Portfolio Optimization Using Markowitz Model and Index Model – A Study on 10 Selected Stocks

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Abstract. Portfolio optimization nowadays plays a significant role in financial industry. This paper aims to analyze the asset allocation of technology, financial services, and consumer defense industries. Ten representative companies were selected from these industries. At the same time, the difference between Markowitz model and Sharpe single index model is observed by using the three constraints of quantification of actual financial market factors. On this basis, this paper optimizes the asset portfolio. This result shows that: first, the correlation coefficient between SPX500 and listed companies is very high, which is a good choice in the portfolio of balancing risk and return. Second, we test several portfolios under 3 constraints, and it is found that a higher return always occurs with a high risk. Third, without the consideration of SP index, investors can get the highest return. The results in this paper may shed light on the financial investors.

Keywords: Return; risk; portfolio.

1. Introduction

Since American economist Markowitz proposed the mean variance model, portfolio theory has been a heated topic in the financial field. Based on Markowitz portfolio theory, investors can determine the least variance portfolio and derive the effective border by computing the average return and covariance matrix of each security in the portfolio [1]. Today, there are an increasing number of investors entering the stock market, and it is very necessary to provide investors with an effective and reasonable portfolio. At the same time, risk asset portfolio is a significant part of financial research. Besides, adding different constraints according to the requirements of different investors also makes portfolio management more effective.

After careful study of financial risk asset portfolio, this paper notices that most of the current industry research focuses on stock portfolio in different specific areas. More research has been done on the combination of different populations in a single area. For example, Ojiambo explored the portfolio optimization in the electricity market from the perspective of individual investors based on Markowitz mean-variance model and constructed the effective frontier based on Markowitz portfolio theory. Ojiambo selected the best portfolio based on investors' investment utility under different risk aversion degrees [2]. Moreover, Murthy constructed an optimal investment portfolio using Index Model. And Murthy conducted a study on nifty metal index [3]. Besides, Wang conducted research about the investment strategy of energy based on Markowitz mean-variance model and Wang selected 8 stocks in the technology industry, constructed the optimal portfolio based on Markowitz portfolio theory [4-5].

However, this paper aims to focus on the asset allocation of ten representative companies from three different fields: technology, financial services, and consumer defense. The specific research process is as follows. First, ten stocks were selected from the fields of science and technology, financial services and consumer defense for the closing price of nearly 20 years; Secondly, the monthly price is filtered from the initial data; Third, use Excel to calculate not only the annual average return and standard deviation, but also the residual standard deviation and the correlation between ten stocks of Sharpe single index model; Fourth, three different constraints are added to build its Markowitz model and Index model, including minimum variance portfolio, maximum Sharpe ratio portfolio, and capital distribution line.

The structure of this paper is listed below. The second section shows data, and the third section shows methods. The fourth section is the results, and the fifth section is the conclusion.

2. Data

Yahoo Finance is world-class in providing researchers with accurate and reliable industry information. Based on this information, all the data in the paper comes from Yahoo Finance (https://finance.yahoo.com/). To gain as more data as possible, the data range in this paper is from Nov 5,2001 to Dec 5,2021. Ten representative companies selected from three industries – technology, financial services, and consumer defensive -- are NVDA, GS, CSCO, USB, TD CN, INTC, ALL, CL, JNJ and PG. As shown in Table 1, this paper converts the date to annualized average return and annualized standard deviation. Overall, these ten stocks have performed stably in the past twenty years. Good investors in the market are always waiting and observing for the market to confirm their judgment about the price changes of stocks. In fact, the performance of these ten stocks in the market can affirm their own advantages. To facilitate the allocation and hedging of risk in this investment portfolio, the stocks selected in this paper are distributed in three fields with a large span.

Table 1. Descriptive statistics of the selected assets

	NVDA	CSCO	INTC	GS	USB
Annual Return	32.802%	9.714%	8.905%	10.825%	9.878%
Annual StDev	55.774%	30.809%	30.503%	29.572%	23.680%
residual StDev	47.405%	23.762%	24.889%	20.881%	18.781%
	TD CN	ALL	PG	JNJ	CL
Annual Return	11.010%	10.080%	9.437%	8.464%	7.105%
Annual StDev	18.134%	24.884%	14.587%	14.785%	15.350%
residual StDev	13.865%	19.317%	13.288%	12.423%	13.787%

As can be seen from Table 1, among these ten stocks, the highest average return belongs to NVDA, and it also has the highest variance. In contrast, CL has the lowest average return and a stable variance. And JNJ, on the other hand, has a good return with the lowest risk factor.

