

Investigating the difference of option's real values between Lookback option and European option based on price models

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Abstract. The concept of pricing model is crucial for global financial markets today. Millions of investors tend to seek options to effectively maximize their returns. The lookback option is based on various exotic options in response to this trend. This paper chooses Amazon's stock as our data to analyze and then simulate its future stock price 1000 times. By calculating the minimum and maximum value of stocks during a given period, we know the returns gained by using the lookback option and then compare the payoffs with those of the European option. We then conclude that the lookback option could lower the uncertainty over the timing of market entry. Also, the lookback option allows buyers to minimize their regret and maximize their profits. The analysis for pricing models in this paper could provide investors with a reference standard when selecting options and help investors avoid a certain degree of risk.

Keywords: Component; Formatting; Lookback Option; Amazon; Volatility; Sensitivity.

1. Introduction

As people's interests in option pricing increase with the development of new options markets, various exotic price options have been used in trading. An option is a security that gives its owner the right to trade in a fixed number of shares of a specifies common stock at a fixed price at any time on or before a given date [1]. Path dependent option, a specific type of exotic option in the trading market, generates values that depend not only on the price of the underlying assets but also the path that the assets were taken from the beginning of the option to the end, or only a part of its life. Path dependence has been offered as an alternative analytical perspective for economics, a revolutionary reformulation of the neoclassical paradigm [2]. One of the outstanding models for the path-dependent option is called the "lookback option", also known as a hindsight option.

Lookback options are path-dependent options whose payoffs depend on the maximum or the minimum attained over a certain period by a security's price [3]. A standard lookback call (put) gives the option holder the right to buy (sell) an asset at its lowest (highest) price during the life of the option [4]. This type of option allows the holder to "look back" or review the historical prices of the underlying assets over the whole lifespan of the option after purchasing it. Then, to maximize his or her profit, the holder may have the probability to choose a specific timing to enter the market when there occurs the widest differential between the strike price and the price of the underlying asset. Lookback options can be broadly classified into two types: fixed lookback and floating lookback. When a fixed strike lookback option is operated, the strike price is fixed at purchase. At the time of exercise, the most beneficial price of the underlying asset over the contract's life is used instead of the current market price, which is different from other types of options trades. When operating a floating lookback option instead of using a fixed basis price against the most favorable price in profit calculations, it uses the widest price difference attained over the whole lifespan of the contract. This article mainly tries to hedge risks on the stock prices of an enterprise, Amazon, by simulating data

using the lookback option pricing model. We construct a lookback option model to help investors avoid risks.

Options are widely used in today's financial field. When combined with different underlying assets. Options generate various derivative instruments, which are crucial components of the financial market [5]. The commonly used ones are the Black-Scholes pricing model, binomial option pricing model, and so on. The Black-Scholes pricing model, one of the most effective models in pricing options contracts, assumes that the perfect capital markets exist and the returns on the underlying asset are log-normally distributed. In addition, the Black-Scholes model is European and can only be exercised at expiration. As a special limiting case of the celebrated Black-Scholes model, the binomial option pricing model was first introduced by Cox, Ross, and Rubinstein [1] as a simple discrete-time model for valuing options, and then improved by Geske and Johnson [6] based on Geske's compound option model to value American put options and utilize convergence acceleration techniques. Richard Breen created a new accelerated binomial option pricing method to illustrate its accuracy and indicate its computational efficiency vis-à-vis other methods [7]. Throughout today's financial market and pricing models, these are just the tip of the iceberg. The connections between these various models always produce new derivative models and new mathematical problems, which are always worth studying.

