

The Influence and Application of Financial Mathematics in Contemporary Financial Markets

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Abstract. The change of times has promoted the rapid development of the economy, and also stimulated the prosperity of the market. As an important component of the economic market, the continuous improvement of the financial economy requires the support of theoretical knowledge in the financial field. As an important theoretical part of financial economics, financial mathematics, relying on its computer and mathematical advantages, conducts a deeper exploration of securities and market equilibrium in the financial market. After decades of research, it has achieved fruitful results, providing strong data support for promoting the normal operation of financial institutions. Starting from the concept and related theoretical composition of financial mathematics, this article mainly analyzes the impact of financial mathematics on contemporary financial markets, and expounds the application of some core theories of financial mathematics at the market level. The article also briefly explains the current situation of financial markets from the perspectives of theoretical innovation and application issues, and describes the prospects of financial mathematics.

Keywords: Financial mathematics, modern financial market, impact and application analysis, mathematical model, theoretical innovation.

1. Introduction

Financial mathematics, also known as analytical finance, mathematical finance, emerged in the late 1980s and early 1990s. It is a new interdisciplinary discipline that gradually adapts to market development and combines mathematics and finance. Financial mathematics is based on finance theory, mainly using modern mathematical theories and methods to analyze modern financial markets. By taking market uncertainty and instability factors as the basis, it constructs mathematical models related to finance and economy to play an auxiliary role in financial market analysis and calculation, thereby achieving the goal of scientifically and effectively solving problems in the financial economy [1].

Financial mathematics is a collection of theoretical and qualitative research, emphasizing the concepts of arbitrage, optimization, and equilibrium. Its core content is the study of portfolios, optimal choices, and asset pricing theory in a random environment [2]. After decades of development, scholars and experts have developed and provided a series of theoretical foundations for selecting the optimal investment portfolio and asset pricing for the financial market. Among them, theoretical achievements that still have significant impact today include: efficient market theory, securities portfolio model, capital asset pricing model, arbitrage pricing model, option pricing model, and so on. These financial economic models are widely used in contemporary financial markets, occupying a central position in modern financial mathematical theory, and have very important practical significance.

Through a specific analysis of the application measures of financial mathematical models in the financial market and their important impact on the development of the financial market, we will understand the unique charm of financial mathematics in the field of financial analysis, and expect it to promote the orderly development of the financial market.

2. Analysis of the Impact of Financial Mathematics on Modern Financial Markets

2.1. Strengthen the Accuracy of Financial Services

In the contemporary financial market, the accuracy and reliability of financial services directly affect their development quality level. Due to the existence of asymmetric information such as moral hazard and adverse selection, financial markets cannot fully, accurately and effectively perform their functions of allocating resources efficiently and reasonably, resulting in inefficient allocation of financial resources and unable to effectively promote high-quality economic development. In order to effectively monitor the risks of the financial system, reduce investment risks, improve returns, and ensure stability, financial mathematics emerged as the times require [3]. Through specific analysis of the industry's development situation, using some efficient and convenient mathematical models, we can conduct a more scientific assessment of investment risks and formulate more scientific options prices. Financial mathematics comprehensively promotes the optimization, improvement, and development of financial markets, laying a solid foundation for the analysis, prediction, and monitoring of financial markets in the international financial field.

Quantitative data analysis results and qualitative market analysis theoretical research not only help the government understand the development status and potential problems of the industry in a timely manner, thereby better formulating and adjusting industry policies, but also promote enterprises to have a clearer understanding of their own development level in the industry, understand their own shortcomings, and make corresponding improvement measures. Individuals can also more accurately grasp the development trend of the market through financial mathematics, and obtain and withdraw profit opportunities that may exist in specific financial markets. With the improvement of science and technology, the upgrading and optimization of modern management theories, precise modeling of large databases has become a reality, and the accuracy of financial mathematics has been further amplified. As a result, technical personnel have fully grasped more accurate and authentic information, accurately improving the service level of the financial market and promoting the beneficial development of academic research [4, 5].

