

Pricing Strategy of Dual-channel Supply Chain with E-commerce Live Broadcast

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Abstract. As a novel sales strategy, live-streamed selling has drawn considerable interest. In this paper, we study a dual-channel live-streaming supply chain consisted of a single manufacturer, a moderate live streamer and a live-streaming platform. We construct two scenarios, the retail live streaming and commissioned live streaming modes, under manufacturer self-live streaming and investigates the optimal mode with the Stackelberg games. The study discovers: (1) Under the commissioned live streaming mode, the price of Internet celebrity live streaming is less expensive than that under manufacturer self-live streaming within a specific commission ratio and is proportionate to the commission ratio. (2) Under the retail live streaming mode, only when consumers' preferences are more concentrated on the manufacturer self-live streaming, the price of Internet celebrity live streaming is less than the manufacturer self-live stream channel. (3) In both scenarios, Internet celebrity live-streaming sales effort is consistently lower than the manufacturer self-live stream channel. Additionally, the consumer's sensitivity coefficient and the trust degree in Internet-celebrity streamer, the personal impact of Internet-celebrity streamer, and the proportion of impulsive consumers are all positively correlated with the price, sales effort, and profit of both channels.

Keywords: Dual Channel Supply Chain; Internet Celebrity Live Streaming; Manufacturer Self-live Streaming.

1. Introduction

With the continuous development of the e-commerce industry, the number of online shopping gradually increased, and a single online shopping can no longer satisfy the needs of consumers, "live broadcast + e-commerce" mode emerged as the times required. Until June 2021, there were 384 million active e-commerce consumers in China.

A dual-channel supply chain for e-commerce Internet casting is created as more and more manufacturers open self-live streaming and invite Internet celebrities to sell commodities in the live streaming. Depending on their following and the exposure provided by the live broadcast platform, the top Internet-celebrity streamers have a tremendous capacity to sell things. On the other hand, the top streamers may possess "bargaining power" over the goods and even take the original market of the manufacturer's live stream due to their influence and dominant advantage. Meanwhile, many manufacturers are discouraged by prohibitive "booth fees". Therefore, more and more manufacturers prefer to invite moderate live streamers to sell products. However, manufacturers inviting Internet-celebrity streamers will present a new challenge. Since Internet celebrity live streaming channel and the original live streaming of their stores produce some level of competition, it is urgently necessary to find a solution to the issue of how to set pricing for various channels of products.

2. Literature Review

The current pricing decision issue in the e-commerce supply chain has received extensive scholarly discussion. Guo, Li and Sun [1] studied the effect of the showroom effect on the price, profit, and service effort of each supply chain member. The paper divided service effort into two stages, ex-ante service, and ex-post service, contrasting no-service strategies. Yang, Zhang and Yan [2] investigated the effects online commentary will have on supply chain online channels. The research concluded

that posting internet reviews is not always profitable unless they are sufficiently positive. Lin, Zhou and Hou [3] explored a new sales mode "online shopping-offline pickup" (BOPS) based on the Stackelberg game theory and explored the impact of opening a BOPS channel on product quality, price, and profitability of manufacturers and retailers from three perspectives. Wei and Chang [4] examined the impact of implementing a price-matching approach when multichannel retailers choose to open online channels. Sarkar & Pal [5] constructed a multichannel supply chain framework with traditional and direct channels and explored their competitive pricing and service decisions. According to Zhou, Zhao & Wang [6], a manufacturer-dominated dual-channel supply chain system based on the Stackelberg model is taken into consideration. It was discovered that the producer should determine the price of the dual channel on elements like demand uncertainty and the sensitivity of demand to price, regardless of the channel.

In recent years, the vigorous development of live broadcasting has attracted the attention of many academics. Pan, Feng & Zhao [7] discussed the effects of live streamers' sales ability, consumers' preferences, and consumer consumption cost on prices and profits. Fan, Wang & Song [8] argued that the live streaming service of streamers affects both the sales of traditional channel manufacturers as well as the return rate when selling products live. He, Chen & Mu [9] looked at three modes for inviting streamers to sell commodities: commission-only, fixed-fee, and a combination of both. The study indicated retailers prefer to work with streamers with high sales capacity in a pure commission mode. Zhang, Tang & Han [10] investigated whether multinational corporations should open live shopping channels on overseas e-commerce platforms. Three popular live commerce sales modes were examined by Yang, Zheng & Hao [11], including the transfer mode (live streaming and transaction on both platforms), the live platform mode (live streaming and transaction based on the live platform), and the e-commerce platform mode (both live streaming and transaction based on the e-commerce platform). To investigate the equilibrium strategies of supply chain participants under various sales agreements, Wang, Zhao & Ji [12] constructed a multi-level supply chain in which upstream suppliers can sell their products through online platforms and live broadcasting sales channels.

