Application of Maximum Sharpe Ratio and Minimum Variance Portfolio Optimization for Industries

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Abstract. Since the start of 2022, expectations regarding the continuous raising of interest rates by the Fed to curb the 40-year-high inflation has been negatively impacting the US stock market, leading to volatility spikes. Thus, portfolio optimization plays an important role to risk-averse investors. This paper investigated an optimal portfolio strategy that aims to maximize the Sharpe ratio to outperform a common benchmark, the S&P500. Eleven US stocks were selected, one from each of the eleven market sectors, based on market capitalization. Then, mean-variance portfolio optimization was conducted to obtain an optimal distribution of stocks weighing in Maximum Sharpe Ratio Portfolio (MSRP) and Global Minimum Variance Portfolio (GMVP), and backtesting was adopted to construct portfolio performance for MSRP, GMVP and Equal-Weight Portfolio (1/N) strategies by using historical data from the first eight months of 2022. The Sharpe ratio was calculated to compare the performance between the three strategies---MSRP, GMVP and 1/N---and the S&P500. Empirical results, supported by a robustness test, suggested that MSRP strategy outperformed S&P500 with the highest Sharpe ratio, which indicated that MSRP achieved a higher return per unit of risk regardless of market volatility presented in different time intervals. Therefore, this research provides insights into portfolio investment decision-making for risk-averse investors.

Keywords: Mean-variance Analysis, Portfolio Optimization; Sharpe Ratio; US Stock Market

1. Introduction

High volatility in the stock market indicates a growing opportunity to profit in a short period of time. To mitigate the high inflation rate in the US, which is over 8%, compared to 5.25% in 2021, and higher than the long-term average of 3.26% [1], the Fed has raised interest rates by .75 points [2], causing volatility to balloon. Hence, at this particular time, portfolio optimization is more critical, especially for risk-averse investors.

Much research has been done to optimize portfolio return for the purpose of investment. Mean Variance Optimization (MVO) is one of the most wildly used methods to optimize portfolios [3-4]. MVO for various assets creates risk/reward data sets, based on which investment decisions then can be made. Recent publications indicate an interest in examining genetic algorithm implementation on MVO [5], which deduces the significance of MVO in portfolio optimization. Meanwhile, there are many portfolio optimizations proposed based on the Sharpe ratio [6]. Sharpe ratio can measure the performance of the portfolio taking risk into account. Research studies have been conducted to improve the Sharpe ratio and apply it using actual data [7-8]. Nevertheless, there are some works that combine MVO and Sharpe ratio to optimize portfolio return [9]. Since these two methods can be applied separately, the combination of the two may bring better results.

In this paper, the aim is to determine an optimal portfolio strategy that maximizes the Sharpe ratio to outperform the benchmark S&P500. A diversified portfolio consisting of 11 US stocks, each from a different industry sector, was selected based on market capitalization. Mean Variance Optimization (MVO) was applied to obtain an optimal combined weight distribution for maximum Sharpe ratio portfolio (MSRP) and global minimum variance portfolio (GMVP). Backtesting was utilized to determine portfolio performance for MSRP, GMVP and 1/N strategies by using historical data in the first eight months of 2022. After that, the three portfolios were compared among each other based on

Sharpe ratio. The one that performed the best was then compared with the benchmark S&P500 based on portfolio return, Sharpe ratio and maximum drawdown (MDD).

This paper is organized as follows. Section 2 shows the chosen stocks as well as the stock prices utilized in the research. Section 3 introduces the methodology applied and financial indicators used to measure optimized portfolios. Section 4 presents the result and discussion of the findings. The conclusion is reached in Section5.

2. Data

The data utilized in this paper were obtained from Yahoo Finance [10]. The stock prices were selected from the corporations with the greatest market capitalization in each of the 11 industries, which is shown in Table 1. The time period chosen is from 1/2/2019 to 12/30/2021, and the data frequency is daily return. Besides, some basis information is shown in the following Table 2.

