

The Correlation Between Different Country Stock Market Indices

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Abstract. The main purpose of this project is to explore the correlation between different stock market indices. Using cointegration test to measure the correlation between different countries' stock market indices to get the results illustrate the strength of the relationship between different countries, thereby helping investors make correct decisions in real investments.

Keywords: cointegration, correlation, stock market index.

1. Introduction

The stock market index is the weathervane of a country's stock market. Correctly predicting the trend of the stock market index can guide investors to make more correct decisions. The correlation between stock market indices in different countries helps investors more reasonably predict the direction of their own country stock market from changes in other countries. The correlation between indices can better help investors understand the world stock market environment and make more precise investment decisions.

The literature this study refers to is "Contagion in international financial markets: A recursive cointegration approach" by Nafeesa Yunus. This article chooses cointegration statistic as the test statistic to prove whether there are cointegration relationship. And use CIV and eigenvalue to measure the strength of the cointegration relationship.

Contagion in international financial markets:
A recursive cointegration approach



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2. Research Process

2.1. Methodology

2.1.1. Unit root test

Unit root tests are conducted on the residuals to determine if the series exhibits a unit root, which means the data are unstable. Common test statistics include the ADF statistic and Phillips-Perron statistic.

The unit root test should be the first step because the cointegration test need to ensure that the data is volatile or unstable in the short term. Only on this basis, the following test can verify the existence of the cointegration relationship.

In the study, PP test is chosen to check the existence of unit root. The Philipps-Perron test is an improvement over the ADF test, as it adjusts for heteroscedasticity in the series through non-parametric methods. Unlike the ADF test, the PP test does not require the selection of lag order. Instead, it employs a kernel estimation approach to directly address any autocorrelation or heteroscedasticity present in the time series.

The final step of the PP test is analyzing the result by following the rules following. If the p-value associated with the test statistic is lower than a predefined significance level (typically 0.05 or 0.01),

the null hypothesis of a unit root is rejected, indicating stationarity. Otherwise, the null hypothesis cannot be rejected, suggesting non-stationarity.

2.1.2. Cointegration test

Cointegration test is a statistical method used to examine the long-term relationship between two or more time series variables.

In cointegration analysis, the primary objective is to determine whether a linear combination of nonstationary variables is stationary or not. ($a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$) If the combination is stationary, it implies that the variables share a long-term relationship and move together over time. This long-term relationship is known as the cointegrating relationship.

2.1.3. AIC test

$$AIC = 2k - 2 \ln(L)$$

Here k is the number of parameters in the model and $\ln$ is the natural logarithm of the likelihood of the model. The term penalizes complexity to avoid overfitting.

The AIC is a tool used for model selection among a set of models. It is particularly useful in the context of statistical modeling and time series analysis to determine the optimal number of lags in autoregressive models or to choose between competing models more generally.

The AIC is designed to balance the complexity of a model with its goodness of fit. A lower AIC value indicates a model that is a better choice, as it suggests a model that achieves a good fit with fewer parameters. It is particularly useful in time series modeling, such as ARIMA, where determining the order of lags (p, d, q) is crucial.

2.2. Data Analysis

2.2.1. Unit root test result

Table 1. The unit root test statistic result sheet

index	Dickey-Fuller A (alpha)	p-value (0.01)	result
S&P 500	-9.225	0.5936	unstable
S&P/TSX Composite Index	-19.086	0.08563	unstable
CAC 40	-12.616	0.4062	unstable
DAX	-23.82	0.03252	unstable
FTSE MIB	-8.2399	0.6502	unstable
Nikkei 225	-9.9083	0.5572	unstable
FTSE 100	-25.932	0.02091	unstable
ShenZhen Index	-18.32	0.09687	unstable

The unit root statistic result sheet shows that under 0.99 significance level, we can reject the null hypothesis all the stock here show instability, which fulfill the condition to do the cointegration test.

2.2.2 Cointegration test result

Eigenvectors, normalised to first column:
(These are the cointegration relations)

	GSPC.Close.l2	GSPTSE.Close.l2	FCHI.Close.l2	GDAXI.Close.l2
GSPC.Close.l2	1.000000e+00	1.0000000000	1.0000000000	1.000000e+00
GSPTSE.Close.l2	4.643017e-02	0.8126008652	-1.7408700319	-1.613771e+00
FCHI.Close.l2	-1.293766e-01	-1.0278577081	-2.1942685106	5.045809e+00
GDAXI.Close.l2	-2.517331e-01	-1.1023846093	2.0377205383	-2.160174e+00
FTSEMIB.MI.Close.l2	1.843567e-01	-0.1569092848	0.1336378848	-1.301124e+00
N225.Close.l2	-4.671714e-01	1.3947154563	0.2357967905	2.524232e-01
FTSE.Close.l2	-2.351613e-01	-0.9313494677	0.1135639505	-1.924354e+00
X399001.SZ.Close.l2	-7.592294e-02	0.0862588577	-0.2026602800	-1.282115e-01
constant	-8.587175e-05	-0.0004216333	-0.0003315906	-1.413393e-05

	FTSEMIB.MI.Close.l2	N225.Close.l2	FTSE.Close.l2
GSPC.Close.l2	1.0000000000	1.0000000000	1.0000000000
GSPTSE.Close.l2	-3.836103415	-1.353360402	-1.2059952881
FCHI.Close.l2	-3.765923386	-2.307139675	-0.5073239613
GDAXI.Close.l2	-7.132513949	-7.477101063	-0.8295562830
FTSEMIB.MI.Close.l2	11.472161538	1.418587984	-0.1675688776
N225.Close.l2	0.765018001	3.368735005	0.0629440609
FTSE.Close.l2	-1.179972542	12.133415646	2.0921921952
X399001.SZ.Close.l2	0.552608323	-6.462101748	0.3910795802
constant	0.001814656	0.001430903	0.0002789314

