Pricing Technique for European Option and Application

Zeyuan Shao*

School of Mathematics and Physics, Xi'an Jiaotong-Liverpool University, China

*Corresponding Author Email: zeyuan.shao@student.xjtlu.edu

Abstract. In financial mathematics, the pricing technique for derivatives is constantly debated. In this paper, the pricing technique of the European Option is mainly discussed, and the binomial tree (BN) model is first applied to the pricing process of European options. The previous results show that carbon credit index can be traded as an option, and BN model can correctly simulate the future price of call option constructed by consuming the carbon credit index. Secondly, the Black-Scholes (BN) model is also a crucial technique for pricing European options, and it is successfully applied to predicting the future three months' CSI 300 index option price. Finally, BN model is compared with BS model, and the result reflects that BN model can perform as well as BS model for pricing European Option When the step reaches 2000. However, the efficiency of the BN model is stable under low volatility. Under higher volatility, such as 1.5 sigmas, the required steps will increase to achieve the same accuracy level. For American options, the BN simulator of a put option is close to the actual value, but the call option simulator will fluctuate. For the stock-pricing process, both models estimate far above Monte-Carlo method. The result of this paper is to provide some clues for pricing European options with different methods.

Keywords: Black-Scholes model; Binomial tree model; European option; Pricing technique.

1. Introduction

1.1. Background

The option is a significant derivative traded among markets and has experienced emerging growth since the establishment of Chicago board options exchange (CBOE) in 1973[1]. As an essential risk management tool, options are widely applied by different characters of market participants. For example, corporate can apply options to protect the company from foreign exchange rate loss, and consumers can apply it to hedge risks. As options have different features, such as different rights to exercise and diversified execution times, there should be related formula to calculate their value. For the option price is a key factor of investment strategy, how to price options properly has always been an essential concern for economists, option providers, and consumers. The Black-Scholes formula is one of the most significant achievements of option pricing theory, known for its simplicity and flexibility. This model has been the coral of European option pricing. Hence, later developments are mainly solving the empirical test of stock price by applying a stochastic process. Although the BS model has solved the European option pricing problem, there are still some limitations with only one theorem. Choosing the right model suitable for different options is critical for simulation accuracy. Hence, this paper will introduce different techniques for pricing options.

1.2. Related research

A survey by He has shown that the Black-Scholes model and binomial tree have the same tendency when pricing European Option. However, the accuracy of the binomial tree is influenced by the volatility of model. According to the simulator result, binomial tree can correctly reflect the change in the American options price. However, both models cannot precisely reflect the change in stock price compared with the Monte-Carlo simulator [1]. Over the past decades, options' pricing technique has significantly advanced after the proposal of the Black-Scholes model. Some research has postulated optimized pricing techniques based on that model [2]. The studies indicate that the Black-Scholes model can reflect the CSI 300 index option price change in the future, and introduction of parameters will help the model reduce errors in simulation [3].

The evidence presented in the former research suggested that the simulator made by the binomial tree model will converge after reaching two thousand steps, and the volatility will influence the stability of estimator. Specifically, the estimator made under lower volatility will be more stable, while higher volatility will increase the required step for the binomial tree model to achieve the same confidence degree [4]. A previous study has established that the carbon credit index can be traded as an Option in the global market. It is straightforward to deduce the future price based on the pricing technique of the binomial tree model [5].

1.3. Objective

In next section, this research will introducing some practical technique for option-pricing by firstly listing some assumption simplify the requirement for application. Then there will be a brief derivation process of model's formula, and there will be a example help demonstrate how to apply those technique into practical cases. Finally, a comparison between model will analysis model's efficiency when estimating different options, and determine some problems when models fail to meet some underlying requirements.

2. Method

2.1. Binomial Tree Model

Cox and Ross first proposed the binomial tree model in their report 'A Survey of Some New Results in Financial Options Pricing Theory. This model is known for its simplicity and intuitive calculation process. The binomial tree model is based on a discrete time interval, and it is also suitable for pricing the American option, which can be executed before maturity. Since this model can reduce the interval into a smaller range, it can handle with complicated model [2].

2.1.1. Basic Assumptions

(1) Only two possible results will occur when the time moves forward; the option price will go up or down.
(2) There is no trading cost in investment.
(3) Investors are receptors of price.
(4) Allow lending at a risk-free rate.

2.1.2 Binomial tree model pricing formula

Firstly, the option price will go to u times and d times of the previous stage, $s \times u$ and $s \times d$. The extent of price change depends on the volatility σ and time t.

$$u = e^{\sigma\sqrt{t}} \tag{1}$$

$$d = e^{-\sigma\sqrt{t}} = \frac{1}{u} \tag{2}$$

Secondly, determine the option's value at each final node. The option's intrinsic value, or execution value, is the price at the option's expiration date.

$$S_n = S_0 \times u^{Nu - Nd} \tag{3}$$

For call option: $\mathrm{Max}[(S_n - K, 0)]$
For put option: $\mathrm{Max}[(K - S_n, 0)]$
N_u is the number that option price rise, and N_d is the number price fall?

