustering characteristics of volatility in financial data and is widely used in areas like asset pricing, risk management, and financial market analysis.

The general form of the GARCH (p, q) model is as follows:

$$\begin{aligned} r_t &= \mu_t + \alpha_t \\ \alpha_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= w + \sum_{i=1}^{p} \alpha_i\, \alpha_{t-i}^2 + \sum_{i=1}^{q} \beta_j\, \sigma_{t-j}^2 \quad t = 1,2,\ldots \end{aligned} \tag{4}$$

Among them, $p \geq 0, q \geq 0, w > 0, \alpha i \geq 0 (i = 1,2,\cdots q), \beta j \geq 0 (i = 1,2,\cdots p)$, ε tis an independent and identically distributed sequence, commonly used statistical distributions include normal distribution, student t-distribution, etc.

In China, scholars have extensively applied this model: after Zheng introduced the concept of Value at Risk (VaR) to China in 1997 [9]. Xu and Huang used the GARCH model to fit the volatility equation and subsequently calculated VaR using a one-step-ahead forecasting method [10]. Tang and Huang used the GARCH model to add volatility issues in the market to the market model considerations and improve them [11]. Zhou and Chen argued that the GARCH-t distribution model effectively reflects market risk in China’s SME Board [12]. Wang applied the asymmetric GARCH model under both normal and t-distributions to study stock market volatility, concluding that the t-distribution provides better model estimation performance than the normal distribution [13]. Xiang suggested that employing multivariate GARCH models to characterize the dynamic features of portfolio returns can offer more effective risk information [14].

To capture the asymmetry between returns and volatility, Nelson introduced the Exponential GARCH (EGARCH) model, an extension of the GARCH model. The conditional variance in EGARCH is expressed in natural logarithmic form, allowing the coefficients to be negative, thereby mitigating restrictions on coefficients in the original model and describing the asymmetric impact of shocks on price volatility [15]. Epaphra predicted Tanzanian exchange rate volatility using GARCH (1,1) and EGARCH models, finding that exchange rate behavior is typically influenced by historical information related to exchange rates [16].

2.2. Real Options Theory

As one of the mainstream theories at present, real options theory was proposed by Myers in 1977 when he found that option pricing theory can be used to guide investment decisions when studying the company's "growth opportunities". It also has a lot of application space in the non-financial field. In a narrow sense, real options are the application of financial options theory to non-financial assets; while in a broad sense, real options are a way of thinking that applies financial management ideas to the company.

Real options theory is an expansion of the existing financial options theory, and extends the theory to the investment decision-making method in the non-financial field. When applied to real assets, it becomes a real option. Specifically, it is a right for a company to change its investment decision when making long-term investments because of the uncertainty risks and operational flexibility brought by the investment.

For example, in a certain investment, the operating decision maker only enjoys rights but does not bear obligations. Such an investment often contains the nature of a real option. In general, it is a way of thinking about decision-making. Investors can adjust and modify past behaviors based on the information they will obtain in the future. This way of thinking greatly reduces the risks caused by investment uncertainty, and value is generated accordingly.

According to the real option theory, a small-scale investment in the early stage of a company is equivalent to establishing options, laying the foundation for large-scale investment in the future, but this investment is not equal to commitment. Therefore, the company will wait until the uncertainty disappears completely before making a full commitment. Therefore, this method can achieve potential huge returns while avoiding vicious losses. The evaluation method is to use the real option method for evaluation.

The real option method can consider certain option characteristics inherent in the company itself. The binary tree real option model divides time into discrete time steps, which cannot accurately capture price changes in continuous time and cannot fit the company case study well. In contrast, the BS model is based on a continuous time model, which can better adapt to the changes in asset prices in the actual market. Compared with the iterative calculation of the binary tree model, it is more efficient and less complex. Therefore, when evaluating the potential value of a company, the continuous time valuation can provide more accurate results. Therefore, it is reasonable to use the B-S model to evaluate the potential value of a company.

For example, in 2014, Arasteh and Aliahmadi used the real option method to evaluate uncertain investment activities [17]. This method can help investors decide how to choose investment plans and improve the financial feasibility of projects.

3. Improvements in Real Option Theory and BS Model

3.1. Improvements to BS Model Volatility

Historical volatility method and empirical value method are common methods for determining volatility in option pricing. Historical volatility can be used as a reference for predicting the future, but it is difficult to fully represent the volatility of the future underlying asset returns by quantifying historical volatility. Although the empirical value method has a given range for reference, the adjustment of volatility within the range is based on a subjective judgment, which will inevitably lead to a decrease in the accuracy of the prediction. In previous research on option pricing, the GARCH model was often used to estimate volatility, but it overemphasized the non-negativity of the value, which led to insufficient dynamic estimation of the model and excessive restrictions on the dynamics of underlying asset volatility.