	X399001.SZ.Close.l2	constant
GSPC.Close.l2	1.0000000000	1.00000000
GSPTSE.Close.l2	0.5795793829	0.8662941
FCHI.Close.l2	0.2245343863	3.8108648
GDAXI.Close.l2	1.8558198210	0.8671926
FTSEMIB.MI.Close.l2	0.5854548247	-0.0982047
N225.Close.l2	1.4028773610	0.3006162
FTSE.Close.l2	-2.6096571870	-7.9869373
X399001.SZ.Close.l2	0.0462155115	2.0391576
constant	-0.0009040899	-29.2054299

The equation $Ax_1 + Bx_2 + Cx_3 + \dots + Hx_8 = 0$ represents a cointegration vector. This equation indicates that although individual indices represented by x_i ($i=1$ to 8) may be non-stationary (their values can increase or decrease arbitrarily over time), their specific linear combination is stationary. For example, a cointegration vector $1.5x_1 - 0.8x_2 + 0.5x_3 - 0.3x_4 + \dots + 0.1x_8 = 0$. If x_1 (e.g., the S&P 500 index) increases and x_2 (e.g., the Nikkei 225 index) decreases, the changes in other indices (e.g., x_3, x_4) must offset the movements in x_1 and x_2 to maintain the stationary combination.

In other words, the cointegration equation suggests that there exists a long-term relationship or equilibrium among the individual indices, despite their short-term fluctuations. This implies that any deviation from this equilibrium will be corrected in the long run, leading to a stable relationship among the indices.

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#####
# Johansen-Procedure #
#####
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Test type: trace statistic , without linear trend and constant in cointegration

Eigenvalues (lambda):

```
[1] 0.4783413 0.4328854 0.3917861 0.3842909 0.3530166 0.3385810 0.3170839 0.2853825
[9] 0.0000000
```

Values of teststatistic and critical values of test:

	test	10pct	5pct	1pct
$r \leq 7$	1240.21	7.52	9.24	12.97
$r \leq 6$	2647.89	17.85	19.96	24.60
$r \leq 5$	4173.63	32.00	34.91	41.07
$r \leq 4$	5780.82	49.65	53.12	60.16
$r \leq 3$	7570.88	71.86	76.07	84.45
$r \leq 2$	9406.15	97.18	102.14	111.01
$r \leq 1$	11499.67	126.58	131.70	143.09
$r = 0$	13901.55	159.48	165.58	177.20

Eigenvalue: higher eigenvalue means higher variance.

Test statistic: It is obvious that the test statistic is much higher than the critical value, which is the condition to reject H_0 : less than 7 indices have cointegrating relationship which shows all these 8 indices have obvious cointegrating relationship.

2.2.3. AIC test result

```
AIC for S&P 500: -34456.471766
AIC for S&P/TSX Composite Index: -34456.471766
AIC for CAC 40: -33831.796205
AIC for DAX: -33900.413065
AIC for FTSE MIB: -33145.634625
AIC for Nikkei 225: -34486.088134
AIC for FTSE 100: -34452.964971
AIC for ShenZhen Index: -32897.570631
>
> # Which index is dominant according to AIC?
> dominant_index <- names(which.min(aic_values))
> cat(sprintf("Dominant impact index as determined by AIC: %s", dominant_index))
Dominant impact index as determined by AIC: Nikkei 225
```

From the AIC test result, which tells Nikkei 225 influence the whole market most obviously, which shows the Japanese market is relatively representative.

3. Summary

Through unit root testing, it has been established that each of the indexes is non-stationary over the short term, which signifies inherent short-term volatility. Subsequent cointegration analysis revealed that these eight indices demonstrate a clear and robust long-term cointegration relationship. Such findings suggest the feasibility of predicting market trends by analyzing the movements of a leading market, particularly during periods characterized by significant events and in markets with pronounced cointegration. Further analysis using the Akaike Information Criterion (AIC) on the logarithmic returns of these eight indices pinpointed the Nikkei 225 as the predominant index, although the inter-index variability is not statistically significant. Each of these markets plays a pivotal role in shaping the long-term equilibrium relationships observed. The insights derived from this study offer substantial value to investors, portfolio managers, pension fund managers, and other institutional stakeholders considering investments in international equity funds, enabling more strategic portfolio allocation decisions. Additionally, these findings are of particular interest to regulators and policymakers tasked with analyzing cross-market dynamics and striving to comprehend the behavior and responsiveness of global equity markets, especially in response to major global events.

References

- [1] Cointegration of international stock market indices. (1994, August 1). IMF eLibrary. <https://www.elibrary.imf.org/view/journals/001/1994/094/article-A001-en.xml>
- [2] Johansen, S. (1988, September). Statistical analysis of cointegration vectors. Just a moment... <https://www.sciencedirect.com/science/article/pii/0165188988900413>
- [3] Phillips, P. C., & Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, 75 (2), 335. <https://doi.org/10.2307/2336182>
- [4] Yunus, N. (2013, October). Contagion in international financial markets: A recursive cointegration approach. <https://www.sciencedirect.com/science/article/abs/pii/S1042444X13000327>