2.2. Promote the Overall Development of the Industry

The rapid development of financial mathematics not only enriches and perfects relevant financial theories, but also cultivates and builds a large number of composite high-quality talents in this process, laying a solid foundation for the orderly development of the financial market [4, 5]. A large number of talents are constantly pouring into the market. They not only have sufficient rational thinking and meticulous reasoning logic, but also have strong financial quantitative analysis and application capabilities. They can also propose their own unique perspectives and solutions to corresponding industry issues on the development of the financial market, and have high professional ethics, attitudes, and other financial literacy. Compared to traditional financial economics talents, this type of composite talents, relying on their superb data analysis capabilities, have become popular in the financial market, promoting talent transformation in the financial market, leading to the overall improvement of the quality of financial market talents, laying a good foundation for the development of the financial market, and providing a solid foundation for the progress of the financial industry.

Financial mathematics also presents us with an obvious conclusion based on its strict assumptions, careful logical derivation, and clear and straightforward data formula results, which guides us to adjust resource allocation plans and improve the efficiency of resource allocation in financial services. Firstly, financial mathematics promotes industry innovation, which in turn promotes the development of financial mathematics. For example, the B-S model obtains the pricing formula for options, promoting the innovation and replacement of financial derivatives, which further develops financial mathematics, prompting financial mathematics scholars to invest in research and achieve more theoretical results in financial mathematics. The continuous innovation of such financial products has given new impetus to the growth and development of the industry, driving more stock resources to

flow from low-income industries to high-income sectors, accelerating the optimization of industry structure and the conversion of old and new drivers, and improving the efficiency of resource allocation [6]. Secondly, the theoretical results of financial mathematics have enabled us to find a balance between reducing costs and increasing returns. In the modern financial market, every transaction is often accompanied by huge risk returns and transaction costs. How to operate to fully hedge risks and achieve good returns is a closely watched issue for every trader. Financial mathematics undoubtedly solves this problem for us. His data derivation results show us all possible risks and returns, while we only need to choose the investment portfolio we are willing to invest based on personal preferences. In this way, each investment is allocated in a market-oriented manner, improving the waste of industry resources, and improving the efficiency of resource allocation.

Financial mathematics provides scientific basis for important economic decisions, guides economists to fully understand global economic development trends, effectively guides the direction of national economic construction, guides the development direction of the industry, and promotes remarkable progress in the financial market [7]. Financial mathematical theory is based on modern mathematical theories and methods. It can deeply explore the root causes behind problems based on their appearance, thereby solving problems at the root level. Therefore, financial mathematics scientifically provides theoretical guidance for financial market management, guiding its development towards a healthy and orderly direction.

2.3. Enrich the Theoretical Foundation of Financial Markets

As mentioned in the previous theoretical component section, there is currently a relatively large theoretical component of financial mathematics, which provides rich theoretical support for the financial market and can effectively improve the practical operation level of the financial market [4, 5]. With the orderly development of economic sectors such as the stock, securities, and insurance sectors, the original model can no longer explain existing market conditions, which has prompted economists to continuously innovate theories and further enrich and improve the theoretical basis of financial markets. For example, martingale theory, optimal stopping time theory, theory of stochastic optimal control types, differential game theory, and other intelligent and empirical theories have created modern mathematical models by integrating financial mathematics, financial markets, and modern information technology, thereby optimizing economic decision-making and ensuring the scientificity and rationality of decision-making.

3. Application Measures of Financial Mathematics in Modern Financial Market

3.1. Efficient Market Theory

The efficient market hypothesis refers to the fact that in the securities market, the price of a security can fully reflect all available information, always equal to its investment value, and no one can continuously obtain excess profits. According to different types of information (historical information, public information, and internal information), securities prices also exhibit different changes, and three different types of information reflection efficiency markets are defined: weak efficient markets, semi strong efficient markets, and strong efficient markets. Although the efficient market hypothesis is only a descriptive concept, after empirical research by many scholars and experts, it has developed into a theoretical system, becoming one of the pillars of the modern securities market theoretical system, and has a very important position [8]. It has not only changed people's past understanding of the chaotic and irregular nature of the securities market, but also supported the widespread and comprehensive acceptance of securities portfolio theory, capital asset pricing theory, and option pricing theory through relevant empirical research results.