Table 1. Selected stocks

Ticker	Company name	Market Cap	Secter
BHP	BHP Group Limited	184.903B	Basic Materials
GOOGL	Alphabet.com, Inc.	1.412T	Communication Services
AMZN	Amazon.com, Inc.	1.299T	Consumer Cyclical
WMT	Walmart Inc.	364.573B	Consumer Defensive
XOM	Exxon Mobil Corporation	398.385B	Energy
BRK-B	Berkshire Hathaway Inc.	612.387B	Financial Services
UNH	UnitedHealth Group Incorporated	482.985B	Healthcare
UPS	United Parcel Service, Inc.	170.302B	Industrials
AMT	American Tower Corporation	116.793B	Real Estate
AAPL	Apple Inc.	2.504T	Technology
NEE	NextEra Energy, Inc.	167.222B	Utilities

Table 2. Descriptive statistics for selected stocks

Stock	Max	Min	Mean	Standard deviation	Cumulative Return	Skewness	Kurtosis
AAPL	0.1198	-0.1286	0.0016	0.0179	3.6549	-0.0770	10.2381
UNH	1.1222	-0.1516	0.0007	0.0156	0.9464	0.1935	18.8742
BRK-B	0.0793	-0.0792	0.0009	0.0154	1.2456	0.1653	5.0333
AMT	0.1494	-0.1648	0.0005	0.0188	0.4395	0.4303	15.4094
XOM	0.1161	-0.0960	0.0004	0.0124	0.4692	-0.0064	21.5824
BHP	0.0962	-0.1163	0.0011	0.0155	1.7982	-0.0012	10.1126
WMT	0.1369	-0.1342	0.0009	0.0149	1.2755	0.1250	20.7832
UPS	0.1280	-0.1728	0.0008	0.0174	1.1224	-0.0735	19.8589
NEE	0.1438	-0.0882	0.0009	0.0163	1.3961	1.1102	14.1366
AMZN	0.1171	-0.0907	0.0005	0.0116	0.6196	1.3579	23.3071
GOOGL	0.1269	-0.1222	0.0002	0.0191	0.0708	0.1936	9.4321

According to the data description form, Apple has the greatest mean and cumulative return, while Google has the smallest mean and cumulative return. Meanwhile, Google also has the greatest standard deviation, but its skewness and kurtosis are not so prominent. However, Amazon has the smallest standard deviation but the greatest skewness and kurtosis. All the selected stocks are rather stable and profitable. Therefore, in these 11 stocks, there is no outlier that should be removed.

The covariance for the selected stocks is as shown in the Table 3. The relatively low covariance among most of the stocks illustrates that the data have diversified risk.

Table 3. Covariance matrix for the stock

	AAPL	AMT	AMZN	BHP	BRK-B	GOOGL	NEE	UNH	UPS	WMT	XOM
AAPL	1.0000	0.4694	0.6549	0.5157	0.5657	0.6701	0.4462	0.4947	0.4697	0.4238	0.3616
AMT	0.4694	1.0000	0.3653	0.3763	0.4738	0.4515	0.6646	0.5095	0.3807	0.4279	0.2877
AMZN	0.6549	0.3653	1.0000	0.3731	0.3389	0.6432	0.3152	0.3478	0.3583	0.3584	0.2024
BHP	0.5157	0.3763	0.3731	1.0000	0.6793	0.5241	0.3925	0.4615	0.3893	0.3331	0.6599
BRK-B	0.5657	0.4738	0.3389	0.6793	1.0000	0.5575	0.5287	0.6200	0.5128	0.4358	0.6857
GOOGL	0.6701	0.4515	0.6432	0.5241	0.5575	1.0000	0.4494	0.5235	0.4160	0.3815	0.4378
NEE	0.4462	0.6646	0.3152	0.3925	0.5287	0.4494	1.0000	0.4909	0.3535	0.4514	0.3347
UNH	0.4947	0.5095	0.3478	0.4615	0.6200	0.5235	0.4909	1.0000	0.4145	0.3607	0.4434
UPS	0.4697	0.3807	0.3583	0.3893	0.5128	0.4160	0.3535	0.4145	1.0000	0.3767	0.3638
WMT	0.4238	0.4279	0.3584	0.3331	0.4358	0.3815	0.4514	0.3607	0.3767	1.0000	0.2198
XOM	0.3616	0.2877	0.2024	0.6599	0.6857	0.4378	0.3347	0.4434	0.3638	0.2198	1.0000

