

A Two-Dimensional Adaptive GGDP-Accounting Model

Junkai Chen *

School of Statistics, University of International Business and Economics, Beijing, China, 100000

* Corresponding author: chenjunkai6666@163.com

Abstract. The inadequacy of GDP to reflect the depletion of natural resources and the environmental impact of economic activities has raised the necessity to replace it with GGDP. This study aims to develop a comprehensive GGDP metric that incorporates diverse indicators from multiple dimensions and can adapt to changes in indicators over time. Specifically, the study develops a Two-Dimensional Adaptive GGDP-Accounting Model (2-DAG Model). The model involves selecting four aspects, namely Economic Development (ED), Environmental Cost (EC), Technological Progress (TP), and Societal Environmental Protection (SEP). Each aspect is associated with corresponding indicators. ED, EC, and TP are categorized under the Dimension-X, while SEP is placed in the Dimension-Y. The Entropy Weight Method (EWM) is utilized to allocate weights to the indicators within each aspect. The logistic equation is employed to describe the coefficient- α in the dimension-x, enabling the construction of the two dimensions. By combining the two dimensions, the final Gross Green Domestic Product (GGDP) value is determined.

Keywords: Green GDP, 2-DAG Model, EWM, Logistic Equation.

1. Introduction

Green GDP (GGDP) is an economic metric that incorporates the environmental expenses of economic growth and development, offering a more accurate representation of economic health than conventional GDP calculations. [1] Material Flow Analysis (MFA) and the Ecological Footprint (EF) framework are effective methods for calculating GGDP and evaluating the environmental impact of human activities.[2-4] Various indices, such as the Happy Planet Index (HPI) and the Genuine Progress Indicator (GPI), extend beyond economic growth to consider social and environmental dimensions. [5-6]The above approach to calculating green GDP is merely a fixed calculation within the same dimension, lacking the integration of multiple indicators from different dimensions and adaptability. Therefore, a model that combines multiple indicators, and has adaptability by adjusting coefficient proportions based on indicator values, is necessary.

2. Construction of 2-DAG Model

2.1. Determination of Four Aspects

Based on the review of existing literature and reference, we make appropriate adjustment to it and finally choose four aspects, as the Table.1 shows.[7]

Table 1. Four aspects of 2-DAG Model

Aspect	Abbreviation	Index
Economic Development	ED	EDI
Environmental Cost	EC	ECI
Technological Progress	TP	TPI
Societal Environmental Protection	SEP	SEI

● Economic Development (ED)

GGDP is an extension of GDP. At the same time, the construction of GGDP in this paper is also based on GDP. Therefore, GDP is adopted to measure economic development.

For Economic Development (ED), we adopt a basket of economic indicators borrowed from the GDP accounting methodology. At the same time, for the follow-up accounting, we made normalization processing when assigning weight, limiting the value of EDI between 0 and 1:

$$EDI = \sum_i^n \theta_i \times GDP(i) \quad (1)$$

Where $GDP(i)$ is the i -th indicator of GDP-calculation, θ_i is its normalized weight.

● Environmental Cost (EC)

Green GDP should incorporate environmental costs for a comprehensive and accurate measurement of economic development, which emphasizes recognizing economic costs of environmental degradation and resource depletion, ignored in traditional GDP measurements.

By selecting relevant indicators from the content of EC and assigning certain weights to them, the corresponding calculated score for EC is ECI:

$$ECI = ECI_1 + ECI_2 \quad (2)$$

Where ECI_1 is environmental-pollution level score and ECI_2 is resource depletion level score.

$$\begin{aligned} ECI_1 &= \sum_{i=1}^n \lambda_i \times E(i) \\ ECI_2 &= \sum_{k=1}^m \lambda_k \times E(k) \\ \sum_{i=1}^n \lambda_i + \sum_{k=1}^m \lambda_k &= 1 \end{aligned} \quad (3)$$

Where λ is the weight, E is the normalized value of the selected index.

The two levels of indicators we find to calculate EC are in Table.2, which can be described by several indexes.