Because the EGARCH model logarithmically processes volatility, it does not need to make non-negative assumptions on parameters like the GARCH model in the treatment of disturbance terms, making the model more dynamic. Therefore, this paper chooses to use the EGARCH model proposed by Nelson as shown below to predict volatility.

$$\ln\sigma^2 = \omega + \beta \ln_{t-1}^{\sigma} + \gamma \frac{\mu_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|\mu_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right], \text{where } g(t-1) = \frac{\mu_{t-1}}{\sqrt{\sigma_{t-1}^2}} [15] \quad (5)$$

Among them, σ^2 is the condition variance, that is, the asset price volatility at the time of T; σ_{t-1}^2 is the price volatility of the asset when the asset is at t-1; μ_{t-1} is the disturbance item of the previous predicted period; $\gamma$is the non-symmetrical term coefficient. It indicates that positive and negative income affects asymmetry on the fluctuation rate; α is the return coefficient, which means the impact of the past fluctuations; β is attenuated factor, which mainly reflects the correlation between the previous phase of the volatility and the volatility of the latter phase; g(t-1) is a white noise that is independent and distributed by zero average, which is continuously distributed. The index can be used to obtain the following formulas:

$$\sigma_\iota^2 = \sigma_{\iota-1}^{2\beta} e^{\varpi-\alpha\sqrt{\frac{\pi}{2}}*} \exp\left[(\alpha+\gamma)^* \frac{\mu_{\iota-1}}{\sigma_{\iota-1}}\right], \mu_{\iota-1} < 0 \quad (6)$$

$$\sigma_\iota^2 = \sigma_{\iota-1}^{2\beta} e^{\varpi-\alpha\sqrt{\frac{\pi}{2}}} \exp\left[(\alpha-\gamma)^* \frac{\mu_{\iota-1}}{\sigma_{\iota-1}}\right], \mu_{\iota-1} \geq 0 \quad (7)$$

Among them, coefficients (α+γ) and (α-γ) indicate that the sensitivity of the positive and negative value of the Egarch model on the positive and negative values of the disturbance items is asymmetric, and the effects of the disturbance items of positive and negative values on the volatility of the target asset price different. In other words, G (T-1) is an improvement of the GARCH model. After the improvement, the Egarch model has different degrees of sensitivity to the positive and negative rate. Therefore, the model has higher dynamics and more fit the real situation of the market.

3.2. Vaguely Processed the B-S Model

Even if the physical option model is widely used in the company's value evaluation, there will still be some parameter settings that have more defects. To weaken the subjectivity of the parameters, by replacing the original only parameters by the fuzzy set, the operating algorithm of the trapezoidal fuzzy number is consumed in the original physical options B-S formula, thereby becoming a new fuzzy physical option model.

The vague set of the price S is S ',S'=(S_1,S_2,α_1,β_1) , that is, the range of S's fluctuations is(S_1,S_2), and S will decrease α1 units under the least ideal situation to S_1, will increase β_1 units to S_2 in the most ideal situation; in the same way, the fuzzy set of the execution price X is x ', X'=(X_1,X_2, α_2,β_2) ,For (X_1,X_2), X will reduceα2units to X_1 under the most unsatisfactory situation, and will increaseβ_2 units toX_2 under the most ideal situation. Therefore, the B-S formula after the improved mathematics improvement is:

$$V_2 = S'N(d_1) - X'^{e^{-rT}(d_2)} = \left[S_1\ N(d_1\ ) - X_2\ e^{-fT}N(d_2), S_2\ N(d_1) - X_1 e^{-fT}N(d_2), \alpha_1\ N(d_1) - \beta_2\ e^{-fT}N(d_2), \beta_1 N(d_1) - \alpha_2\ e^{-fT}N(d_2)\right] \quad (8)$$

$$\text{Where} \begin{cases} d_1 = \dfrac{\ln\frac{E(S')}{E(X')} + T\times\left(r+\frac{\sigma'^2}{2}\right)}{\sigma\sqrt{T}} \\ d_2 = d_1 - \sigma'\sqrt{T} \\ \sigma' = \frac{D(\acute{S})}{E(\acute{S})} \\ E(S') = \frac{S_1+S_2}{2} + \frac{\beta_1-\alpha_1}{6} \\ E(X') = \frac{X_1+X_2}{2} + \frac{\beta_2-\alpha_2}{6} \\ D(S) = \frac{(S_2-S_1)^2}{4} + \frac{(S_2-S_1)(\alpha_1+\beta_1)}{6} + \frac{(\alpha_1+\beta_1)^2}{24} \end{cases} \quad (9)$$

Among them, e^{-rT} is the current value coefficient of continuous compound interest; $N(d_1)$ and $N(d_2)$ are the cumulative probability of less thand1andd2when the normal distribution is standard. The current price; X is the execution price, that is, the required investment cost; T is the length of the execution period, that is, the length from the evaluation benchmark date to the period to the date of date; Variable value; E (S') and D (S') are the mean and variance of fuzzy subsets, respectively.