3. Methodology

After sorting all the selected data and adding restraints, Mean Variance Optimization (MVO) [3], the process of measuring an asset's risk against its expected return and investing based on that risk/return ratio, was applied to find the optimal combined weight distribution of maximum Sharpe ratio portfolio (MSRP) and global minimum variance portfolio (GMVP). MVO is a method to generate different sets of risk and reward allocations for a portfolio based on risk preferences, which acts as a reference for investors to make investment decisions. As indicated in the formula of MVO, an optimal portfolio can be obtained by minimizing volatility or maximizing Sharpe ratio. As it only considers the portfolios that are on the efficient frontier, MVO simplifies the selection problems for the portfolios. Moreover, this mathematical framework also quantifies the concept that risks are reduced by diversification, because lower risk can be obtained with portfolios moving opposite to the right end of the efficient frontier [11]. The formula for Mean Variance Optimization is shown as follows:

Portfolio Expected Return:

$$\mathrm{E(R_p)} = \sum_{\mathrm{i}} \mathrm{w_i E(R_i)} \tag{1}$$

where R_p denotes the return on the portfolio; R_i denotes the return on asset i; w_i denotes the weighting of component asset i

Portfolio Variance:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{2}$$

where σ_i is the (sample) standard deviation of the periodic returns on an asset i, and ρ_{ij} is the correlation coefficient between the returns on assets i and j.

Moreover, to increase reliability of practical applications, exponential moving average (EMA)[12], and covariance was employed as expected return and variance for MVO. EMA is the exponentially weighted mean of (daily) historical returns [13]. This method is a simple improvement over the mean historical return as it gives more credence to recent returns and thus aims to increase the relevance of the estimates. One of the major benefits of EMA is that it is more retroactive to the latest change of the stock price, because data in recent times account for a higher weight than data in older times. The formula for EMA is shown as below:

$$\mathrm{EMA} = \mathrm{Closing\ price} \times \mathrm{multiplier} + \mathrm{EMA(previous\ day)} \times (1 - \mathrm{multiplier}) \tag{3}$$

Other than that, based on the closing price of 2022, backtesting [14], a method to test investing strategy, was applied to the three portfolios and S&P 500. Backtesting simulates a portfolio strategy, with historical prices, to test the how well the performance of the strategy would have been in the past [15]. It allows trading strategies to be tested without risking capital. Then, based on the result of backtesting, the performance of MSRP, GMVP and portfolio of equal weight (1/N) are compared. The portfolio with the best performance will be compared with the S&P 500, in terms of performance indicators such as return, Sharpe ratio and maximum drawdown (MDD) [16-17].

Sharpe ratio measures the performance of the portfolio by considering risk. The value of Sharpe ratio is in direct proportion with the performance of the investment regarding risk-adjusted returns. It gives investors a good knowledge of the risk level they are assuming and whether they will earn a good return for the risk or not [6]. The formula for Sharp ratio is shown as below:

$$S_a = \frac{E(R_a - R_b)}{\sigma_a} \tag{4}$$

where S_a denotes Sharpe ratio, E denotes expected return, $R_a$denotes asset return, R_b denotes risk free rate, σ_a denotes standard deviation of the asset excess return.

MDD, which is an indicator that demonstrates the amount of loss of the value of an investment from its last value that is at its zenith, also plays an important role in this paper. MDD measures the maximum fall of the investment, as given by the difference of the value of the lowest point and that of the apex before that point. If the value of an investment or an asset goes through several boom-bust cycles, it will calculate over a long period of time. In this paper, MMD is used to measure the performance of the portfolios. The formula for MDD is shown below:

$$\mathrm{MDD} = (\mathrm{LP} - \mathrm{PV}) / \mathrm{PV} \times 100\% \tag{5}$$

where MDD is a percent; LP denotes the lowest value after peak value; PV denotes Peak value.

