

Improvements of portfolio performances by bitcoin based on Markowitz Model and Index Model

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Abstract. Contemporarily, with the rapid development of society, investors are noticing the importance of finance and asset allocation and actively participating in the market. Achieving greater returns with controlled risk is the common goal of the vast majority of investors, and thus there has been a lot of researches focusing on portfolio optimization. As Bitcoin has become known and its high volatility has captivated people who want to achieve high returns, it seems feasible to add it to the investment portfolios to enhance the performance. However, it is crucial to investigate the impacts of Bitcoin on portfolio. On this basis, this study uses weekly data of five companies from different industries and Bitcoin to construct two portfolios with and without Bitcoin based on Markowitz model and index model to calculate annual return, annual volatility and Sharpe ratio. According to the analysis, the implementation of Bitcoin has boosted both the Sharpe ratio and annual return of the portfolio for the optimal parameter, i.e., indicating there are positive impacts for the Bitcoin. These results shed light on guiding further exploration of improving the performances of portfolios based on different types of underlying assets.

Keywords: Bitcoin; Markowitz Model; Index Model; Portfolios optimization.

1. Introduction

With the rapid development of society and the increase in huamn beings' financial awareness, investment has become one of the hottest topics in people's daily life. More and more people are realizing the importance of asset allocation and management and are actively participating in the market, so a reasonable and effective investment portfolio becomes increasingly necessary. In order to invest properly and optimize the portfolio and return, there are many strategies and theories such as the Markowitz Model and Single Index Model are significant. After Harry Markowitz posted the paper on his model [1], there are lots of scholars investigate, explore and improve the models. Mangram describes the usefulness of Markowitz's model for modern portfolio theory and proposes a concise computer-based approach to perform these operationally complex calculations [2]. Moreover, Based on the study from Markowitz's recommendations, Lintner concluded that no possible diversification approach would be sufficient to eliminate all the risks of holding common stocks and the goal of diversification is to produce the most favorable combination of risk and expected return [3]. Besides these, there are also a lot of studies regarding the application of portfolios using the Index Model [4], which also suggests stock portfolios can recover losses from other securities included in their portfolio if some securities have negative returns. Mandal uses a single index model to construct the optimal portfolio by including securities in the portfolio and their respective weights [5], thus observing that it is easier and more comfortable to construct the optimal portfolio using Sharpe's single index model as a simple mechanism for constructing the optimal stock portfolio. Moreover, Shah uses the Sharpe single index model to obtain the exact number of securities and investment weights to obtain specific portfolios and weights to invest in different securities [6]. Achieving more attractive returns while controlling risks is important to every investor, therefore, there are plenty of people working on various portfolio strategies and their characteristics. Kim and Wagman analyze startup accelerators and portfolio size and show that portfolio size is less than efficient in a rational-expectations equilibrium [7]. Besides, there is also research that examined the relative popularity of various portfolio approaches and concluded that the financial approach was the most popular [8]. There is a lot of existing research on the optimization problem today, demonstrating many important

theories and strategies, but those that focus more on a single model. based on two different models and using real historical data from several different sectors of equities, there are not many studies that analyze the impact of bitcoin on a portfolio, and even fewer studies that analyze the impact of bitcoin on a portfolio.

Bitcoin is a digital currency created in 2009 that uses cryptography to ensure security. One of its main features is that it is decentralized, which means it is not controlled by any central institution such as banks or governments. Its transactions take place directly between users, which makes it highly flexible and free, and also allows it to resist censorship. Another key feature of Bitcoin is that it is based on the distributed ledger technology of blockchain. This decentralized database is maintained by a network of users and records all bitcoin transactions. The blockchain allows for transparency and security, as it is public and the information recorded on it is difficult to change. Bitcoin can be used as a medium of exchange for goods and services, similar to traditional currencies. It can also be bought and sold on exchanges, much like stocks or other investments. Over the years, the value of bitcoin has been very volatile, with many instances of its price fluctuating dramatically in a short period of time, and such high volatility has been one of the biggest reasons for interest and concern about it. In the paper by Pichl and Kaizoji [9], they explored the volatility of bitcoin prices from several aspects of multiple methods to learn predictions of log returns. They conclude that the time series volatility of the bitcoin price is high and the chances of market bubbles and crashes are relatively high, but there are also plenty of arbitrage opportunities because of this. After the analysis of Baur and Dimpfl [10], they found that the volatility of bitcoin is extreme by a factor of thirty compared to other major currencies, and therefore they concluded that bitcoin cannot be played as a currency. In addition, Chaim and Laurini studied the dynamics of bitcoin's daily returns and volatility and showed that bitcoin's return jumps can significantly affect the structure of losses and returns associated with it [11].