The concept of the lookback model has existed for a long time, and it evolved from other exotic options at the beginning. Goldman et al. [8] examined the properties of European options with exercise prices that are functions of the realized sample path of the stock, which generates the analytic price formulas for European one-asset lookback options. Also, Antoine Conze and Viswanathan [3] derive explicit formulas for various European lookback options by using probabilistic tools and categorized lookback options into several definitions, like "standard lookback call", "call on maximum", and "partial lookback call", etc. The pricing of lookback options poses interesting mathematical challenges [9]. For other new classes of options, the payoffs of double lookbacks depend on the maximum and/or minimum prices of one or two traded assets analyzed by Hun He et al. [10]. Meanwhile, Babsiri and Noel [11] provide an approach for pricing multivariate and partial look-back options by using Monte Carlo simulations. So far, lookback models of various patterns have been studied, but most of these models still have a lot of unsolved problems due to "analytic complexity in analytic expressions"[9]. Hong Yu et al. [12] presented the partial differential equation formulation of both floating strikes. They fixed strike American lookback option models by using appropriate similarity transform with the realized extremum of the asset price can be monitored continuously or discretely. More recently, Min Dai et al. [9] came up with an analytic price formula for two types of European Quanto lookback options and examined how the lookback option interacts with American early exercise features Russian option an American option with a lookback feature. It is clear that the lookback option is closely related to the European option. Therefore, the comparison between the two options will be discussed in this paper by using Amazon stocks. After analyzing the existing literature, we mainly research establishing a lookback option based on stock options to propose recommendations to the decision maker.

The core method of this research is to select the stocks that need to be analyzed, (it can be one stock or multiple stocks to do comparable trials) to simulate its stock price in the future. Then calculate the value of Lookback options and European options to compare their differences to get the characters of the characteristics of Lookback options. (The various formulas used will be described in detail below) The entire process requires the use of Excel and Python software for calculation and enumeration. Calculations are repeated thousands of times to increase the stability and reliability of the data. And we do multiple simulation calculations and draw charts to show the results.

According to our research results, we believe that because lookback options are calculated based on the maximum and minimum stock prices, their overall value is generally higher than European options. This reflects the advantages of lookback options in providing investor benefits. At the same time, it allows investors to trade at the maximum and minimum values, increasing flexibility rather than relying on the final stock price. However, according to our analysis results, lookback fixed call

options are more sensitive than other options, which increases the difficulty for investors to purchase and use lookback options. The floating lookback option is relatively less sensitive and can give investors more stable returns. Therefore, in general, lookback options are more flexible than European options and may bring higher income, but prices and high sensitivity will also negatively impact. Therefore, we are more inclined to recommend investors looking for stability to use lookback options. And the whole process we used can also be used to simulate more other options for comparison.

In section 1, “introduction”, we introduced the relevant information about Lookback options and European options and differences between them. As options related to paths, the value of Lookback options depends on the highest and lowest prices in the path of stock. The value of European options is based on the final price. In section 2, “data”, we will introduce the data source used in the article: Amazon stock price and returns in the past year and introduce our processing of these data. Section 3 is “method”, we will introduce all the formulas and variables used in the article. Here we describe the various methods we use to simulate and estimate and analyze the data. Later in section 4, “result”, we will actually take the collected data into the steps described above. We will divide Lookback options into “fixed call option”, “fixed put option”, “floating call option”, and “floating put option”, and analyze one by one to simulate the Lookback options and European options for amazon stocks. We then make a sensitivity analysis for the results. In section 5 “discussion”, we will summarize the entire process and explain the practical uses. For example, Lookback options allow investors to sell assets at the highest price or buy stocks at the lowest price within a period to get the most profit. Finally, in section 6, “conclusion”, we will summarize all the main points of this article to provide a reference standard for investors to choose options.

2. Data

Amazon is an American multinational company, and its primary business is e-commerce, cloud computing, digital and artificial intelligence. Amazon has the largest online marketplace in the world and has the highest brand valuation in 2020. We are interested in Amazon’s historical financial performance and want to allow investors to choose the option to maximize the return.

The option contract is a tool to reduce or eliminate the risks of holding an underlying stock for investors. Also, the correlation between the two assets should be negative. In this way, when the value of one investment decreases, the value of other investments will rise, which is the working process of hedging. To help the investors control and reduce the risks of holding Amazon’s stock, we simulate the stock prices for 10 days based on parameters from historical data, calculate the different option values, and compare their payoffs to find the optimal choice.