Table 2. Indicators of environmental cost

First-level Indicator	Second-level Indicator	Abbreviation	Unit
Environmental Pollution	Air Pollution Index	API	/
	Annual Emission of CO_2	AEC	m^3
	Annual Discharge of Industrial Wastewater	ADIW	m^3
	Annual Discharge of Industrial Solid Waste	ASW	t
Resource Consumption	Annual Amount of Deforestation	AAD	m^2
	Annual Coal Consumption	ACC	t
	Annual Oil Consumption	AOC	t
	Annual Gas Consumption	AGC	m^3
	Biodiversity Loss	H	/
	Difference of Land Desertification Degree	DLD	%

For biodiversity, we use Shannon-Wiener index (H) to measure. It provides a single value that represents the diversity of species present in the habitat, with higher values indicating greater diversity. [8] The formula for the Shannon-Wiener index is:

$$H = -\sum (p_i) * \ln(p_i) \quad (4)$$

Where p_i is the proportion of individuals of a given species in the habitat.

● Technological Progress (TP)

Capacity for innovation and human capital are key measures of technological progress, which in turn influence economic and societal outcomes. We calculate TPI in the similar way to CPI, as the Table.3 shows.

Table 3. Indicators of technological progress

First-level Indicator	Second-level Indicator	Abbreviation	Unit
Innovation Capacity	Number of New Patents	NNP	/
	Research and Development Spending	RDS	%
	Number of Scientific Publications	NSP	/
Human Capital	Proportion of Higher Education	PHE	%
	Literacy Rate	LR	%

● Societal Environmental Protection (SEP)

Societal Environmental Protection can be divided into two levels public and the government The two levels of indicators we found to calculate SE are in Table.4.

Table 4. Indicators of Societal Environmental Protection

First-level Indicator	Second-Level Indicator	Abbreviation	Unit
Public	Environmental Democracy Index	EDI	/
	Number of New Environmental Organizations	NNEO	/
	Number of New Environmental Companies	NNEC	/
Government	Number of Environmental Regulations	NER	/
	Environmental Law Index	ELI	/

Among them, the Environmental Democracy Index (EDI) includes a sub-index on public participation, which measures the extent to which countries provide opportunities for public participation in environmental decision-making. In the most recent edition, the top three countries for public participation were Uruguay, Costa Rica, and Norway.

2.2. The Dimension-X

Upon analyzing the four indexes, it is apparent that ED, EC, and TP belong to the same index level while SE shows no direct impact on former indexes. Therefore, we synthesize the first three into one dimension and separate the last one into the other.

EDI is selected as the positive index, TPI is the corresponding positive index, and ECI is the negative index. Then the value X on Dimension-X can be calculated as follows:

$$X = \frac{1}{\beta_1} \times (\alpha_1 \times EDI - \alpha_2 \times ECI + \alpha_3 \times TPI), \quad \beta_1 = 3\sqrt{2} \quad (5)$$

Where α_i (i =1,2,3) is coefficient of every index for Dimension-X and β is normalization coefficient of Dimension-X.

2.3. The Dimension-Y

If EDI in Dimension-X is a positive index, index SEI in Dimension-Y is still a positive index, then the value Y on Dimension-Y can be calculated as follows:

$$Y = \frac{\alpha_4}{\beta_2} \times SEI, \quad \beta_2 = \sqrt{2} \quad (6)$$

Where α_4 is the coefficient of index for Dimension-Y and β_2 is normalization coefficient of Dimension-Y.

3. Calculation of 2-DAG Model

3.1. Weights Calculation for Four Aspects Based on EWM

The Entropy Weight Method (EWM) is commonly used as a weighting method that measures value dispersion in decision-making. It assumes that the greater the degree of dispersion, the greater the degree of differentiation, and more information can be derived. Thus, higher weight should be given to the indicator. We use the EWM method to calculate the weight of indicators above.