4. Evaluation Case Analysis

In the study of scholars Lu Qiuyu and Zhan Haoyong, Xinhua Wenxuan Publishing Media Co., Ltd. (hereinafter referred to as "Xinhua Wenxuan") used its B-S model to improve parameters to analyze its potential value and use EVIEWS13 to fit the Egarch model [18].

Xinhua Wenxuan is the first large -scale joint -stock system publishing and distribution enterprise established in accordance with the standards of listed public companies. Established in 1999, it was listed on the Hong Kong and Shanghai Stock Exchange in 2007 and 2016. It is the first domestic publishing company in China to be listed on the "A+H" shares. Choosing Xinhua Wenxuan as the objective value evaluation object can not only verify and evaluate the effectiveness of the value model of this type of enterprise, but also provide valuable reference information for other related parties and investors.

As shown in Table 2 below, all parameters are significant at a level of 1%. Among them, $\beta = 0.93$ reflects the dependence of the current volatility on the previous period of volatility, which is significant at a level of 1%, indicating that during this predicted period, the current volatility is significantly affected by the previous period of volatility. It also illustrates these fluctuations. The rate is predictable. $\gamma = 0.09$, asymmetric items greater than 0, reflect the leverage effect of the market, showing that the market's response to positive messages during the observation period is more sensitive.

Table 2. EGARCH model fitting results

Variable	Coefficient	Standard error	Z-statistic	Significance level
ω	-0.742013369	0.080719118	-9.192535592	0.0000
α	0.280151063	0.022606177	12.39267764	0.0000
γ	0.090892761	0.015214368	5.974139657	0.0000
β	0.928466972	0.00976931	95.03915483	0.0000
R2	-0.000900	Sample mean		0.000180709
Adjusted R2	-0.000900	Sample standard deviation		0.024756582
Regression standard error	0.024767716	Akaike Information Criterion		-4.778221414
Residual sum of squares	0.744715841	Schwartz criterion		-4.757223474
Logarithmic likelihood estimation	2907.769509	HQ guidelines		-4.770316502
DW inspection	2.078168619			

By substituting the above parameters into the B-S option pricing model, the potential value of Xinhua Wenxuan is calculated to be 3.732 billion yuan [18]. By substituting the parameters into the fuzzy real option model, V2 '= [43.52, 71.90, 6.28, 34.66] can be obtained:

$$E(V2') = \frac{43.52+71.90}{2} + \frac{34.66-6.28}{6} = 62.44 \tag{10}$$

With reference to the DCF model, the current value of the free cash flow of enterprises is 4.656 billion yuan, and the overall value is 10.9 billion yuan.

According to the Flush Data, the receiving price of the A -share closing price of Xinhua Wenxuan evaluation is 9.92 yuan, and the total number of equities is 79190,3900 shares, and the value of Xinhua Wenxuan A shares is 7.856 billion yuan. According to Sina Finance data, the H -shares closing price is 5.27 Yuan, the total number of equities is 44193,7100 shares, and the value of Xinhua Wenxuan H shares is 2.329 billion yuan. Therefore, the general equity value of Xinhua Wenxuan was 10.185 billion yuan. According to Xinhua Wenxuan's 2022 annual report, the company's interest liabilities of the company on the benchmark date were 514 million yuan. The sum of the company's equity value and interest liabilities is the value of the overall corporate, so the overall corporate value of Xinhua Wenxuan is 10.699 billion yuan.

Improved the valuation and evaluation of the real corporate value of the valuation and assessment of the benchmark date is relatively small, and the corporate value assessment under the options of the real property is the closest to the real value of the enterprise. By replacing single parameters through the fuzzy interval, the difference is further reduced. It is in line with the continuous growth trend of the enterprise. Therefore, choosing to improve the DCF-fuzzy BS model is more reasonable and effective for valuations for enterprises.

5. Conclusion

The actual power law can identify the potential value of the company. Through case research and analysis, the feasibility of this method is verified

Sexuality proves that the use of physical rights law can meet the company's value characteristics, and consider uncertain factors and flexibility, thereby rationally evaluating the company's overall value.

The fuzzy physical rights law applies to the evaluation of the company's value. Combined with the B-S model of fuzzy mathematics, to a certain extent, the original model parameters set a single and fixed defect. Combined with the actual development of the company in the market, through fuzzy processing, it obtains a more reasonable value interval, so that the company’s value evaluation is more scientific and accurate.

Opatement is limited to the basic attributes of the expiration date, and only when the current price of the target asset exceeds the exercise price, the exercise of the options has practical significance. If the enterprise is regarded as a target of rights, only when the future cash flow of the enterprise is obviously higher than its investment, the exercise can obtain substantial value after performing exercise operations. In addition, the value created by digitalization and information empowerment cannot accurately reflect the financial statements of the corporate. This is consistent with the characteristics of the physical options. It is necessary to evaluate the current value of the current free cash flow of enterprises as the current price of the target asset. However, no matter whether it is the original BS model or the improvement of the evaluation, it has failed to use a more typical company as a case in this article.

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