Table.1 Amazon Stock & Option Related Parameters

St.dev.	Spot Price	Dividend	Strike Price	Current time	Time	Risk-free rate
0.346	3027.99	0	3027.99	March 18 th 2021	0.0397	0.08%

Table 1 shows some calculating results and parameters that we prepare to have further calculation. We input Amazon’s stock price from March 2020 to March 2021, to calculate the daily returns. Based on the result of returns, we multiply the standard deviation of daily returns by the square root of 252 to get year-basis volatility, 0.346. Amazon does not pay dividends, so the dividend is equal to 0. We consider March 18th, 2021 as the current time in the calculation when Amazon’s stock price (spot price) is \$3027.99. Since in the initial state, the strike price is equal to the spot price, which is \$3027.99. In addition, the risk-free rate is 0.08%, which is the daily treasury yield curve rate for one year selected from the U.S. Department of the Treasury. In terms of the time, 0.0397, which is the division result that 10 days out of 252 weekdays in one year.

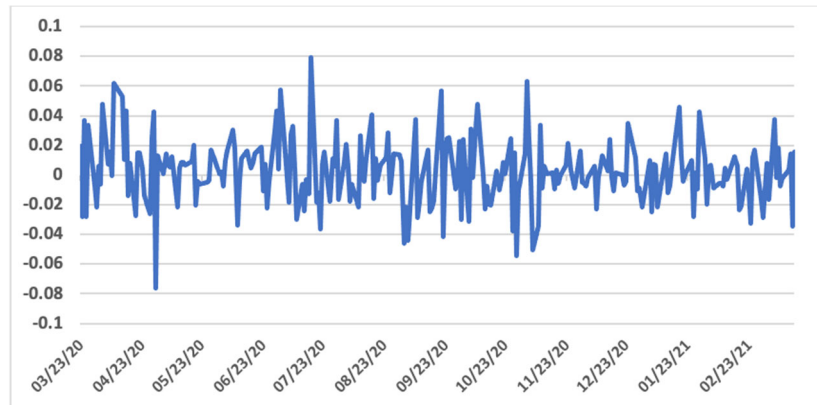


Fig. 1 Change of Amazon Stock Daily Returns (2020-2021)

Figure 1 shows the change of Amazon daily stock returns from March 2020 to March 2021, from which we can see the overall trend of stock price return fluctuates around 0%. During May 2020, July 2020, and November 2020, there were three big fluctuations of stock returns. The highest rise in the stock return was about 8% in July 2020. In May 2020, the change of stock price return dropped to around -8%.

3. Method

Lookback options can be divided into “fixed call option”, “fixed put option”, “floating call option”, and “floating put option”. This part will show how to simulate future stock prices and list the calculation methods of different types of payoff. All variables are shown in this section.

3.1 Lookback Option

The advantage of Lookback options is to provide payoff based on the maximum and minimum values in the stock price path, allowing investors to choose to use options or not to use options. Our article will provide a simulation of future stock prices and show how Lookback options work and compare them with European options.

As introduced in the data section, we collected the price of Amazon stock in the past 1 year, got the strike price, calculated its standard deviation, and simulated the risk-free rate. Once we have these data, we can start to simulate future stock prices. During the calculation, we simulated the stock changes of amazon in the next 10 days and repeated the simulation 1000 times to increase the reliability of the data. We are using the Black-Scholes Model to simulate option pricing [13]:

$$S_t = S_0 * e^{(r-\delta-0.5\sigma^2)T+z\sigma\sqrt{T}} \tag{1}$$

Where “St” is the final strike price, “S0” is the initial strike price, “e” is the natural logarithm, “r” is the risk-free rate, “δ” equals to 0 here, “σ” is standard deviation, “T” is next 10 days and “z” is a random number of normal distributions. “S0” is the current stock price, and “St” is the price 1 day later. By using this formula, we can simulate amazon’s stock price day by day to analyze. The entire simulation process is a total of 1000 times.