In each dimension, for the i 'th country and its j 'th index, the weight of it, f_{ij} is calculated as the following.

$$f_{ij} = \frac{r_{ij}}{\sum_{i=1}^m r_{ij}} \quad (7)$$

Where $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$

$$e_j = -\ln\left(\frac{1}{n}\right) \sum_{i=1}^m \ln(f_{ij}) \quad (8)$$

Here, e_j means that the greater the differentiation degree of indicator j is, the higher weight should be given to the indicator. Therefore, the weight ω_j of indicator j is calculated as follows.

$$\omega_j = \frac{1 - e_j}{m - \sum_{i=1}^m e_j} \quad (9)$$

Then, we get the weight value vector of each dimension. The comprehensive performance of sample country j by considering the total n indicators can be obtained as follows:

$$S_i = \sum_{j=1}^n \omega_j \cdot f_{ij} \quad (10)$$

The internal weight calculation of different layers is shown in the Table.5

Table 5. Weights of indicators for four aspects

Aspect	First-level indicator	Second-Level Indicator	Weight
EC	Resource Consumption	DLD	4%
		H	7%
		AGC	10%
		AOC	9%
		ACC	7%
		AAD	13%
	Environmental Pollution	API	13%
		AEC	20%
		ADIW	10%
		ASW	7%
TP	Human Capital	LR	19%
		PHE	14%
	Innovation	NSP	15%
		RDS	19%
		NNP	33%
SEP	Government	NER	13%
		ELI	22%
	Public	EDI	22%
		NNEO	13%
		EEEC	30%

3.2. Determination of Coefficient- α : Logistic Equation Fitting

We want to fit the change of coefficient- α occurring with the change of economic size, and consider the adjusted value of GDP as the independent variable. Considering that it was between 0 and 1 and presented nonlinear changes, we adopted Logistic Equation fitting based on empirical research.[9] Since EDI itself is weighted and describes the proportion of economic development in GGDP, the determination of it should be measured by normalization operation after the determination.

$$\sum_{i=1}^4 \alpha_i^2 = 1 \quad (11)$$

The logistic equation is a non-linear differential equation that can be described as follows.

$$\frac{dx}{dt} = kx(\partial - x) \quad (12)$$

We can obtain the general solution of the equation by using the method of separating variables and performing bilateral integration.

$$x(t) = \frac{\partial A e^{bt}}{A e^{bt} + 1} = \frac{\partial}{1 + B e^{-bt}} \quad (13)$$

Where B, b, ∂ are all constants determined by initial conditions.

After that, we assign values to specific parameters based on the Group Decision Method. [10]

$$\alpha_i(t) = \frac{\partial}{1 + B e^{-bt}} \quad (i = 2, 3, 4) \quad (14)$$

Where α_i is the value of the i-th coefficient- α and t is adjusted value of GDP for certain nation.

We get input from multiple experts with different perspectives and areas of specialization. The results can be shown as Figure1.

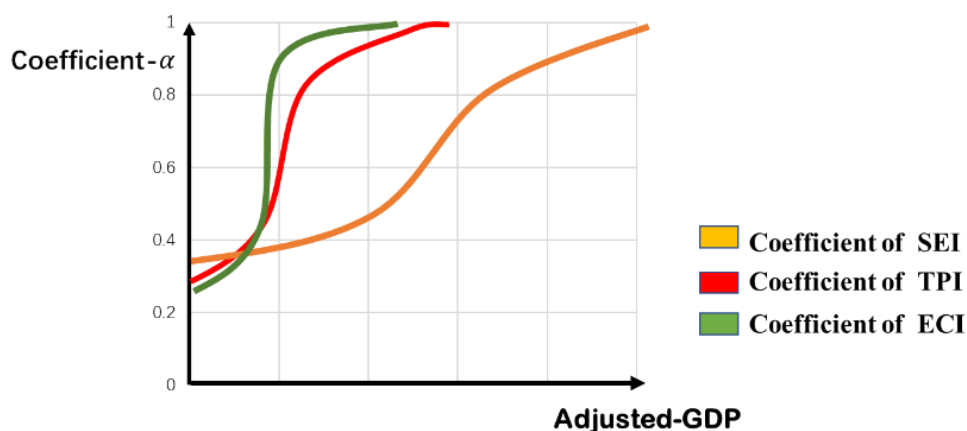


Figure 1. Coefficient function

It can be seen that the coefficient of SEI changes the slowest with the growth of GDP, while the coefficient of ECI changes the fastest. Among them, the coefficient changing relationship between ECI and TPI is similar.