In summary, the current academic research on the e-commerce live supply chain focuses on the decision problem of the three-level e-commerce supply chain of live streamers, platforms, and manufacturers, or the optimal decision when the live channel and other channels are adopted at the same time in a comprehensive manner. This paper discusses an online dual-channel that includes a manufacturer live-streaming, an Internet celebrity live streaming, as well as a platform, based on the pricing decision and live streaming-related studies mentioned above. In view of the different types of rewards charged by Internet celebrities for live streaming, the paper constructs two supply chain live streaming structure, retail live streaming mode and commissioned live streaming mode. Furthermore, the essay builds the profit models for each member in the two modes separately and assesses each member's best course of action under the two modes using the Stackelberg game theory.

3. Problem Description and Hypothesis

3.1 Model Description

Under the retail live streaming, the manufacturer determines its own live sales price and sales effort, while Internet-celebrity streamer wholesale goods from the manufacturer, determines his own sales price, and then utilize the live broadcast platform to sell goods through his traffic. And for the two live broadcast channels, the live broadcast platform levies a fixed service cost. And Internet-celebrity streamer is equivalent to acting as a retailer. In TikTok, Kuaishou, it is common where many stores to support streamers to sell one piece.

Under the commissioned live streaming mode, the manufacturer invites the streamer to live stream and pays a range of "booth fees" and commissions to him. Internet celebrity live streaming is comparable to the manufacturer's "brand salesman", which is responsible for the sale of the items but does not have the authority to determine the price. The live-streaming platform not only charges a

service fee but also takes an additional commission from the streamer. It is common on the Taobao platform that manufacturers will publish relevant live-streaming tasks in the "Ali V Task", after which Internet celebrities will take orders and fulfil the tasks.

The live streaming of the manufacturer and Internet-celebrity streamer are both presumed to be on the same platform in this essay. The platform will levy a certain technical service cost c for the live stream and, in addition, will charge a specific commission rate ε for any Internet celebrities who use the platform to take orders and carry out live broadcast tasks. The two modes mentioned above don't consider the manufacturer's production cost, distribution cost, or fixed "booth fees" that celebrities charge for live streaming. Set the product's market demand as a . There will be competition between two different live channels, this paper defines μ as the price competition coefficient and sets range, where $0 \leq \mu \leq 0.4$. Given the nature of live streaming, the live sales effort has the potential to directly affect sales volume on the day of the broadcast, the paper defines f as the sales effort of the live streamer and β as the sensitivity coefficient of consumers to the sales effort of the live streamer, where $0 \leq \beta \leq 1$. Meanwhile, for the peculiarities of Internet celebrity live broadcast, the streamer typically has his impact, which can draw viewers. The essay makes the following assumptions: n represents the personal influence of the streamer; k represents consumers' trust degree in Internet-celebrity streamer, where $0 \leq k \leq 1$; and σ represents the proportion of impulsive customers in Internet celebrity live streaming, where $0 \leq \sigma \leq 0.2$.

Table 1. Symbolic description of relevant parameters

Symbol	Description
p_i	Product sales price
f_i	Sales effort of the live-streaming
a	Market demand for products in live streaming
β	The sensitivity coefficient of consumers to the sales effort
μ	The price competition factor of the product
λ	The commission proportion of Internet celebrity live streaming
k	Consumers' trust degree in Internet-celebrity streamer
N	The viewer in Internet celebrity live streaming
w	Wholesale price
c	Technical service cost for unit products of the platform
ε	The commission rate levied by the platform for Internet celebrity live streaming
n	The personal impact of Internet-celebrity streamer
θ	Consumers' preference
σ	The proportion of impulsive customers in Internet celebrity live streaming

Remark: subscript $i = 1,2$ represent Internet celebrity live streaming; subscript $i = s_1, s_2$ denote the manufacturer live.