4. Results and Discussion

To follow a rational and practical portfolio investment framework, diversification is applied by setting another portfolio constraint that the weights for each stock should not exceed 20%.

$$W_i \leq 20\% \tag{6}$$

where W_i denotes the weights for stock i. Mean-variance portfolio optimization was performed to generate optimal weights for each stock. According to Table 4, the distribution of optimal portfolio composition varied between MSRP and GMVP strategies as expected, reflecting a trade-off between expected return and risk in making investment decisions. As shown in Figure 1, MSRP consisted of five stocks --- AAPL, GOOGL, NEE, UNH and UPS --- each with a weight that reached the 20% limit, which suggested stock returns of these five constituents outperformed others, especially in 2021 as the portfolio’s expected return was calculated based on exponentially weighted mean of historical returns. For GMVP strategy, as shown in Figure 2, seven stocks were included with weights ranging from 0.2% to the 20% limit. Specifically, AMZN, BRK-B and WMT together accounted for 60% of the portfolio, which suggested that stock prices of the three were the least volatile especially in 2021, i.e., limited stock returns were presented in the three.

Table 4. Optimal Portfolio Composition under Different Investment Strategies

Portfolio Constituent	Optimal Composition in Maximum Sharpe Ratio Portfolio (MSRP) (%)	Optimal Composition in Global Minimum Variance Portfolio (GMVP) (%)
AAPL	20.0%	0.0%
AMT	0.0%	8.0%
AMZN	0.0%	20.0%
BHP	0.0%	0.0%
BRK-B	0.0%	20.0%
GOOGL	20.0%	0.2%
NEE	20.0%	14.7%
UNH	20.0%	0.0%
UPS	20.0%	11.3%
WMT	0.0%	20.0%
XOM	0.0%	5.9%