Fixed Lookback option formula is shown below (K is strike price):

$$P_{fixed_C} = Max(S_{max} - K, 0) \tag{2}$$

$$P_{fixed_P} = Max(K - S_{min}, 0) \tag{3}$$

Where “Pfixed_C” is the call option payoff of the fixed Lookback option, “Pfixed_P” is the put option payoff of the fixed Lookback option, and K is the strike price. This formula means that the call option payoff is the maximum between the maximum value during the path minus strike price and zero. The put option payoff is the maximum between the strike price minus the minimum value during the path and zero. Following this process, we can calculate the payoff for both call and put options.

After finding the payoff, we defined the option value, which is the average of the payoffs of all the 1000 simulations. “Vfixed_t” means the final option value, and “Vfixed_0” means the initial option value.

$$V_{fixed_t} = P_{fixed_C} / 1000 \quad (4)$$

Looking for the option value at time 0, we used the formula:

$$V_{fixed_0} = V_t * e^{-rt} \quad (5)$$

3.2 Floating and European option

Similarly, we can repeat this process for Floating Lookback call and put option and European option. The formulas for floating Lookback options shown below (“Pfloating_C” is the payoff of floating Lookback call option and “Pfloating_P” is the payoff of floating Lookback put option):

$$P_{floating_C} = S_t - S_{min} \quad (6)$$

$$P_{floating_P} = S_{max} - S_t \quad (7)$$

A European option is different from the Lookback option in that it is not a path-dependent option, so its payoff formula is based on the final value of the stock price in the time period and strikes price. European option requires investors to use the option on the day the option expires, so it does not have the function of providing investors with the best time to enter or exit the market. The formula for the European option shown below means the difference between the Final price minus the Strike price and zero (“Pe_C” is the payoff for the European call option and “Pe_P” is the payoff for the European put option).

$$P_{e_C} = Max(S_t - K, 0) \quad (8)$$

$$P_{e_P} = Max(K - S_t, 0) \quad (9)$$

4. Result

4.1 Results

In this section, we predict a series of Amazon’s strike prices for 10 days, from which we choose 5 results to analyze and find the maximum and minimum strike prices. Based on the formulates in the method section, we calculate and compare the option payoffs.

Table 2. Simulation 1-5: Predictive Striking Price for 10 days

Simulation	Day1	Day2	Day3	Day4	Day5	Day6	Day7	Day8	Day9	Day10
1	3046.42	3166.36	3217.82	3237.71	3105.22	3117.64	3091.63	3100.85	3124.27	3116.76
2	3019.45	3036.26	3018.63	2978.92	2923.23	2723.63	2723.63	2726.94	2672.32	2665.45
3	2848.45	2868.49	2896.30	2997.40	2925.74	2984.30	3033.64	3047.58	3122.34	2933.47
4	3001.79	2996.11	2954.67	2832.56	2845.62	2832.86	2807.07	2938.91	2860.93	2819.00
5	3026.06	3081.92	2984.42	2947.85	2980.93	3027.94	2871.46	2877.29	2952.62	2991.13

Table 3. Max & Min Predictive Stock Price in Simulation 1-5

Simulation	Min Price	Max Price
1	3046.42	3237.71
2	2665.45	3036.26
3	2848.45	3122.34
4	2807.07	3001.79
5	2871.46	3081.92

We take 5 results from the simulation to show the calculating process and results, which repeats for 1000 times. Table 2 shows the results of predictive striking price for 10 days using the formula (1). Table 3 shows the maximum and minimum strike prices for 10 days.

Table 4 shows the payoff results of six different options. We use the formula (2) (3) (6) (7) (8) (9) to gain the option payoff correspondingly. For 1st simulation, the payoff of the fixed call option is highest than that of the floating call option and European call option. In terms of the 5th simulation, the payoff of the fixed put option is the highest. Therefore, for different strike prices that we predict in table 2, the returns of options fluctuate. Investors could refer to the results from table 4 to choose the optimal option to maximize the returns.