3.2 Commissioned Live Streaming Mode

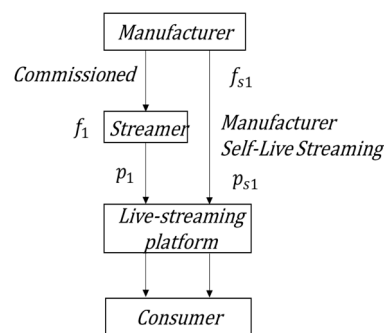


Fig 1. Commissioned live streaming mode

As shown in Figure 1, the manufacturer acts as the leader in the Stackelberg game and contracts with Internet-celebrity streamer. While Internet-celebrity streamer is responsible for selling the merchandise and is not engaged in determining the pricing decision of the products. The manufacturer decides the price p_{s1} of the manufacturer self-live streaming channel and the sales effort f_{s1} as well as the price p_1 of Internet celebrity live streaming channel. Internet celebrity live streaming, which is a follower, determines the sales efforts of Internet celebrity live streaming f_1 and the platform determines the commission ratio ε well as the cost of the unit service c . The demand functions in the commissioned live streaming mode are displayed as follows.

Demand for Internet celebrity live streaming channel:

$$D_1 = (1 - \theta)a - p_1 + \mu p_{s1} + \beta f_1 + k(n + \sigma N). \quad (1)$$

Demand of manufacturer self-live streaming channel:

$$D_{s1} = \theta a - p_{s1} + \mu p_1 + \beta f_{s1}. \quad (2)$$

In the commissioned live streaming mode, the manufacturer is required to give Internet-celebrity streamer a set amount in commission. The commission factor in the paper defines as λ . Considering that the platform must charge service cost, and the commission ratio charged by different products is different, so sets the commission ratio range is (0.1,0.5). The profits of the manufacturer and the streamer are shown as follows.

The profit function of manufacturer:

$$\Pi_{s1} = (1 - \lambda) p_1 D_1 + (1 - c) p_{s1} D_{s1} - \frac{1}{2} f_{s1}^2. \quad (3)$$

The profit function of Internet celebrity live streaming:

$$\Pi_1 = (\lambda - c)(1 - \varepsilon) p_1 D_{s1} - \frac{1}{2} f_1^2. \quad (4)$$

The paper uses the inverse solution method. We first calculate the sales effort decision of Internet celebrity live streaming. The second-order derivatives of f_1 in equation (4) are as follows.

$$\frac{\partial^2 \Pi_1}{\partial (f_1)^2} = -1 < 0, \text{ so there exists an optimal solution. Let } \frac{\partial \Pi_1}{\partial f_1} = 0, \text{ We can get } f_1 = \beta p_1 (\lambda - c)(1 - \varepsilon);$$

Substituting into formula (3) yields.

$$\Pi_{s1} = (1 - \lambda) p_1 \left((1 - \theta)a + p_1(-1 + \beta^2(\varepsilon - 1)(c - \lambda)) + p_{s1} + k(n + N\sigma) \right) + p_{s1}(1 - c)(\theta a - p_{s1} + \mu p_1 + \beta f_{s1}) - \frac{1}{2} f_{s1}^2. \quad (5)$$

Take partial derivatives of the p_1 , p_{s1} , f_1 and set them equal to 0. The optimal solution is obtained simultaneously:

$$p_1^* = \frac{(-1+c)(a(2+\beta^2(-1+c))(-1+\lambda)(-1+\theta)-a(-2+c+\lambda)\theta\mu-(2+\beta^2(-1+c))k(-1+\lambda)(n+N\sigma))}{2(2+\beta^2(-1+c))(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))(-1+\lambda)+(-2+c+\lambda)^2\mu^2},$$

$$p_{s1}^* = \frac{-(-1+\lambda)(-2a(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))\theta a(-2+c+\lambda)(-1+\theta)\mu+k(-2+c+\lambda)u(n+N\sigma))}{2(2+\beta^2(-1+c))(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))(-1+\lambda)+(-2+c+\lambda)^2\mu^2},$$

$$f_{s1}^* = \frac{-b(-1+c)(-1+\lambda)(2a(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))\theta a(-2+c+\lambda)(-1+\theta)\mu-k(-2+c+\lambda)\mu(n+N\sigma))}{2(2+\beta^2(-1+c))(-1+c)(-1+b^2(-1+\varepsilon)(c-\lambda))(-1+\lambda)+(-2+c+\lambda)^2\mu^2},$$

$$f_1^* = \frac{\beta(-1+c)(-1+\varepsilon)(c-\lambda)(a(2+\beta^2(-1+c))(-1+\lambda)(-1+\theta)-a(-2+c+\lambda)\theta\mu-(2+\beta^2(-1+c))k(-1+\lambda)(n+N\sigma))}{2(2+\beta^2(-1+c))(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))(-1+\lambda)+(-2+c+\lambda)^2\mu^2}.$$

