Study and Application of Game Theory

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Abstract. Game Theory is branch of applied mathematics that provides tools for analyzing situations, which has better solved the operability analysis of competition and other problems, and has become a research field in economics. Many kinds of behaviors and phenomena around us can be summarized by game theory. This paper mainly introduces the concept of game theory, the application classification of game theory, the classical model of game theory and the application of game theory in life.

Keywords: Game Theory; Classical Game Theory Model; Prisoner's Dilemma; Nash Equilibrium.

1. The Background of Game Theory

According to different characteristics, games can be classified, the most obvious of which is the number of players. So, the game can be designated as one player, two player, or n-player game. Each category of game has its own unique characteristics. In addition, the participants may not be a person, but may be a country, a company or a team with a community of interests [7].

1.Constant sum game is a completely conflicting game, which is also known as pure competition game. For instance, poker is a type of constant sum game. This is because that the player's wealth combination remains unchanged, even though its distribution will change during the game. They often have completely opposite interests to the players in the game. In mutability and games, they may both be winners or losers.

2.Variable sum games can be further distinguished as cooperative or non- cooperative games. Players are able to communicate and reach binding agreements in a cooperative game; Besides, in a non-cooperative game, players have an ability to communicate, but they cannot reach binding agreements, such as enforceable contracts.

2. The Classification of Game Theory

A game can be described in one of three ways: in extensive, normal, or characteristic-function form. (Sometimes these forms are combined, as described in the section Theory of moves.) Most parlour games, which progress step by step, one move at a time, can be modeled as games in extensive form.

Extensive-form games can be described by a “game tree,” which means each turn is a vertex of the tree and the players’ successive choices are indicated by the branch.

The normal (strategic) form is primarily used to describe two-person games. Payoff matrix can be a example for this form. And each row describes the strategy of one player and each column describes the strategy of the other player.

The characteristic-function form is generally used to analyze games with more than two players. It indicates the minimum value that each player alliance (including a single player alliance) can guarantee for itself when playing against an alliance made up of all other players.

3. The Classical Models of Game Theory

1. The Tree Model

Most indoor games are played step by step and can be used as a wide range of games. A wide range of games can be described by a "game tree", in which each round is a vertex of the tree, and each branch represents the player's continuous selection [8].

Although according to the original rules, there are cases where neither player wins (such as (C) above), for the sake of simple analysis, A condition is added: when all nine squares are marked and no three symbols of the same kind are on the same line, the first player A is judged to win. Thus, when the game is over (i.e., at the top of the leaf) there must be a winner.

The vertex of the leaf corresponds to the final state of the game, which is marked as 1 or 0, corresponding to player A winning or losing, respectively; For player B, the numbers mean the opposite.

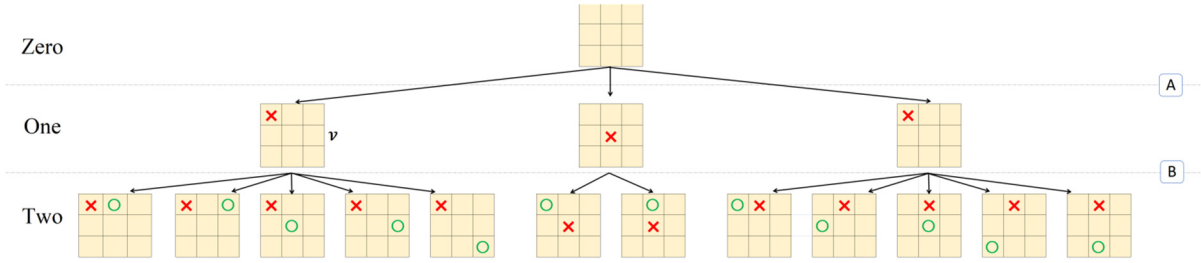


Fig 1. Game Example

For the situation in Figure (A) below, A has only one choice and will win later, so the vertex in the tree corresponding to figure (A) can also be marked as 1 -- indicating that A will win.

A brief analysis of figure (B) shows that no matter what choice B makes, he will lose and A will win. Therefore, the vertex corresponding to Figure (B) in the tree can also be marked as 1 -- indicating that no matter what move B makes, A will win.

In the case of (c) below, he can win whatever he chooses, so he can also mark the vertices in the tree corresponding to (c) as 0 -- indicating that B can definitely win.