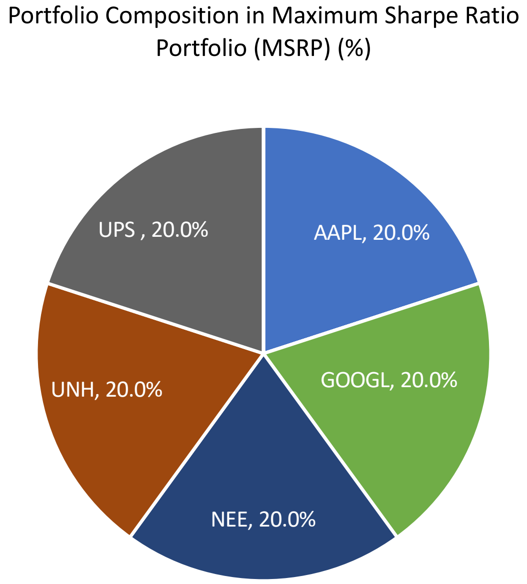

Figure 1. Visualization of Portfolio Composition in MSRP

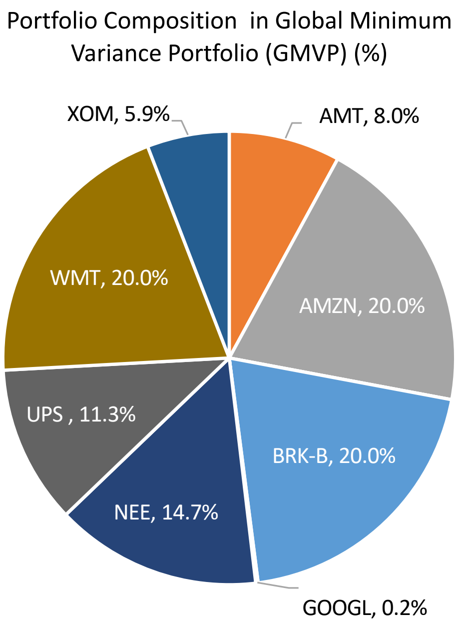

Figure 2. Visualization of Portfolio Composition in GMVP

Backtesting was employed to portfolio compositions of MSRP, GMVP and 1/N on historical data from January 3rd to September 2nd, 2022. Compared to GMVP and 1/N, MSRP obtained the best performance under all indications, which included returns, volatility, Sharpe ratio and drawdown, as shown in Table 5. MSRP was the only one with a positive Sharpe ratio, which is 0.38, indicating that the strategy obtained an excess return of 0.38 per unit of risk (volatility). Apart from that, MSRP was the least volatile with an annual volatility of 21.3%, which outperformed GMVP strategy by 1.8 percentage points. MSRP also presented the lowest level of drawdown of -13.0%, which outperformed the benchmark (S&P500) by 10.6 percentage points. This indicated that investing in MSRP would have a lower downside risk and a smaller loss than investing in the S&P500. In addition, MSRP achieved a 31.1% relative return against benchmark (S&P500), indicating that it outperformed the benchmark by 31.1% in portfolio return, as shown in Figure 3. GMVP and 1/N strategies also outperformed the benchmark with relative returns of 13.5% and 18.4%, respectively.

Table 5. Portfolio Performance under Different Investment Strategies

Investment Strategy / Indicators	Maximum Sharpe Ratio Portfolio (MSRP)	Global Minimum Variance Portfolio (GMVP)	Equal Weight Portfolio (1/N)	S&P500 (Benchmark)
Simple Return (%)	5.9%	-11.7%	-6.8%	-25.2%
Cumulative Return (%)	3.9%	-8.0%	-4.6%	-17.7%
Annual Volatility (%)	21.3%	23.1%	21.8%	23.7%
Relative Return (%)	31.1%	13.5%	18.4%	N/A
Sharpe Ratio	0.38	-0.42	-0.22	-1.10
Maximum Drawdown (%)	-13.0%	-22.2%	-18.7%	-23.6%

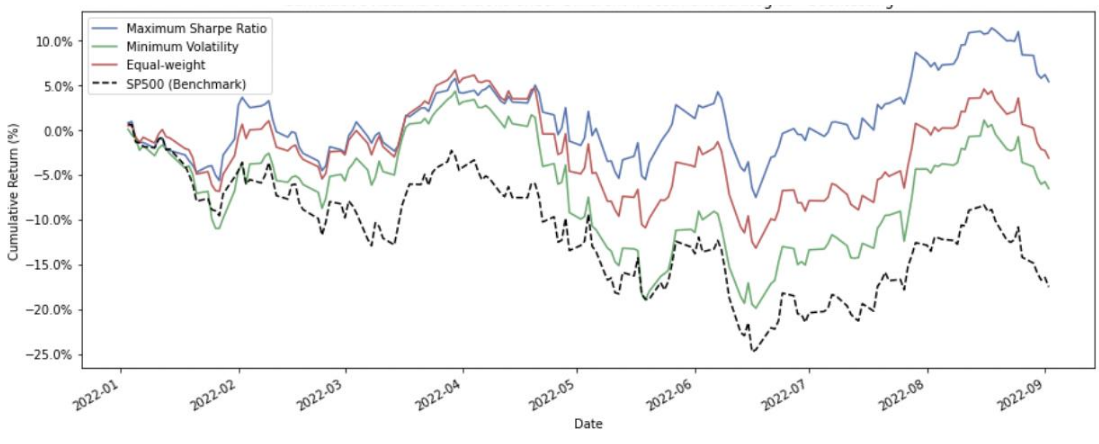

Figure 3. Cumulative Portfolio Return under Different Investment Strategies

Other than that, a robustness test was conducted in different time periods using the same method above to verify results. Since the spread of COVID-19 negatively impacted the US stock market, with the lowest point occurring during March 2020 for all market sectors, market volatility, measured by the volatility index (VIX), has increased significantly after 2019, resulting in investors' panic attack that created havoc in the market. Thus, the robustness test justifies the methodology and tests the results in three time-settings reflecting different levels of market volatility, which are 2019, 2020 and

2021. This corresponds respectively to the time before, during and after the first year of the COVID-19 pandemic. Accordingly, historical data were collected in the three-year period of 2016 to 2018, 2017 to 2019, and 2018 to 2020 respectively.