3.2. Securities Portfolio Model

The mean variance analysis method is Markowitz's portfolio theory [9]. In order to solve problems such as how to determine the size of risks in the market, how to maximize returns and minimize risks, he concluded that investors should diversify their investments by summarizing the probability distribution of investment losses and the degree of deviation of possible returns. Assuming that there are n types of risky securities available for investors to choose from in the market, and the yield vector is recorded as $R = (r_1, r_2, \dots, r_n)^T$, the portfolio vector is recorded as $W = (w_1, w_2, \dots, w_n)^T$, and the covariance matrix of the returns of the two types of securities is $V = (\sigma_{ij})_{n \times n}$, then the return of the

securities portfolio is $R_p = W^T R = \sum_{i=1}^n w_i r_i$, and the total risk is recorded as $\sigma_p^2 = W^T V W$. The optimal decision-making model for portfolio investment can be written as the following quadratic programming problem [10]:

$$\begin{cases} \min \sigma_p^2 = W^T V W \\ W^T e = 1 \\ E(R_p) = W^T \cdot E(R) \end{cases} \quad (e = (1, 1, \dots, 1)^T) \quad (1)$$

Markowitz's portfolio investment idea caters to the needs of investors to avoid risks, and is widely accepted by investors. It has obvious guiding significance for the development of financial mathematics.

3.3. Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) is a set of theories established by William Sharpe, John Lintner, Jack Treynor, and Jan Mossin based on securities portfolio theory [11].

The capital asset pricing model is based on the core assumption of investors' homogenization with different initial wealth and risk preferences. The model can be expressed as follows:

$$CAPM : R_i = \beta_i (R_m - R_f) + R_f \quad (2)$$

Including:

R_f : Risk free yield, generally refers to the one-year treasury bond interest rate;

R_i : the expected yield of the i -th security;

β_i : the β Coefficient of the i -th security;

R_m : expected return rate of market securities portfolio;

The above formula indicates that the return on risky assets consists of two parts, namely, the return on risk-free assets R_f and market risk compensation $(R_m - R_f)$. Risk assets can only be compensated for by systemic risk, and market portfolios have the lowest non systemic risk. Therefore, market portfolios are optimal portfolios. Market portfolio returns minus risk-free asset returns are expressed as the returns generated by assuming unit system risk. The capital asset pricing model is extremely concise and operable. It perfectly solves the risk management and measurement issues in financial investment decisions that are closely watched by market investors and academia. It has been widely used in investment risk analysis, stock return prediction, and many other issues. It is the core foundation of modern capital market equilibrium theory and plays a pivotal role in the field of financial mathematics.

3.4. Arbitrage Pricing Model

The core assumption of arbitrage pricing theory is that the market does not allow arbitrage opportunities, that is, the market price will be adjusted so that investors do not have opportunities to obtain benefits from investment without risk and capital. According to this theory, risk can be generated by multiple factors, not just a market factor. The expected return of a security has an approximate linear relationship with risk factors. The arbitrage pricing model can be expressed as [12]:

$$APT : E_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \dots + \lambda_k b_{ik} \quad (3)$$

Including:

E_i : expected yield of security i;

λ_0 :Expected return of securities under zero system risk;

λ_j : Risk premium for factor j;

b_{ij} : Sensitivity coefficient for factor j.

The APT model is a complex and diversified CAPM model, which establishes the relationship between securities returns and other factors in the macro economy in a broader sense. It develops the capital asset pricing model from a single factor model to a multi factor model, which is suitable for any combination of asset portfolios, and is more easily extended to the situation of multi period returns. Therefore, APT model has a broader significance in connotation and practicality, and together with CAPM model, it constitutes the core of modern capital market theory.

3.5. Option Pricing Model

The option pricing model, also known as the B-S model, believes that the price of options is closely related to current and future forecasts and will not change due to investor preferences. The B-S option pricing model can be used to formulate the prices of various financial derivatives, is an effective tool for the valuation of various derivatives, and can also provide an effective basis for risk prediction. The theory assumes that the price of a stock follows a lognormal distribution, there are no transaction costs, and there is no arbitrage opportunity in the market. The partial differential equation that must be satisfied for the price of a European call option that does not pay dividends at time t is as follows [9]:

$$PDE : \frac{\partial u}{\partial t} + rx \frac{\partial u}{\partial x} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 u}{\partial x^2} = ru$$

$$s.t \begin{cases} u(T, X) = (X - K)^+ = \max \{(X - K), 0\} \\ u(t, 0) = 0 \\ \frac{\partial u}{\partial x} = 1 \quad (x \rightarrow +\infty) \end{cases} \quad (4)$$