Based on these existing studies, this research is focusing on the impacts of the BTC on portfolio based on Markowitz Model and Index Model model. Since the BTC has high volatility, this paper would like to find that if adding BTC could enhance the performance of the portfolio. The remainder of this paper is organized as follows. Section 2 shows the data and method, while Section 3 contains the results and discussion. Section 4 displays the limitations and prospects, and Section 5 is the conclusion.

2. Data & Method

This research used the data of weekly prices of five different stocks, SPX500(Standard & Poor's 500) and BTC(bitcoin) from Jan 1, 2020 - Dec 18, 2022, in investing.com. The five stocks are TSLA (Tesla), META(Facebook), GOOG(Google), AMZN(Amazon), and MSFT(Microsoft), which are the representative companies belonging to different sectors of Consumer Discretionary, Communication Services, and Information Technology.

In order to find the impacts of portfolio performances by bitcoin and optimize the portfolio, the Markowitz Model and Index Model were used. The risk of an investment is determined not only by the volatility of its returns but also by its relevance to the other assets in the portfolio. An investor can reduce the overall risk of a portfolio by including a variety of assets with different risk and return characteristics while still maintaining a high level of expected return, striking the right balance between risk and return. The Markowitz model, also known as the mean-variance model and modern portfolio theory, is a financial model used to construct an optimal portfolio of assets that minimizes risk while maximizing expected returns. According to the model, a portfolio that is diversified across a range of low or negatively correlated assets has a lower overall risk. Investors should diversify their portfolios to include a mix of assets with different risk and return characteristics. Since the performance of one asset may not be correlated with the performance of another, this diversification can help reduce the overall risk of a portfolio. The Markowitz model is widely used in finance and investment management and has had a significant impact on the development of modern portfolio

management and other financial models and investment strategies. Besides, Index model is also a portfolio construction method that advocates holding a portfolio of securities that reflects the composition of a financial market index (e.g., the S&P 500 used in this paper), to match the performance of the index as closely as possible to achieve average market returns, based on the IDEA that it is difficult for investors to consistently outperform the market. It does not require a professional fund manager to manage the portfolio and has a relatively low expense ratio, so it can provide broad diversification at a lower cost, which has made the index model increasingly popular. However, both the Index Model and the Markowitz Model have their own limitations, so this paper goes beyond using a single model and explores the impact of Bitcoin on portfolio performance based on both of these two important models.

After loading all the data into the program, including the weekly prices of SPX, TSLA, MSFT, META, GOOG, AMZN and BTC from Jan 1, 2020 to Dec 18, 2022, the values of the annual return, max drawdown, annual volatility, sharpe ratio and correlation of these assets were calculated. Under the two model, the portfolio were solved with and without the value of the BTC using Python, getting the values of maximum sharpe, minimum standard deviation, market optimal, annual return, annual volatility and the sharpe ratio before and after. The sum weights of the assets is set to be 1 and the weight of each assest should be varied from [-1, 1], (i.e., allowed shorting with maximum leverage ratio of 1).

3. Results & Discussion

3.1. Description of the Underlying Assets

Seen from Table. 1, the annual returns for most of the underlying assets are relatively low, with the exception of TSLA, which has a significantly higher return. The maximum drawdowns for most of the assets are also relatively high, with META having the highest maximum drawdown and MSFT having the lowest. The annual volatilities for most of the assets are moderate to high, with TSLA and BTC having the highest volatility and SPX having the lowest. The Fig. 1 also shows that the TSLA and BTC have the highest accumulative return within these selected stocks. The Sharpe ratios for most of the assets are moderate, with TSLA having the highest Sharpe ratio and META having the lowest. Based on these data, it seems that TSLA has the highest risk-adjusted return among the assets, followed by MSFT and BTC. On the other hand, META has the lowest risk-adjusted return among the assets, followed by AMZN.