Table 4. Results of Option Payoff

Options	P_{fixed_c}	P_{fixed_p}	$P_{floating_c}$	$P_{floating_p}$	P_{e_c}	P_{e_p}
1	209.72	0	70.33	120.96	88.76	0
2	8.27	362.54	0	370.81	0	362.54
3	94.35	179.54	85.02	188.88	0	94.52
4	0	220.92	11.94	182.78	0	208.99
5	53.93	156.53	119.68	90.79	0	36.86

4.2 Sensitivity Analysis

Table 5. Option Values

Option	P_{fixed_c}	P_{fixed_p}	$P_{floating_c}$	$P_{floating_p}$	P_{e_c}	P_{e_p}
Day10	129.1539	134.9002	122.9706	123.8458	80.6839	85.1508
Day 1	129.1498	124.8960	122.9667	123.8412	80.6813	85.1481

Table 5 shows the option values on day 10 and day1. We use formula (4) to calculate the value of the last day and formula to gain the value of the first day. Also, we can conclude that all the option values of day 1 is less than that of day 10.

We do sensitivity analysis for option return values and volatility separately.

Returns: Call Option & Put Option

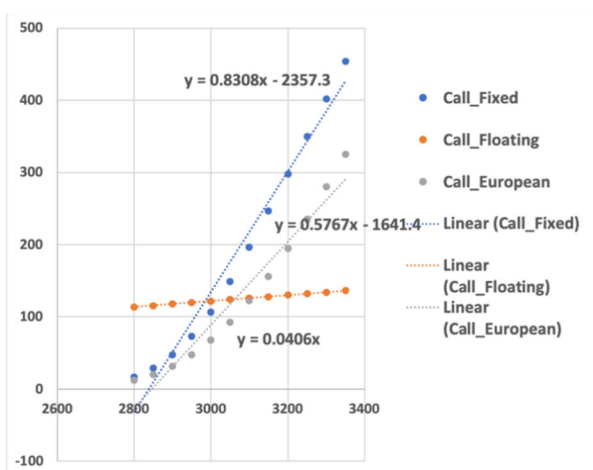


Fig. 2 Relationship of Call Option Values and Strike Prices

We use linear equations to show the relationship between stock price and option values. Strictly speaking, they are not linear relationships, but we can use linear to analyze the slope and changes in general as the interval is small enough.

In Fig. 2, as Amazon's stock price is less than around \$3,000, the call option with floating strike price has the highest value among the three options. However, when the stock price is above \$3050, the lookback floating call option becomes the highest. Regarding the change, the slopes of a lookback call option (fixed strike price), lookback call option (floating strike price), a European call option is

0.8308, 0.0406, 0.5767. For the lookback call option (fixed strike price), when Amazon’s stock price increases by \$50, the call option price (fixed strike) goes up by around 0.8308, which is 0.64%. When the lookback call option (floating strike) increases by \$50, the call option price goes up by 0.0406, which is 0.033%. When the European call option increases by \$50, the option price goes up by 0.5767, which is 0.71%. Based on those percentage changes, we can conclude that the lookback fixed call option is more sensitive than the other call options as the stock price changes the same amount.

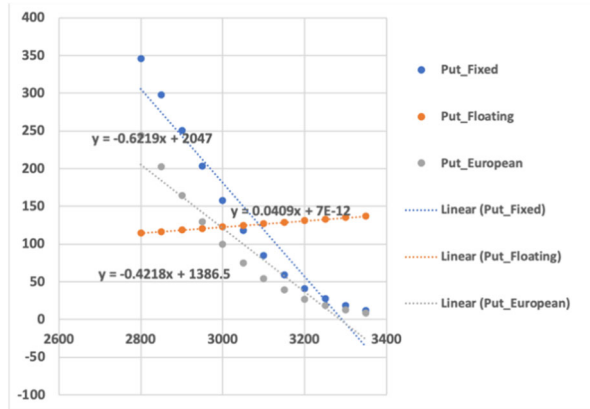


Fig. 3 Relationship of Put Option Values and Strike Prices

Figure 3 shows the relationship between put option values and strike prices. The line of a floating lookback put option has almost the same slope as a floating lookback call option, while the slopes of fixed lookback put options and European put option are negative. When the stock prices are less than \$3000, the lookback fixed put options have the highest value. When the stock price is higher than around \$3100, the values of a fixed lookback put option and European put option decrease sharply. Therefore, we get the conclusion that the lookback fixed put option is the most sensitive to the fluctuation of strike prices.