3. Method

Markowitz's portfolio (mean variance, efficient boundary) model employs the expected rate of return (mean) of both the risky and non-risky assets represented by variance (or standard deviation) to study investment portfolio. Theoretically, unsystematic risk can be avoided through investment portfolio. The connection between the market portfolio and non-risky assets is called the capital market line. Markowitz's portfolio theory has following assumptions: (1) Investors are rational, and their behavior is to maximize their investment within a limit budget. (2) Investors can correct their information about returns and risks [6]. Investments focus on two key elements: expected return and risk level. Consequently, market investors need to solve the question of how to quantify risk level and return on investments. To solve portfolio optimization problems, the Markowitz portfolio optimization model [7-10] is a good reference. Although a great number of theories and models have been developed and can be used in terms of asset allocation nowadays, Markowitz's theory can be regarded as the foundation of asset allocation. In this paper, the returns of assets are described as follows:

$$\vec{r} = \{r_1, r_2, \ldots, r_n\}^T. \quad (1)$$

In addition, the averages of the above asset's returns can be written as:

$$\vec{m} = \{\mu_1, \mu_2, \ldots, \mu_n\}. \quad (2)$$

when $\mu_i = \langle r_i \rangle$. If the set $\vec{w} = \{w_1, w_2, \dots, w_n\}$ is a set of unknown portfolio weights, then $R = \vec{r}^T\vec{w}$ is a portfolio return and $M = \vec{\mu}^T\vec{w}$ is its average return. Besides, when $\Sigma = \langle (\vec{r} - \vec{m})^T(\vec{r} - \vec{m}) \rangle$, the variance-covariance matrix of the assets returns, then the portfolio variance can be expressed in the following form:

$$Var = \vec{w}^T\Sigma\vec{w} \tag{3}$$

In terms of Markowitz mode, there are two significant methods: the mean-variance analysis method, and the portfolio efficient frontier model. Markowitz's portfolio theory has been widely employed in portfolio selection today. Markowitz focused on two significant factors: risk level and return. And after that, market investors pay more attention to these two factors risk and return when describing reasonable and achievable investment goals.

According to Markowitz, the return rate is defined as the expected value of the investment return, while the return risk is defined as the variance or standard deviation. In this way, the issue of measuring the risk of assets is solved. There is another import assumption is that Markowitz believes that typical market investors are risk averse and prefer low risky assets with high expected returns. In this way, Markowitz provides a set of portfolio theories based on the analysis of the mean variance of utility maximization.

In this paper, it use the following notations: $\vec{\mu} = \{\mu_1, \mu_2, \dots, \mu_n\}^T$is the set of assets' average returns; $\vec{w} = \{w_1, w_2, \dots, w_n\}$is the unknown set of assets' weights; $\vec{\sigma} = \{\sigma_1, \sigma_2, \dots, \sigma_n\}^T$is the set of assets' standard deviations; $\vec{\beta} = \{\beta_1, \beta_2, \dots, \beta_n\}^T$ is the set of assets' betas; $\{\sigma(\varepsilon_1), \sigma(\varepsilon_2), \dots, \sigma(\varepsilon_n)\}^T$ is the set of the residuals' standard deviations; $\vec{v} = \{w_1\sigma_1, w_2\sigma_2, \dots, w_n\sigma_n\}^T$is an auxiliary vector; and

$$\mathrm{P} = \begin{Bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{Bmatrix} \tag{4}$$

is the cross-correlation coefficients. This paper also uses the following formula for the MM portfolio return:

$$r_P = \vec{w} \cdot \vec{\mu}^T \tag{5}$$

And the formula for the MM portfolio standard deviation:

$$\sigma_P = \sqrt{\vec{v}P\vec{v}^T} \tag{6}$$

Like above, in the Index model, this paper uses formula for the IM portfolio return:

$$r_P = \vec{w} \cdot \vec{\mu}^T \tag{7}$$

And the formula for the IM portfolio standard deviation:

$$\sigma_P = \sqrt{(\sigma_M\beta_P)^2 + \sum_{i=1}^{n} w_i^2\sigma^2(\varepsilon_i)} \tag{8}$$

4. Results

This paper chose three constraints for portfolio optimization, and three constraints of are designed considering the variety of circumstances that may arise in the actual stock transaction. First, this paper reports the correlation between the assets, and the results are shown in the following table 2.