Substituting the optimal solutions p_1^* , p_{s1}^* , f_{s1}^* , into Eqs. (4), (5), and (6) yields.

$$\Pi_1^* = \frac{\left\{ \begin{aligned} & -(-1+c)^2(\varepsilon-1)(c-\lambda)(a A_1(\lambda-1)(\theta-1)-a A_4\theta\mu-k A_1(\lambda-1)(n+N\sigma)) \\ & (a(A_1 A_2(\lambda-1)(\theta-1)+A_3(c-\lambda)\theta\mu+2 A_4(\theta-1)\mu^2)-k(A_1 A_2(\lambda-1)+2 A_4\mu^2)(n+N\sigma)) \end{aligned} \right\}}{2(2 A_1 A_5(c-1)(\lambda-1)+A_4^2\mu^2)^2},$$

$$\Pi_{s1}^* = \frac{\left\{ \begin{aligned} & (c-1)(\lambda-1)(a^2(2-2\lambda+4(\lambda-1)\theta-2 A_4s^2+\beta^2(c-1)(1+\theta(-2+\theta+2c(\varepsilon-1)\theta)+\lambda(-1+\theta(2+\theta-2\varepsilon\theta)))) \\ & +2 A_4(\theta-1)\theta\mu)+2ak(A_1(\lambda-1)(\theta-1)-A_4\theta\mu)(n+Nz)-k^2 A_1(\lambda-1)(n+Nz)^2 \end{aligned} \right\}}{2(2 A_1 A_5(c-1)(\lambda-1)+A_4^2\mu^2)^2},$$

where $A_1 = 2 + \beta^2(c - 1)$; $A_2 = -2 + 3\beta^2(\varepsilon - 1)(c - \lambda)$; $A_3 = -2 + \beta^2(\varepsilon - 1)(2 + c - 3\lambda)$; $A_4 = c - 2 + \lambda$; $A_5 = -1 + \beta^2(\varepsilon - 1)(c - \lambda)$;

3.3 Retail Live Streaming Mode

Figure 2 depicts the retail live-streaming mode. The manufacturer remains the leader in the Stackelberg game. First, the manufacturer sells the product to Internet-celebrity streamer at the wholesale price w ; sells the product to the consumers at the sales price p_{s2} of the manufacturer self-live streaming, while determining the sales effort f_{s2} . Then, depending on the wholesale price established by the manufacturer, the streamer establishes his own live-streaming sales price p_2 and sales effort f_2 . The platform determines the unit service cost c for the two channels. The demand functions and profit functions under the retail live streaming mode are as follows.

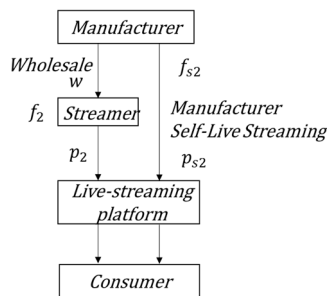


Fig 2. Retail live broadcast mode

Demand for Internet celebrity live streaming channel:

$$D_2 = (1 - \theta)a - p_2 + \mu p_{s2} + \beta f_2 + k(n + \sigma N). \quad (6)$$

Demand of manufacturer self-live streaming channel:

$$D_{s2} = \theta a - p_{s2} + \mu p_2 + \beta f_{s2}. \quad (7)$$

The profit function of the manufacturer:

$$\Pi_{s2} = wD_2 + p_{s2}(1 - c)D_{s2} - \frac{1}{2}f_{s2}^2. \quad (8)$$

The profit function of Internet celebrity live streaming:

$$\Pi_2 = ((1 - c)p_2 - w) D_2 - \frac{1}{2}f_2^2. \quad (9)$$

The paper uses the inverse solution method. Firstly, we consider the sales effort decision of Internet celebrity live streaming. The second partial derivatives of p_2 and f_2 are obtained according to formula (9):

$$\frac{\partial^2 \Pi_2}{\partial (p_2)^2} = -2c - 2 < 0, \quad \frac{\partial^2 \Pi_2}{\partial (f_2)^2} = -1 < 0, \text{ so there exists an optimal solution.}$$