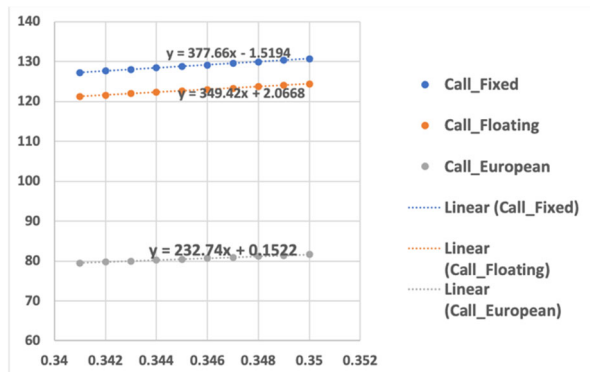


Fig. 4 Volatility, Sensitivity Analysis of Call Option

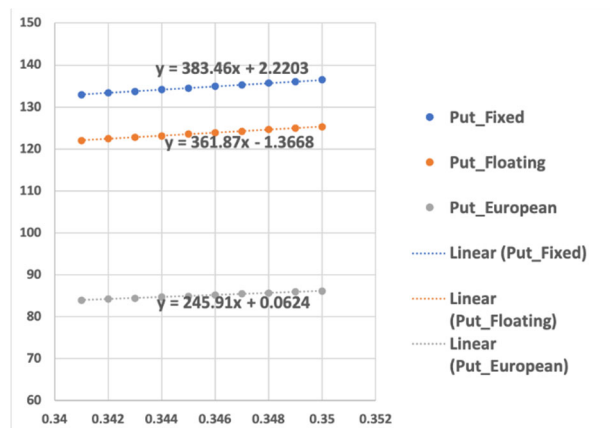


Fig.5 Volatility, Sensitivity Analysis of Put Option

We simulate the option price by adding a 0.001-unit standard deviation. In Figure 4 and Figure 5, the range of standard deviation is from 0.341 to 0.350. In figure 4, for the lookback call option (fixed strike), when the standard deviation increases by 0.001, the call option value increases by 3.88%. For the call option with the floating strike, the option value increases by 3.58% with 0.001 addition of standard deviation. The option value will increase by 2.39% for the European call option as the standard deviation goes up by 0.001. Therefore, the call option (fixed strike) is the most sensitive in terms of volatility, while the European call option is the least sensitive. Furthermore, the overall price of lookback options is higher than the European option. We can have a similar conclusion regarding the volatility sensitivity analysis for the put option from figure 6.

In summary, for both call and put options, the option with a fixed strike price is the most sensitive to changes caused by stock price fluctuation. In real life, Investors choose to hold options to reduce the risks of holding Amazon's stock. Investors who prefer low volatility should consider holding the floating lookback option, which is less sensitive to changes but has relatively low returns. Investors who prefer higher returns should consider lookback fixed options, which have more potential growing space for option values along with higher volatility.

5. Discussion

Based on the aforementioned methods and data analysis, the lookback option is undoubtedly attractive to investors, and its advantage is clear and obvious. The first to mention is that an option with a lookback feature would guarantee the investor's fantasy of buying at the low and selling at the high [4]. Everyone who trades in the financial markets is trying to seize the opportunity to get the most out of it. With the permission to review the history of prices in the market, the uncertainty will be minimized, and returns will be maximized. Holders could choose the moment that the market is at the lowest monthly average index level to enter and thus purchase the asset at its lowest price during the contract's life by taking advantage of knowing the whole historical information.