Table 2. Correlation coefficient between 10 stocks

	SPX	CSCO	NVDA	INTC	ALL	USB	TDCN	PG	GS	JNJ	CL
SPX	1	0.638	0.528	0.579	0.631	0.610	0.645	0.413	0.709	0.543	0.441
CSCO	0.638	1	0.488	0.615	0.298	0.329	0.411	0.221	0.488	0.240	0.166
NVDA	0.528	0.488	1	0.525	0.158	0.161	0.339	0.061	0.344	0.166	0.070
INTC	0.579	0.615	0.525	1	0.287	0.281	0.413	0.137	0.412	0.326	0.110
ALL	0.631	0.298	0.158	0.287	1	0.540	0.418	0.231	0.416	0.453	0.408
USB	0.610	0.329	0.161	0.281	0.540	1	0.540	0.335	0.471	0.235	0.217
TD CN	0.645	0.411	0.339	0.413	0.418	0.540	1	0.231	0.495	0.273	0.212
PG	0.413	0.221	0.061	0.137	0.231	0.335	0.231	1	0.173	0.494	0.484
GS	0.709	0.488	0.344	0.412	0.416	0.471	0.495	0.173	1	0.297	0.202
JNJ	0.543	0.240	0.166	0.326	0.453	0.235	0.273	0.494	0.297	1	0.527
CL	0.441	0.166	0.070	0.110	0.408	0.217	0.212	0.484	0.202	0.527	1

After calculating and observing the correlation coefficient of 10 stocks, this paper can see in the table 2 that these 10 stocks have positive correlations with each other, and their correlations are between 0 and 1. Apart from each stock with their own correlation of 1, among these 10 stocks, the highest correlation is between GS and SPX, and their correlation is 0.708, which is convenient to researchers and investors to conduct research in the same filed with the same interest relationship invest. And the lowest correlation is between PG and NVDA, and their correlation is only 0.060. In other words, if investors buy and want to short these two stocks at the same time, it can help them to reduce the overall risk in the portfolio and achieve risk hedging.

Case 1. This extra optimization restriction mimics the FINRA Regulation T (https://www.finra.org/rules-guidance/key-topics/margin-accounts), which permits brokers-dealers to permit their clients to hold positions that are at least 50% backed by their account equity. The results are shown in Table 3.

$$\sum_{i=1}^{11} |w_i| \leq 2 \tag{9}$$

Table 3. Markowitz model & Index Model under constraint 1

MM constraint1	Variance	Return
Min SD	10.95%	11.00%
Max Sharpe Ratio	30.00%	23.70%
IM constraint1	Variance	Return
Min SD	9.64%	10.00%
Max Sharpe Ratio	30.00%	22.41%

Case 2. This optimization constraint is made to emulate some arbitrary "box" weight constraints that the customer might supply. The results are shown in Table 4.

$$|w_i| \leq 1 \text{, for } \forall i \tag{10}$$

Table 4. Markowitz model & Index Model under constraint 2

MM constraint2	Variance	Return
Min SD	10.95	11%
Max Sharpe Ratio	32.55%	32.55%
IM constraint2	Variance	Return
Min SD	9.64%	11.00%
Max Sharpe Ratio	30.00%	29.4%

Case 3. Finally, in order to determine if the wide index's inclusion in the portfolio has a good or negative impact, this article would want to take into consideration a further optimization constraint, which means the weight of S&P 500 is equal to zero, and the results are shown in Table 5.

$$w_1 = 0 \tag{11}$$

Table 5. Markowitz model & Index Model under constraint 3

MM constraint3	Variance	Return
Min SD	11.18%	11.19%
Max Sharpe Ratio	80.00%	80.00%
IM constraint3	Variance	Return
Min SD	9.76%	9.63%
Max Sharpe Ratio	50.00%	50.00%

After calculating and observing the performance of Markowitz model and Index model respectively, this paper has some observations. First, the result from Markowitz model and Index model are very close. Second, the result using Index model are slightly higher than that of Markowitz model. And the explanations are that macro environment factor does not affect the result at a high level, instead, the firm-specific factors influence the result more. However, macro factors still have some influences on the result. Overall, these 10 stocks from three fields are good and worth investors investing their money in.

5. Conclusion

Today, most portfolio research is based on analysis of general market conditions or specific industries. This paper aims to conduct portfolio analysis on various fields such as science and technology, financial services, and consumer defensive to guide investors to make wise investments. This paper uses Markowitz model and Index model to find the optimal portfolio of these stocks, and in this process, its advantages and disadvantages are obvious. At the same time, three constraints with practical significance are added to the two models to understand how factors in the real financial market affect the portfolio. Different models under the same constraint and different constraints on the same model are analyzed. Under most constraints, the performance of the index model on the selected stocks is better than that of the Markowitz model.

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