Table 1. Descriptions of different underlying assets

	Annual Return	Max Drawdown	Annual Volatility	Sharpe Ratio
SPX	0.054	-0.370	0.238	0.228
TSLA	0.450	-0.456	0.735	0.613
MSFT	0.131	-0.263	0.290	0.451
META	-0.205	-0.920	0.443	-0.461
GOOG	0.076	-0.329	0.314	0.242
AMZN	-0.033	-0.464	0.368	-0.090
BTC	0.247	-0.535	0.737	0.335

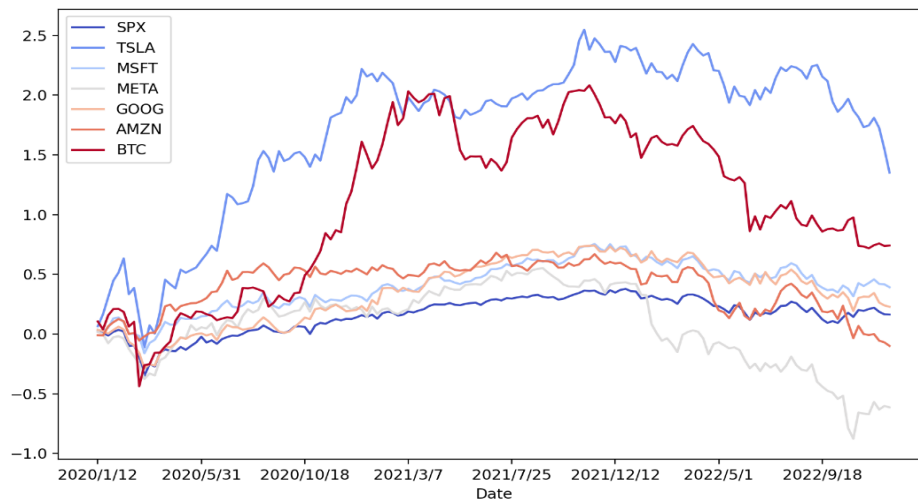


Fig 1. The accumulative Returns for different underlying assets.

The Fig. 2 displays the correlation form of these underlying assets. The correlation coefficient indicates the degree of correlation between the two variables. A coefficient of 1 indicates a perfectly positive correlation, meaning that when the value of one variable increases or decreases, the value of the other variable will also increase or decrease by the same amount, while a correlation coefficient of -1 is the opposite, indicating a perfectly negative correlation. A correlation coefficient of 0 means there is no correlation. In Fig. 2 one can see that the trend of Bitcoin (BTC) has the strongest correlation with the trend of Tesla (TSLA) with a correlation coefficient of 0.36, and it has the least correlation with the trend of Meta (META) with a correlation coefficient of 0.1.



Fig 2. Correlation Heatmap of different underlying assets.

3.2. Portfolio With and Without Bitcoin

The effective frontier with 20000 random distributed weights for the combinations of the assets without portfolio are shown in Fig. 3. Besides, the capital assets line (CAL, red line) and the optimal portfolio (the stars) is also given in the Fig. 3. The summary of different optimal conditions (i.e., different models) are given in Table. 2 with weights of underlying assets, annual return, volatility as well as Sharpe ratio. According to the results, the annual return ratios are 0.661, 0.069 and 0.613 for maximum Sharpe, minimize volatility and market optimal models, respectively, while the Sharpe

ratios are 1.089, 0.253 and 1.087, respectively. Similar results for the portfolio with Bitcoin are shown in Fig. 4 and Table. 3. According to the results, the annual return ratios are 0.673, 0.074 and 0.622 for maximum Sharpe, minimize volatility and market optimal models, respectively, while the Sharpe ratios are 1.112, 0.279 and 1.110, respectively.