In addition, it helps to lower the uncertainty over the timing of the market entry. It is human nature that people psychologically tend to favor things that give them complete control over something as uncertain as the price of the stock or the unpredictable volatility in the financial market, not to mention the grueling decision of timing an entry or exit. People could simply choose an appropriate period to hold the stock based on the historical price, which, to a large extent, allows a buyer to minimize his or her regret of selling too early and regret holding on for too long to be suddenly swept by a correction. The lookback option is like insurance against regret as lookback call options would allow investors to buy at the lowest price during the life of the options. In contrast, lookback put options would allow investors to sell at the highest price. It lowers the chances of an option going to waste, or in other words, the option is unlikely to expire worthlessly.

However, there are drawbacks to this model. The cost of the lookback option is too high to be ignored. A seller of such an option would price it at or near the biggest expected differential based on historical volatility. Also, the seller will consider the demand of the options when pricing them. All these costs to purchase the option would be taken upfront.

Moreover, lookback options cannot be publicly traded. Since an agreement of the standardized pricing of lookback options is not reached yet, this model is only available over the counter and not on any major exchanges worldwide. Even though the disadvantages exist, this model's theoretical and practical significance indicates that its advantages outweigh its disadvantages, and scholars and researchers are gradually improving those shortcomings.

As mentioned before, the lookback option has a close relationship with the European option, and the comparison between these two pricing models is worth discussing. A European option limits rights to exercise an option contract to only the expiration day. Lookback option generates payoff as the difference between the strike price and the maximum or the minimum price during the life of the contract depending on whether it is a call option or put option. The payoff of a European option can be simply calculated as the difference between the price in its expiration day and the strike price.

Through the calculation of the two options, the difference can be found easily. If the price on the expiration day is the highest or lowest price over the whole lifespan of the option contract, then a European option makes no difference with the lookback option. The lookback option increases the chance that people could gain returns if the price on expiration day is not the extreme value. The European option is like gambling, where the payoff is entirely dependent on the final price on the day of expiration.

6. Conclusion

As the options market is growing, investors choose to hold options to hedge risks of holding other assets. The lookback option allows the investors to observe the historical values of the underlying assets over the lifespan of an option, and the investors have the right to determine when to exercise the option.

Through the analysis process shown above, we simulated the future stock price path and defined the value of stocks. By comparing the Lookback option with the European option, we found that the overall value of the Lookback option is higher than the European option. This is because the lookback option, as a path-dependent option, uses the maximum and minimum value of stocks during the period, while the European option uses the final price. This proves that the Lookback option often brings larger payoffs and an advantage for investors to choose the most suitable time to enter or exit the market. On the other hand, the value also shows that the Lookback option is more sensitive to the changes in stock prices, standard deviations, etc. The lookback option can either cause higher losses or bring higher returns. If the stock price fluctuates too much when it goes down, investors may give up the use of options.

Combined with the analysis results of the simulated data, in general, the high value of the lookback option is reflected in its use of the maximum and minimum stock prices within a time period, which determines the trading timing for investors and helps investors buy at low prices and sell at high prices. However, high costs and inconvenient transactions have reduced people's desire to use lookback options. So, lookback options are more suitable for those who want to seek low-risk and stability in the stock market and are willing to spend cost and time to earn income, rather than those who are expecting to get rich overnight like gambling.

There are still drawbacks in this paper. Firstly, for the data analysis, the only stock price data that we consider here is the stock of Amazon with only ten days. More days of data and more stocks of companies in various industries should be collected to increase the randomization and lower the contingency. Secondly, more data analyzed will add greater credibility to this paper. Furthermore, the lookback option and European option model analysis in our research is only in the theoretical stage. It has not been well put into practice in the real capital market. This is a limitation to verify our conclusions about lookback models. For future research, more comparisons between lookback and other exotic options should be made to identify the comparative advantage and absolute advantage of the lookback option pricing model. The standardized pricing of the lookback option model needs to be settled, therefore generalizing this model to be applied to a wide range of transactions, not just available over the counter (OTC).

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