Let $\frac{\partial \Pi_2}{\partial p_2} = 0, \frac{\partial \Pi_2}{\partial f_2} = 0$, we can get: $p_2 = \frac{a(c-1+\theta-c\theta)+(c-1)p_{s2}\mu+(-1+\beta^2-\beta^2c)w+(c-1)k(n+N\sigma)}{(2+\beta^2(c-1))(c-1)}$ and

$$f_2 = \frac{\beta(a(c-1)(\theta-1)+p_{s2}\mu-cp_{s2}\mu-w-(c-1)k(n+N\sigma))}{2+\beta^2(c-1)}. \text{ By substituting into formula (8), we can find the}$$

partial derivatives of p_{s2}, w, f_{s2} and make them equal to 0, the optimal solution can be obtained.

$$p_2^* = \frac{-a(6+\beta^4(c-1)^2(1+\theta(\mu-1))-2\mu^2+2\theta(\mu-1)(3+\mu)+\beta^2(c-1)(5+\theta(-5+3\mu)))+k(6+\beta^2(5+\beta^2(c-1))(c-1)-2\mu^2)(n+N\sigma)}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$p_{s2}^* = \frac{2a(-2-\beta^2(c-1))\theta+a(4+\beta^2(c-1))(\theta-1)\mu-(4+\beta^2(c-1))k\mu(n+N\sigma)}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$w^* = \frac{(c-1)(a(-(2+\beta^2(c-1))^2(\theta-1)+(2+\beta^2(c-1))^2\theta\mu-\beta^2(c-1)(-1+\theta)\mu^2)+k(4+\beta^2(c-1))(4+\beta^2(c-1)+\mu^2))(n+N\sigma)}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$f_2^* = \frac{\beta(c-1)(a(2(\theta-1)(-1+\mu^2)+\beta^2(c-1)(1-\mu^2+\theta((-1+\mu)\mu-1)))-(2+\beta^2(c-1))k(-1+\mu^2)(n+N\sigma))}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$f_{s2}^* = \frac{-\beta(c-1)(-2aA_1\theta+a(4+\beta^2(c-1))(\theta-1)\mu-(4+\beta^2(c-1))k\mu(n+N\sigma))}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)}.$$

Substituting $p_2^*, p_{s2}^*, w^*, f_2^*, f_{s2}^*$ into equation (8), equation (9). We can get the optimal profit:

$$\Pi_2^* = -\frac{A_1(c-1)\left\{\frac{(a(2(\theta-1)(\mu^2-1)+\beta^2(c-1)(1-\mu^2+\theta((-1+\mu)\mu-1)))^2}{-A_1k(-1+\mu^2)(n+N\sigma)}\right\}}{2(\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2))^2},$$

$$\Pi_{s2}^* = \frac{\left\{\frac{((1-c)(a^2(-A_1(1+\theta(-2+3\theta))+2(4+\beta^2(c-1))(\theta-1)\theta\mu-2(\theta-1)^2\mu^2))}{-2ak(\beta^2(c-1)(1+\theta(\mu-1))+2(1-\theta(\mu-1)^2+\mu^2))(n+N\sigma)}\right\}}{2(\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2))},$$

4. Property Analysis of Equilibrium Solution

According to the equilibrium solutions of commissioned live streaming mode and retail live streaming mode in Section 2. The following conclusions are drawn from the comparative analysis.

Proposition 1. In the commissioned live streaming mode, (1) comparing the sales effort of the manufacturer self-live streaming channel and Internet celebrity live streaming channel, there exists $f_{s1} > f_1$; (2) contrasting the price of them, there exists a threshold value λ_0 , when $\lambda < \lambda_0$, $p_1 < p_{s1}$; (3) and the price, sales effort and profit of Internet celebrity live streaming are positively correlated with the commission ratio λ .