Results for robustness test, as presented in Figure 4-6, demonstrated consistency with the previous findings and further verified that MSRP outperformed not only the benchmark (S&P500) but also the two other commonly used strategies, GMVP and 1/N.

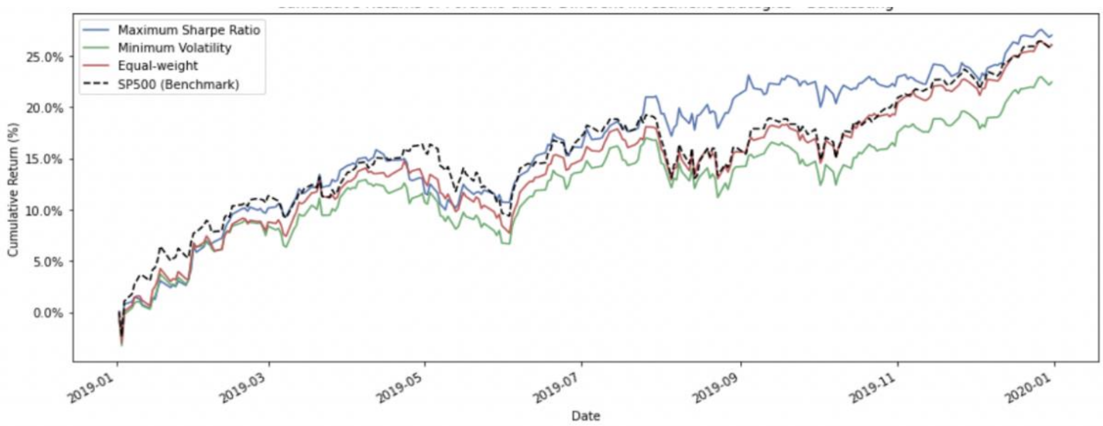

Figure 4. Annual Portfolio Return under Different Investment Strategies in 2019

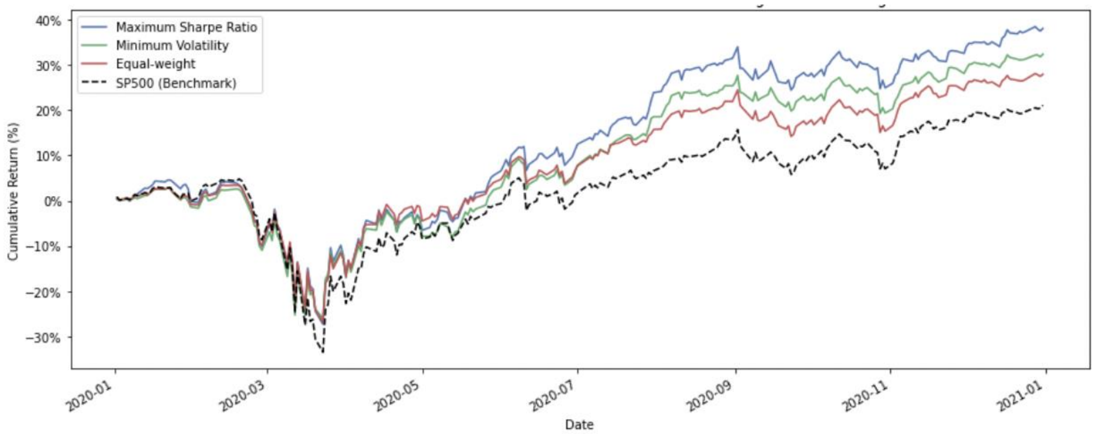

Figure 5. Annual Portfolio Return under Different Investment Strategies in 2020

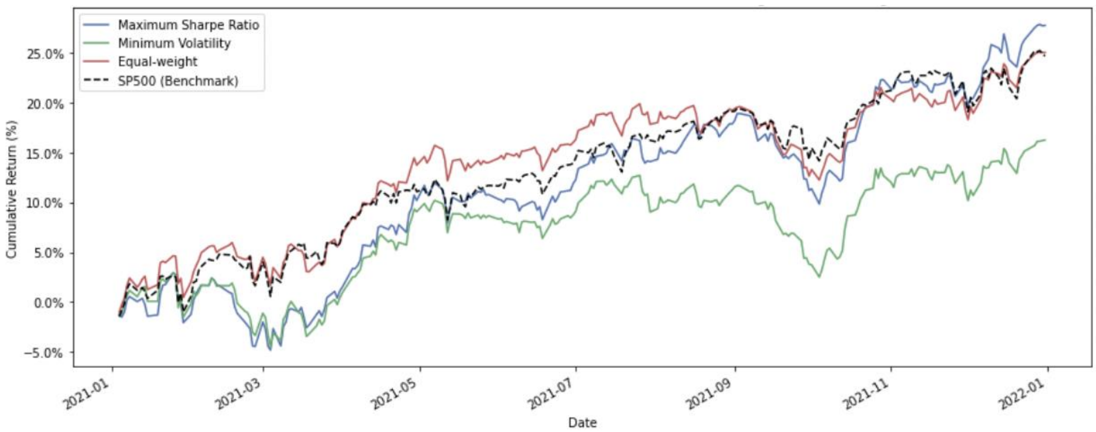

Figure 6. Annual Portfolio Return under Different Investment Strategies in 2021

Last but not least, as shown in Table 6, a summary of the above empirical results demonstrated that MSRP outperformed S&P500 in 2019, 2020, 2021, and the first eight months of 2022, regardless of significant market volatility within those periods, which further verified the consistency and robustness of the empirical findings.

Table 6. Comparison of Portfolio Performance between Maximum Sharpe Ratio Strategy and the Benchmark S&P500

Year / Indicators	2019		2020		2021		2022 (First 8 Months)	
	MSRP	S&P500	MSRP	S&P500	MSRP	S&P500	MSRP	S&P500
Annual Return (%)	30.3%	28.9%	36.9%	16.2%	30.7%	26.9%	3.9%	-17.7%
Annual Volatility (%)	10.7%	12.5%	36.0%	34.4%	13.9%	13.1%	21.3%	23.7%
Relative Return (%)	1.4%	N/A	20.7%	N/A	3.8%	N/A	31.1%	N/A
Sharpe Ratio	2.53	2.10	1.05	0.61	2.00	1.88	0.38	-1.10
Maximum Drawdown (%)	-5.7%	-6.8%	-29.7%	-33.9%	-8.7%	-5.2%	-13.0%	-23.6%

5. Conclusion

In conclusion, taking into account market volatility caused by rising interest rates in the US, this paper investigated an optimal portfolio investment strategy in the US stock market with the goal of outperforming S&P500, assessed by the Sharpe ratio. Mean-variance analysis was applied to identify an optimal portfolio composition in MSRP and GMVP strategies, followed by a backtesting for MSRP, GMVP and 1/N strategies to obtain corresponding portfolio performance. Then, an empirical result that MSRP strategy outperformed S&P500 the most was obtained, after comparison of the Sharpe ratio between the three portfolio investment strategies and the benchmark S&P500. Moreover, a robust test, conducted in three different time intervals using the same method, further demonstrated consistency with the empirical results. This finding suggests that risk-averse investors could adopt a MSRP approach, instead of GMVP or 1/N, to achieve a higher portfolio return for every unit of risk tolerated.

This study was conducted under certain portfolio constraints, such as a 20% weight-limit and the stock selection of one from each sector, which resulted in a reduced reliability in representing portfolio investment strategies and decisions of investors in practice. Thus, the reliability and validity of empirical results could be improved by increasing the data sample size to reduce random errors, adding technical indicators in stock selection such as MACD, and backtesting using various estimation models.

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