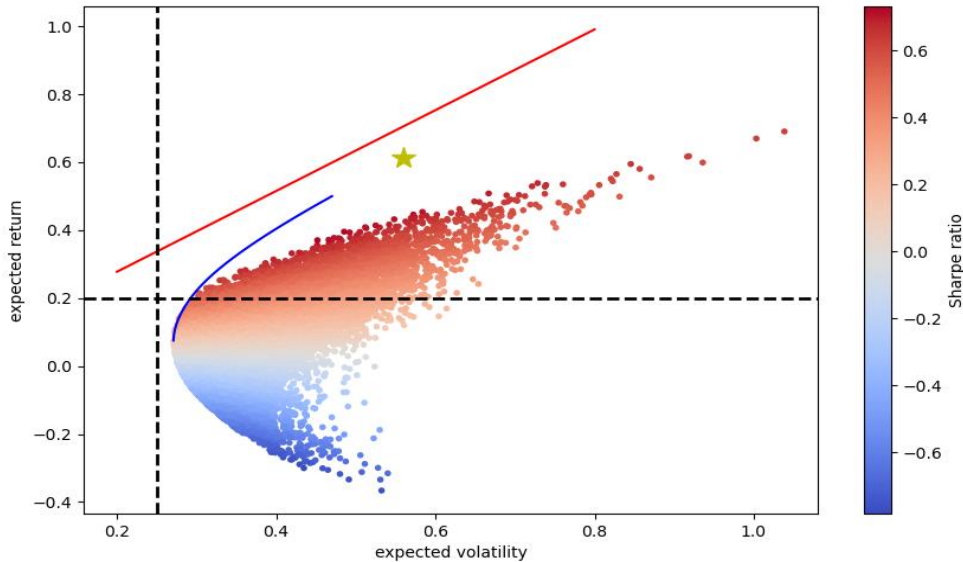


Fig 3. Effective frontier and capital market line for portfolio without BTC.

Table 2. Summary of the portfolio without BTC under different optimal conditions

	Maximum Sharpe	Minimize std	Market Optimal
TSLA	-0.117	0.509	0.432
MSFT	0.688	1.000	1.000
META	-0.048	-0.972	-0.911
GOOG	0.335	1.000	1.000
AMZN	0.143	-0.537	-0.522
Annual Return	0.661	0.069	0.613
Annual Volatility	0.607	0.271	0.564
Sharpe Ratio	1.089	0.253	1.087