Proof: Make a comparison between the price and sale effort of the two live stream channels, respectively.

$$f_1 - f_{s1} = \frac{\left\{\frac{\beta(c-1)(a(c(\lambda-1)(2+2\varepsilon(\theta-1)-4\theta-\beta^2(\varepsilon-1)(1+\lambda)(3\theta-1))+(-1+\lambda)(2\theta+(\varepsilon-1)\lambda(2-2\theta+\beta^2(3\theta-1))))+c(1-\lambda+(-3+2\varepsilon+\lambda)\theta)\mu+(-2+\lambda)(1-\theta+\lambda(\varepsilon\theta-1))\mu+c^2(\varepsilon-1)(\beta^2(\lambda-1)(3\theta-1)-\sigma\mu)}{-k(\lambda-1)((2+\beta^2(c-1))(\varepsilon-1)(c-\lambda)+(-2+c+\lambda)\mu)(n+N\sigma)}\right\}}{2(2+\beta^2(c-1))(c-1)(1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)-(-2+c+\lambda)^2\mu^2}$$

As we known that $0.1 < \lambda < 0.5$, $0 < \mu < 0.4$. So, we can figure out $f_1 - f_{s1} < 0$;

$$-a(c-1)(\lambda-1)(2-4\theta+\beta^2(-1+c+\theta+c(-3+2\varepsilon)\theta-2(\varepsilon-1)\lambda\theta))-a(-2+c+\lambda)(1-\lambda+(-2+c+\lambda)\theta)\mu$$

$$p_1 - p_{s1} = \frac{+k(\lambda-1)(2-\beta^2(c-1)^2-2c+(-2+c+\lambda)\mu)(n+N\sigma)}{2(2+\beta^2(c-1))(c-1)(-1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)+(-2+c+\lambda)^2\mu^2}$$

Existence of threshold λ_0 ,

$$\lambda_0 = \frac{(a(c-1)(2-4\theta+\beta^2(c-1+(-1-3c+2(1+c)\varepsilon)\theta))+a(3-c+2(c-2)\theta)\mu+k(-2+\beta^2(c-1)^2-c(\mu-2)+3\mu)(n+N\sigma)-(1-c)\sqrt{(B_1+B_2)}}{4a\beta^2(c-1)(\varepsilon-1)\theta-2a(\theta-1)\mu+2k\mu(n+N\sigma)}$$

Where

$$B_1 = a^2(\beta^4(c-1)^2(1+(-3+2\varepsilon)\theta)^2+2\beta^2(c-1)(2+2\theta^2(6+4\varepsilon(\mu-1)-5\mu)-(-5+2\varepsilon)\theta(\mu-2)-\mu)+(\mu-2)^2+16\theta(\mu-1)-16\theta^2(\mu-1))$$

$$B_2 = 2ak(\beta^4(c-1)^2(1+(-3+2\varepsilon)\theta)-\beta^2(c-1)(2+(-5+2\varepsilon)\theta)(\mu-2)+(\mu-2)^2+8\theta(\mu-1))(n+N+k^2(2+\beta^2(c-1)-\mu)^2(n+N\sigma)^2), \text{ and } \lambda < \lambda_0, p_1 < p_{s1}.$$

$$\frac{\partial p_1}{\partial \lambda} = \frac{\left\{\frac{(-1+c)(a(2\beta^2(2+\beta^2(c-1))^2(c-1)(\varepsilon-1)(\lambda-1)^2(\theta-1)+2(2+\beta^2(c-1))(c-1)(1-c+\beta^2(\varepsilon-1)(-2+c^2-2c\lambda-(-4+\lambda)\lambda))\theta\mu+(2+\beta^2(c-1))(c-\lambda)(-2+c+\lambda)(\theta-1)\mu^2+(-2+c+\lambda)^2\theta\mu^3)-2(2+\beta^2(c-1))k(2\beta^2(2+\beta^2(c-1))(c-1)(\varepsilon-1)(\lambda-1)^2+(c-\lambda)(-2+c+\lambda)\mu^2)(n+N\sigma))}{(2(2+\beta^2(c-1))(c-1)(-1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)+(-2+c+\lambda)^2\mu^2)^2}\right\}}{2(2+\beta^2(c-1))(c-1)(-1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)+(-2+c+\lambda)^2\mu^2)^2},$$

$\frac{\partial p_1}{\partial \lambda} \geq 0$, and the same method can be used to confirm the other conclusions.