3.3. GGDP Calculation Formula

We limit the value of i (i= 1,2,3,4) to between 0 and 1, where the specific calculation method will be given later. Therefore, we can define the value of GGDP as follows.

$$GGDP = \sqrt{X^2 + Y^2} \quad (15)$$

4. Conclusions

4.1. Evaluation of 2-DAG Model

The Two-Dimensional Adaptive GGDP-Accounting Model serves as a tool to facilitate the calculation of Green GDP, which is deemed a more comprehensive indicator that takes into account the expenses associated with environmental degradation, technological advancement, and societal factors. The model encompasses a broad spectrum of indicators from various dimensions and is capable of accommodating changes in indicators over time. By utilizing four aspects and two dimensions, the proposed model provides a more comprehensive evaluation of a country's economy, rather than a narrow focus on economic development. The model's strong predictability and robustness render it practically valuable in application.

4.2. Application of 2-DAG Model

Upon constructing the model metrics and quantitatively solving for the coefficients within the model, a 2-DAG model is obtained. Subsequently, this methodology will be employed to compute the Green Gross Domestic Product (GDP) for multiple countries. Since the model includes environmental cost and technological progress of a country, as well as some social factors into the accounting scope of GGDP. Therefore, the GGDP ranking among countries may be very different from the GDP ranking.

As an example, let's consider Brazil and Poland. When GDP ranking is adopted, Brazil ranks 12th in the world and scored 0.82 in EDI, slightly higher than Finland (EDI score 0.76), which ranks 43rd. However, after GGDP ranking is adopted, Brazil's GGDP score is 0.391 and Finland's is 0.77, showing a huge difference in GGDP levels between the two countries. This reflects the huge gap in

environmental costs and scientific and technological level behind the economic development of the two countries, which can be shown as Figure2.

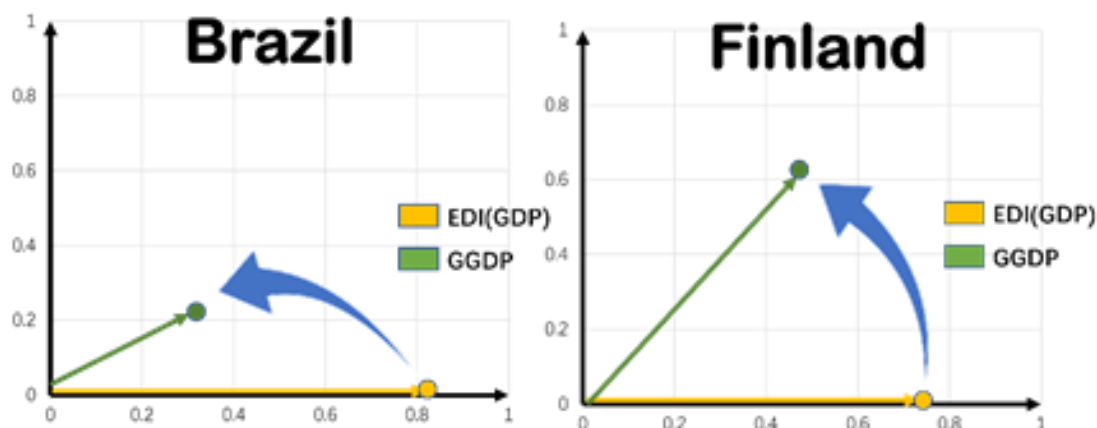


Figure 2. From GDP to GGDP

The aforementioned alterations distinctly manifest that the utilization of the GGDP accounting approach enhances the comprehensive and scientific assessment of a nation's economic status. Consequently, the application of the 2-DAG model can aid in promoting the virtuous adjustment of a country's economy, rendering it highly significant.

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