Including:

r : Risk free interest rate; x : Target stock price at time t;

K : The executive price of the stock at the time of T; X :Target stock price at time T;

σ^2 : Volatility of underlying stock price

Then solve the partial differential equation and obtain the option pricing formula:

$$u(t, x) = xN(d_+) - e^{-rT} KN(d_-) \quad (5)$$

Including:

$$d_{\pm} = \frac{\ln \frac{x}{k} + (r \pm \frac{1}{2} \sigma^2)(T-t)}{\sigma \sqrt{T-t}}, \quad N(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \quad (6)$$

As a very important trading tool in the financial derivatives market, options can effectively measure and manage volatility risk. Compared to other derivative tools, options are more sophisticated risk management tools, with unique functions and roles in risk management, risk measurement, and other aspects. As soon as the B-S model came out, it inspired the establishment of modern financial theory based on the principle of no arbitrage. Its development triggered a large number of studies, leading to a revolution in financial practice. Option pricing models provide beneficial guidance for financial innovation and are one of the main contents of financial mathematical theory.

4. Current Situation of Financial Market

In recent decades, financial mathematical theory has been continuously innovated and developed, not only having a direct impact on the innovation of financial instruments and the effective operation of financial markets, but also widely used in the investment decisions of companies, the evaluation of research and development projects, and the risk management of financial institutions. However, in the actual operation process of financial markets, there is often a large amount of randomness and volatility, especially in the case of large fluctuations in the financial market. Simply using theory cannot better explain the characteristics of market operation. In addition, the theory itself has certain limitations, leading to problems in the practical application of the theory.

4.1. Innovation Theories Emerge Endlessly

With the continuous improvement of mathematical theory, mathematics has become more widely used in financial theory, and scholars and experts have conducted more in-depth and comprehensive research on financial mathematics. For example, by directly introducing martingale theory into modern financial theory and using the concept of equivalent martingale measures to study the pricing of derivative securities, the results obtained not only can profoundly reveal the operating laws of the financial market, but also can provide a set of effective algorithms to solve the pricing and risk management problems of complex derivative financial products [13]; The stochastic optimal control theory developed by combining measurement theory and functional analysis methods has effectively overcome stochastic problems in finance, and has become an important theoretical basis for dealing with stochastic problems in financial economics theory [2]; The progress of information technology has also provided a good prerequisite for the innovative development of financial mathematics. The integration and supplementation of traditional financial methods with computational methods such as genetic algorithms, simulated annealing algorithms, artificial neural networks, and wavelet analysis have achieved good results in the fields of risk control and investment decision-making [13].

4.2. Existing Application Problems

Limited by the strict assumptions of theory, actual financial market conditions often contradict theory, resulting in limited practical use of theory. Secondly, there are many uncertain factors in the financial market, which often exhibit irregular and highly volatile characteristics. Simple financial mathematical models cannot effectively explain the phenomenon. At the same time, the imperfect market supervision and management mechanism will also weaken the application effect of financial mathematics. The non-standard standards and requirements for the use of financial mathematics in the market field cannot ensure the high-quality application of modern technology, and even generate analytical work problems, making it difficult to provide assistance for the efficient development and good progress of the financial industry and market. From a national perspective, an imperfect

financial mathematics system is also difficult to allow financial mathematics to play its due value in the financial market, and cannot effectively meet the development needs of the industry [5].

5. Conclusion

The characteristics of financial markets such as volatility, unexpected events, incomplete markets, and asymmetric information make it particularly difficult to analyze financial systems. Financial mathematics relies on complex databases to establish mathematical models with high accuracy, providing more accurate and scientific estimates and predictions, and promoting the in-depth development and progress of market reform and market analysis theory. If the rise of financial mathematics is a revolution in financial economics, the in-depth development and widespread application of financial mathematics is a historical necessity of the times and market development. With the continuous transformation of the financial market, financial mathematics will also greatly break through its theoretical constraints and application issues, bringing tremendous vitality and development opportunities to the financial economy.

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