Proposition 1 indicates that under the commissioned live streaming mode, there is a threshold value of commission ratio. Price of Internet celebrity live streaming will be more advantageous when the ratio is below this limit. Because the products are typically only on sale in Internet celebrity live streaming for a limited time or even just one day, the manufacturer prefers to sell at a concessional price during the live streaming and hopes that Internet-celebrity streamer would boost sales. However, as the commission ratio rises, the more products that are sold during Internet celebrity live streaming, the more profits of Internet streamer will be made. As a result, the streamer selects to step up sales effort to increase revenues. Due to cost increases, the manufacturer will increase the price of Internet celebrity live streaming, rendering the commodity price of Internet celebrity live streaming

uncompetitive. Therefore, the manufacturer will only invite Internet-celebrity streamer to sell stuff in live streaming when the commission range is appropriate. With the help of the existing traffic of Internet-celebrity streamer and the gimmick of low price to attract consumers, the manufacturer can not only increase sales in a short period of time but also promote commodities and enhance the visibility of brands. Additionally, consumers can purchase the goods for less money. While both channels use live streaming to sell stuff, Internet-celebrity streamer has a wide range of products and a larger consistent following, necessitates less effort on the part. By contrast, manufacturer only sells products which from own shop and the manufacturer live self-live streaming channel typically has longer time and higher frequencies, they typically put more effort into increasing sales.

Proposition 2. In the retail live streaming mode, (1) when contrasting the sales efforts of manufacturer self-live streaming and Internet celebrity live streaming channels, there exists $f_{s2} > f_2$. (2) comparing with the price of live streaming channels of the manufacturer and Internet-celebrity streamer, there is a threshold θ , when $\theta > \theta_0$, $p_{s2} > p_2 > w$. (3) while the pricing and sales effort of the manufacturer are positively connected with θ , the price, sales effort, and wholesale price of Internet celebrity live streaming are negatively correlated with θ .

Proof: Make a subtraction between the sales effort of Internet celebrity live streaming and the sales effort of manufacturer self-live streaming. Make a comparison between the price of Internet celebrity live streaming and the price of manufacturer self-live streaming, and the wholesale price, respectively.

$$f_2 - f_{s2} = \frac{-b(c-1)(a(2-2\mu(2+\mu)+2\theta(\mu-1)(3+\mu)+\beta^2(c-1)(1-\mu(1+\mu)+\theta(-3+\mu^2))) - k(-2+2\mu(2+\mu)+\beta^2(c-1)(-1+\mu+\mu^2))(n+N\sigma))}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)} < 0$$

$$p_{s2} - w = \frac{k(4(-1+c+\mu)+\beta^2(c-1)((4+\beta^2(c-1))(c-1)+\mu+(c-1)\mu^2))(n+N\sigma)}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)} ;$$

$$p_2 - p_{s2} = - \frac{\alpha(\beta^4(c-1)^2(1+\theta(\mu-1))+2(\mu-1)(-3+5\theta+(\theta-1)\mu)+\beta^2(c-1)(5-7\theta-\mu+4\theta\mu)) + k(2+\beta^2(c-1)-2\mu)(3+\beta^2(c-1)+\mu)(n+N\sigma)}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)} ;$$

Existence of threshold value θ_0 , $\theta_0 = \frac{-(2+\beta^2(c-1)-2\mu)(3+\beta^2(c-1)+\mu)(\alpha+k(n+N\sigma))}{\alpha(\beta^4(c-1)^2(\mu-1)+2(\mu-1)(5+\mu)+\beta^2(c-1)(-7+4\mu))}$ satisfying

$\theta > \theta_0, p_{s2} > p_2 > w$. $\frac{\partial p_{s2}}{\partial \theta} = \frac{a(b^2(-1+c1)(-2+u)+4(-1+u))}{b^2(4+b^2(-1+c1))(-1+c1)(-2+u^2)+8(-1+u^2)}$. Other conclusions can be drawn in the same way.

Proposition 2 indicates that the retail live streaming mode is different from the commissioned live streaming mode. Taking into consideration the wholesale cost, the streamer will rise the price for the commodities to maximize profits. Customers favor the live channel of the streamer when θ is low. For the manufacturer, the sales volume is concentrated in Internet celebrity live streaming channel, and the sales volume of the manufacturer self-live streaming channel is bleak. To lure customers with low pricing and capture the market, the manufacturer will decide to raise the wholesale price while lowering the price of the manufacturer self-live streaming. Naturally, the price of Internet celebrity live streaming is more expensive than the price of the manufacturer self-live streaming. When θ grows gradually, the manufacturer is willing to offer the live streamer a cheaper wholesale price for the manufacturer's channel that already dominates most of the market for the products at this point and anticipates that broadening the live channel would boost sales. Additionally, Internet-celebrity streamer offers its products for a low price to draw in more clients. In the end, the price of Internet celebrity live streaming commodities is less than that of the manufacturer's own channel.