Thirdly, the actual price of one-day derivative equals its risk-free profit based on risk-neutral assumption. The expecting option price can be calculated by the expectation of discounted future earnings.

$$c_{t-\Delta t,i} = e^{-r\Delta t}[p \times C_{t,i+1} + (1-p) \times C_{t,i-1}] \tag{4}$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d} \tag{5}$$

2.1.3 Applications based on the binomial tree model

In the carbon finance market, traders are unfair with the asymmetry of the rights and obligations, for buyers are obligated to pay the premium, and their most significant loss is the premium. At the same time, sellers should fulfil the systematic obligation [3]. In this case, carbon emission caps can also be used as important financial assets for carbon, which helped the market's emergence of financial derivatives, including options, futures, equities, and others. However, the price of one ton of carbon dioxide equivalent cannot be uniform across markets because there are still differences between transactions. As a result, there exists the exchange rate conversion ratio of the carbon price between different markets. Hence, trading carbon credits based on the market index can be compared to trading currencies using options [4].

Supposing a company participating in EU ETS purchases X unit of carbon credit index call option from CCX with strike price K to prevent possible loss caused by the rising carbon price.

Assume CCX, a carbon index indicator's risk-free return is q, then ICCX changed to $1 \times (1 + q \times t)$ CCX during the transition period and define $\eta = (1 + q * t)$. In the binomial tree model, the CCX index changes during the period $[t, t + t]$ as below:

$$Ct(EU\ ETS) \left\langle \begin{matrix} Ctu\eta(EU\ ETS) \\ Ctd\eta(EU\ ETS) \end{matrix} \right. \tag{6}$$

According to the binomial tree pricing theory for European option, assuming the initial original asset price is $C = C_0$, then the possible premium price at t=T can be expressed as:

$$V_T = X(C_T - K) += X(C_0 u^{N-\alpha} d^{\alpha - K}) +), (0 \leqq \alpha \leqq N) \tag{7}$$

2.2. Black-Scholes Model

In 1973, Fisher Black and Myron Scholes firstly introduced the Black-Scholes model. They found that CAPM is not necessarily used in option pricing. In this model, Black and Scholes used geometric Brownian motion to simulate asset pricing change [5]. Compared with the Binomial-tree model, this model applies continuous variables instead of discrete ones. Therefore, these two models can be implemented when pricing a standard European option.

2.2.1 Basic assumption

(1) The risk-free rate and volatility of underlying assets are known as constant
(2) The market cannot be predicted, and there are no transaction costs for buying options.
(3) The Option should be European and only exercised at maturity.
(4) Assuming the asset price as S_t, it satisfies the Stochastic differential function:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{8}$$

Expected return rate μ and volatility σ are constant, W_t is a standard Brownian motion.

2.2.2 Black-Scholes pricing formula

The European call option's price is:

$$C(S_t, t) = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \tag{9}$$

$$d_1 = \frac{\ln(S/X) + [r + (\frac{\sigma^2}{2})](T-t)}{\sigma\sqrt{T-t}} \tag{10}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{11}$$

X is the exercise price?
The European put option's price is:

$$P(S_t, t) = Xe^{-r(T-t)} N(-d_2) - S_t N(-d_1) \tag{12}$$

2.2.3 The application based on the Black-Scholes model

Since the Black-Scholes model is a classical pricing technique for European options, a typical application is to simulate or predict a particular price, such as a well-known Chinese CSI 300 index option. Due to the research carried out by Li [6], the actual market cannot comply with the assumptions that the BS model makes, and the main difficulty is estimating volatility according to the endless variety of markets. Therefore, researchers take the data from the past three months and calculate the σ and μ based on the log-return r_t calculated by (1).

$$r_t = \ln\frac{P_t}{P_{t-1}} \tag{13}$$

$$r = \ln\left(\frac{1}{T}\sum_{i=1}^{T} r_i + 1\right) \tag{14}$$

$$\mu = \frac{1}{T}\sum_{t=1}^{T} r_t \tag{15}$$

$$\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)^2 \tag{16}$$

Where P_t is the day price, and P_{t-1} is the last day price. Then researchers simulate the expected risk-free return rate r from the recent 3-month given risk-free rate by (2). After getting all parameters, applying the Black-Scholes formula to calculate the option's value is straightforward and helpful to investors who are wise to manage risk.

3. Comparison

The binomial tree and Black-Scholes models can correctly reflect European call and put option pricing curves. Specifically, the Binomial tree model's performance will be as well as the Black-Scholes model when the steps reach 2000. However, for two reasons, Black-Scholes is more suitable for pricing the European option than a binomial tree. Firstly, the volatility of the model influences the binomial tree's accuracy. The estimation made by the binomial tree model will be more stable for a model under 0.5 sigmas than for a model of 1.5 sigmas. Secondly, the simulation steps to achieve the same accuracy will also be affected by volatility. More giant steps are required to achieve the same accuracy for the higher volatility model, which works for both call and put options [7]. The advantage of applying the binomial tree model can be shown in pricing American option, which can be early executed ahead of the maturity date, because all possible option prices will be shown at nodes of the binomial tree. However, the Black-Scholes model can hardly support the behaviour of early exercise. After all, one point that should be conscious of is that the performance of the Binomial tree model for the pricing call and put option is different. Specifically, the put option price is close to the actual value, but the call option price could fluctuate [8].