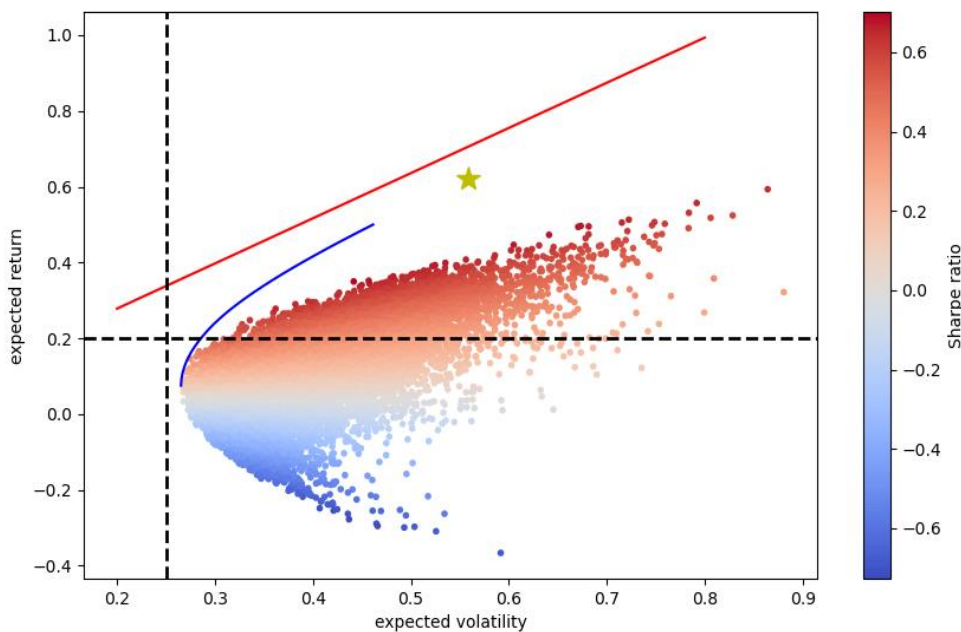


Fig 4. Effective frontier and capital market line for portfolio with BTC

Table 3. Summary of the portfolio with BTC under different optimal conditions

	Maximum Sharpe	Minimize std	Market Optimal
TSLA	0.436	-0.138	0.363
MSFT	1.000	0.684	1.000
META	-0.975	-0.042	-0.912
GOOG	1.000	0.298	1.000
AMZN	-0.630	0.122	-0.604
BTC	0.169	0.075	0.153
Annual Return	0.673	0.074	0.622
Annual Volatility	0.606	0.265	0.561
Sharpe Ratio	1.112	0.279	1.110

3.3. Comparison

Based on the results mentioned above, it can be seen that the portfolio with BTC has better performance than the portfolio without BTC in both annual return ratio and Sharpe ratio for the three models. In this case, it means that the implementation of BTC has the ability to improve the performance of portfolio.

4. Limitations & Prospects

This study explores the impact of bitcoin on portfolio performance under the Markowitz model and the Index model, mainly using data on bitcoin and the five selective assets from 2020 to 2022, but there are still many limitations. In terms of model selection, using the Markowitz model and the Index model may not be appropriate for all investors or portfolios. These two models are based on specific assumptions and therefore may not take into account other factors that may affect portfolio performance and may not capture the full complexity of the market or the specific characteristics of bitcoin. In addition, the choice of data and timing, using only data from 2020 to 2022 may not provide a long enough time frame to accurately assess the impact of bitcoin on portfolio performance to accurately represent market conditions or bitcoin's behavior over a longer time horizon. Market conditions during the study period may not be representative of market conditions during other time periods, which may affect the results of the study. In terms of asset selection, there is a limit to the number of industries that can be represented using only five selective assets, which is still not diverse enough, so this may not fully represent the impact of bitcoin on portfolio performance. A more diverse set of assets may be able to more accurately assess the impact of bitcoin on portfolio performance. In addition, by focusing solely on the impact of bitcoin on portfolio performance, the study may ignore other factors that may affect portfolio performance, such as changes in economic conditions or the performance of individual assets.

For further study, perhaps data including longer time periods should be used, such as 10 years or more, and can be combined with stocks, bonds, and other diversified asset classes to get a more complete picture of the impact of bitcoin on portfolio performance. In addition to this, other different models such as the Capital Asset Pricing Model (CAPM) or the Three Factor Model are also recommended to be tried and can provide more information on the impact of Bitcoin on portfolio performance and how it compares to other assets under a variety of different models. Also, this study focuses on only one cryptocurrency of Bitcoin, therefore, in future studies, the impact of other cryptocurrencies such as Ether or Litecoin on the portfolio could also be analyzed, and these impacts could be compared to provide more insights on the unique characteristics and behavior of each cryptocurrency and how they affect portfolio performance.

5. Conclusion

In summary, this study investigates the feasibility of improving the performances of portfolio by implementation Bitcoin. To be specific, five traditional underlying assets (i.e., TSLA, MSFT, META, GOOG, AMZN) are selected as the benchmark portfolio and the performances before and after implementation of Bitcoin are compared. According to the analysis, for the three optimal models (i.e., maximum Sharpe, minimize volatility and market optimal models), the portfolio with Bitcoin has better performances in annual return as well as Sharpe ratio. Nevertheless, this study uses low frequency data (weekly) in recent three years, which might bring some instability of the results. In the future, longer time period with higher data sampling frequency with more underlying assets ought to be used in order to check the long-term stability and effectiveness of adding cryptocurrency to improve the performances of portfolio. Overall, these results offer a guideline for improvements portfolio performances by adding new type underlying assets.

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