But for (D) below, player B wins if he makes the right choice (bottom left), and loses if he makes the wrong choice (top right). To analyze the game tree, it will be assumed that none of the players make mistakes.

Under this assumption, the vertices in the tree corresponding to (d) below can also be marked as 0 -- indicating that there is a winning strategy for player B.

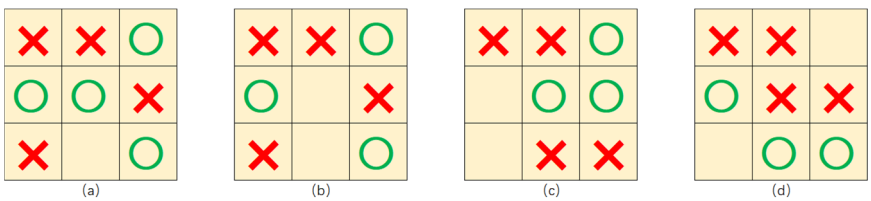
If the subtrees rooted in (b)~(d) are expanded (see (E) below), it can be found that the behavior and logical operation "and" of the vertex of odd layer corresponding to B's turn to label itself according to the label of its child vertex, so it can be denoted by $\wedge$.

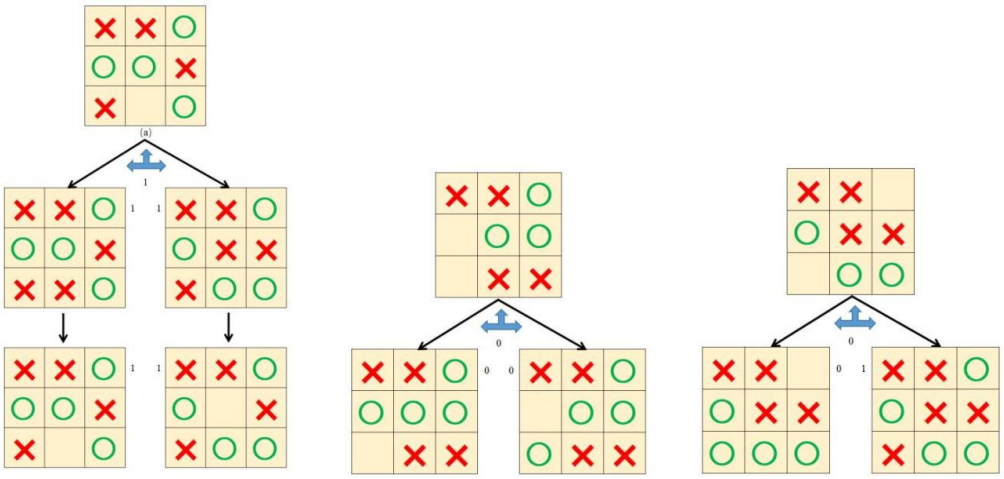
Similarly, it can be analyzed that the even level vertex corresponding to the turn of A moves to label itself according to the label of its child vertex is consistent with the logical operation "or", so it can be expressed by.

Therefore, each branch point in the game tree can be divided into $\vee$

vertices and $\vee$ vertices according to the number of layers. For an $\vee$

vertex, if it has a subvertex labeled 1, the vertex is labeled 1; If all its child vertex labels are 0, then its label is 0. For a $\vee$ vertex, if it has a child node marked with 0, then the vertex marked with 0. It has a label of 1 if all its child vertex labels are 1. This kind of tree is often called an and or tree.





(e)

Fig 2. Game Example

The interpretation of 0-1 marks of branching points in this game tree is given below. If there exists A winning strategy for player A regardless of player B's moves, the node is marked as 1; The node marked 0 means that no matter how player A moves, there is always A winning strategy at the vertex of later should choose any child vertex that will make her win. However, at the vertex of the odd layer, her fate is in the hands of player B.

When the game tree is known, since the 0-1 labeling of branching points is determined by the labeling of their child vertices, they can be computed using the post-order traversal of the tree.