Proposition 3. Compared that in the two modes, (1) the price and sales effort of Internet celebrity live streaming, the price and sales effort of manufacturer self-live streaming, and the wholesale price are all positively correlated with the sensitivity coefficient β of consumers to sales effort, the trust degree of Internet-celebrity streamer k , the personal influence of Internet-celebrity streamer n , and the proportion of impulsive consumers σ .

Proof: $\frac{\partial p_1}{\partial k} = \frac{-(2+\beta^2(c-1))(c-1)(\lambda-1)(n+N\sigma)}{2(2+\beta^2(c-1))(c-1)(-1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)+(c+\lambda-2)^2\mu^2} > 0$. The consistent approach can be used to draw other conclusions.

Proposition 3 illustrates that as consumers become more sensitive to sales effort, Internet-celebrity streamer and the manufacturer will consequently raise the sales effort as a result, and correspondingly the cost increase, at which point streamers in both modes will appropriately raise their sales prices for their own benefit. Nevertheless, the most effective approach for the streamer is still to strive to improve the sales effort. Only when consumers perceive the streamer's sales effort will they choose to purchase commodities, which in turn increase demand and, ultimately, sales revenue of the live streaming. As the public's trust in Internet-celebrity streamer grows, so many fans are willing to buy merchandise from Internet celebrity live streaming, and product pricing deciders of both live-stream channels will raise the prices. At the same time, the manufacturer will inevitably increase the amount of sales effort on his own live channel to improve competitiveness, and the pricing will reflect this. Because consumers are driven by the external factor of the personal influence of Internet-celebrity streamer and will be less price-sensitive, the price of products in each channel will rise in accordance when the personal influence of Internet-celebrity streamer is positive and gradually expands.

5. Summary

In this paper, we investigate an online dual-channel supply chain system with a manufacturer, a Internet-celebrity streamer, and a live platform, and consider the optimal decision price, sales effort level, and profitability of the supply chain members. The following results are reached after analyzing the impact of the commission ratio, consumers' sensitivity to the sales effort of the live stream, the trust in Internet-celebrity streamer, and the effect of streamers' individual influence on each supply chain member's decision-making:

Under the commissioned live streaming mode, Internet-celebrity streamers' sales effort is never as strong as those of the manufacturer self-live streaming. When the commission ratio is kept within a specific range, the price of Internet celebrity live streaming is typically cheaper than the price of the manufacturer self-live streaming and is proportional to the commission ratio. In contrast to the channel of manufacturer self-live streaming, the commission ratio is more responsive to fluctuations in pricing and sales effort on Internet-celebrity streamer.

Under the retail live streaming mode, the amount of sales effort put forth by Internet-celebrity streamer fewer sales effort than that of the manufacturer self-live streaming, and the price of Internet celebrity live streaming is only lower when consumers' preferences are concentrated more in the manufacturer channel. The price of Internet celebrity live streaming is higher in the retail live mode than in the commissioned live mode.

The pricing, sales effort, and profit of each subject are positively correlated with the sensitivity of consumers to live sales effort, trust in Internet-celebrity streamer, the personal influence of Internet-celebrity streaming, and the proportion of impulse consumption, according to the two modes. The optimal decision of Internet-celebrity streamers is inversely proportional to consumers' preference and directly proportional to the manufacturer's channel. In contrast, the manufacturer self-live streaming is more affected by the sensitivity coefficient than the Internet celebrity live streaming channel.

For manufacturers, inviting Internet-celebrity streamers in the way of signing commissioned contract can promote sales more. And for Internet-celebrity streamers, the way of wholesaling merchandise can obtain more profits.

In this paper, the study of live dual-channel supply chains only considered how the moderate streamer as well as the manufacturer make pricing decisions when the manufacturer is dominant. In the future, we can examine and investigate how manufacturers and top Internet-celebrity streamers decide on prices (either when both sides are equally powerful or when Internet celebrities are in the dominant position).

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