As for the pricing of stocks, the Black-Scholes model and binomial tree model have been reported to achieve a higher estimation than the Monte-Carlo estimation, possibly caused by the stock price's random jump and discontinuous distribution [9].

4. Summary

In this paper, the binomial tree model and Black-Scholes model are introduced for pricing the European Option. The binomial tree model helps list all possible future carbon credits and straightforwardly calculates the required date price by calculating the expectation. Then, the Black-Scholes model can help calculate the future three months CSI 300 index option after getting all required parameters. Finally, the results show that the BN model can perform just as well as the BS model for pricing European options. Although the efficiency of the BN model is steady under low volatility, the step approached 2000. More steps will be needed to maintain the same degree of accuracy under higher volatility, such as 1.5 sigmas. The BN put option simulator for American options is accurate to real value, although the BN call option simulator will fluctuate. Both models estimate inaccurately for the stock pricing process.

Furthermore, these theorems can also be applied to various option pricing, such as future, currency, and interest rate options. In addition, the pricing of options is also vital for handling corporate risk management strategy and evaluating insurance contract value [10]. With the further development of the Chinese market system, the application of options will be more frequent than past. The pricing theorem will be helpful for underlying investors, risk managers, and company executives to draw up a suitable investment strategy and avoid potential loss caused by market price change.

References

[1] Arnold, G. and Lewis, D.S. (2019) Corporate financial management. Sixth edition. Pearson. Available at: https://search-ebscohost-com.ez.xjtlu.edu.cn/login.aspx?direct=true&db=cat01010a&AN=xjtlu.0001617527&site=eds-live&scope=site (Accessed: 9 April 2023)

[2] Cox, J.C. and Ross, S.A. (1976) 'A Survey of some New Results in Financial Option Pricing Theory', Journal of Finance (Wiley-Blackwell), 31(2), pp. 383–402. doi:10.1111/j.1540-6261. 1976.tb01893. x.

[3] Korn, R. and Müller, S. (2010) Binomial Trees in Option Pricing—History, Practical Applications and Recent Developments. Heidelberg: Physica-Verlag HD. doi:10.1007/978-3-7908-2598-5_3.

[4] Ning Zhe, Wang Yan and Huang Yingli (2011) 'European Option Binomial Tree Method in the Carbon Trading Exchange on the Condition "Carbon Finance"', 2011 Asia-Pacific Power and Energy Engineering Conference, Power and Energy Engineering Conference (APPEEC), 2011 Asia-Pacific, pp. 1–4. doi:10.1109/APPEEC.2011.5749040.

[5] Black, F. and Scholes, M. (1973) 'The Pricing of Options and Corporate Liabilities', Journal of Political Economy, 81(3), pp. 637–654. Available at: https://search-ebscohost-com.ez.xjtlu.edu.cn/login.aspx?direct=true&db=edsjsr&AN=edsjsr.1831029&site=eds-live&scope=site (Accessed: 22 March 2023).

[6] Li, L. (2019) 'The simulation of CSI 300 Index Option Pricing Based on Black-Scholes Model', 2019 4th IEEE International Conference on Cybernetics (Cybconf), Cybernetics (Cybconf), 2019 4th IEEE International Conference on, pp. 1–5. doi:10.1109/Cybconf47073.2019.9436608.

[7] He Yi-ruo (2021) Comparison of Black-Scholes and Binomial Pricing Model on European Option', Journal of Inner Mongolia University for Nationalities (Social Sciences), 47(5), pp. 88–94. doi: 10.3969/j.issn.1671-0215.2021.05.013.

[8] Bendob, A. and Bentouir, N. (2019) 'Options Pricing by Monte Carlo Simulation, Binomial Tree and BMS Model: a comparative study of Nifty50 options index', Journal of Banking & Financial Economics, (1), pp. 79–95. doi: 10.7172/2353-6845.jbfe.2019.1.4.

[9] Srivastava, A. and Shastri, M. (2023) 'A Study of Black–Scholes Model's Applicability in Indian Capital Markets', Paradigm, (Preprints), pp. 73–92. doi:10.1177/0971890720914102.

[10] Bodie, Z., Kane, A. and Marcus, A.J. (2010) Essentials of investments. 8th ed. McGraw-Hill Irwin (McGraw-Hill/Irwin series in finance, insurance, and real estate marketplace). Available at: https://search-ebscohost-com.ez.xjtlu.edu.cn/login.aspx?direct=true&db=cat01010a&AN=xjtlu.0000178084&site=eds-live&scope=site (Accessed: 9 April 2023).