2. Prisoner's Dilemma

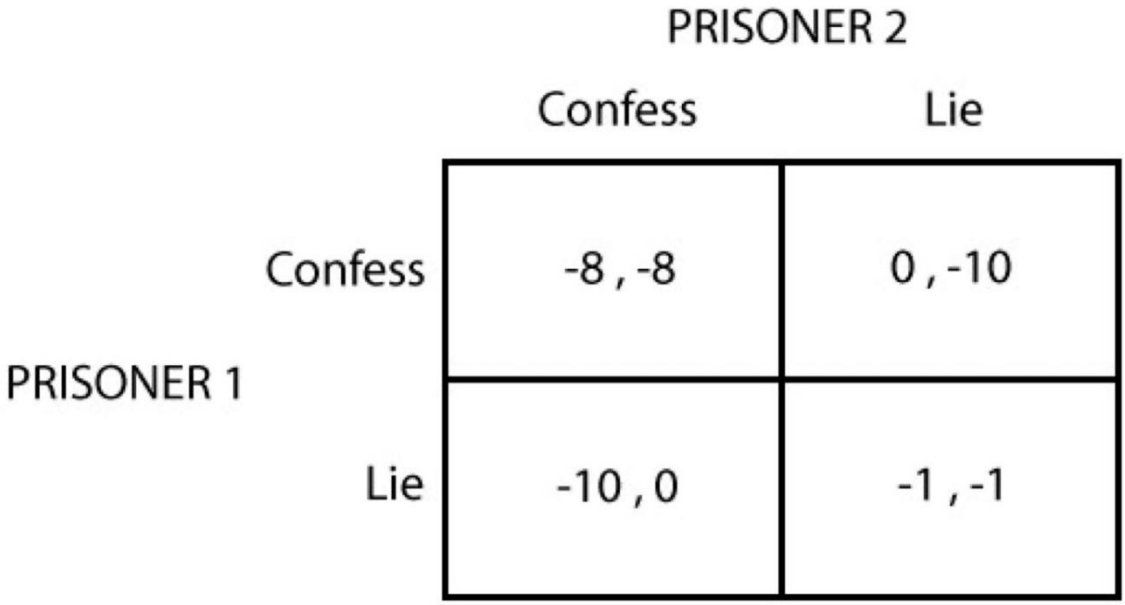


Fig 3. Prisoner's Dilemma Example

The matrix entries at the intersection of each row and column give the results of each player's selection of the corresponding strategy. The revenue of each player related to this result is the basis for determining whether the strategy is "balanced" or stable [9].

Players and all their possible moves are placed on adjacent sides of the matrix. Put the payoff into the matrix. In this case, all returns are negative because they represent time spent in prison. From the matrix above, we can see that if both prisoners admitted, they would each spend eight years in prison. If only one prisoner confesses, he does not go to jail, while the other player must serve 10 years. If both prisoners lie, they will each serve only a year in pris3.Nash Equilibrium.

If after considering the strategies of all other participants, no participant has the motivation to change his game strategy, then the non-cooperative game is called Nash equilibrium. ***Prisoner's dilemma*** is a classic example of Nash equilibrium. Let me remind you that prisoner's dilemma means that two prisoners are convicted as accomplices to a crime. Prisoners are held incommunicado, so they have no contact with each other. Then make the following suggestions to each of them:

When both prisoners confess, the game is Nash equilibrium. Why? Because in this case, no prisoner will benefit from changing strategies. If prisoner 1 changes his strategy and keeps silent, he will receive a longer sentence than when he confesses, and prisoner 2 will be acquitted.

Prisoner 2 should use the same logic to maintain his strategy. Although the best strategy for the whole group is to keep silence on both sides, it is better for individuals to confess, because they cannot know the strategy of another prisoner in advance, and keeping silence when another prisoner confesses will lead to 10 years of imprisonment.

The Nash equilibrium can be applied to a variety of real-life situations. The prices of fruit stalls in the wet market are basically the same with little difference.

In fact, this is also a Nash equilibrium. Although for individual bosses, they can obtain short-term profits by lowering prices, they may be retaliated by peers collectively or cause price wars, which will eventually lead to the reduction of the overall profit rate and the total profit, which is not worth the loss.

4. The Conclusion

In fact, game theory is widely used in people's life. For example, in the pricing problem, the most appropriate price should be determined according to the price of the nearby merchants. This paper introduces a lot about game theory, such as prisoner's dilemma and Nash equilibrium. Understanding these can help us not only learn knowledge, but also apply game theory to life, which can bring convenience.

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