Forecast Research on the Fluctuation Risk of Green Financial Market in China based on the GARCH-LSTM Mixed Model

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Abstract. With the rapid development of China's green financial market, there is also the problem of uncertain risk of fluctuation in the transaction price of green financial products. Therefore, it is imperative to give play to the role of price fluctuations in early warning of risks in green financial markets. Based on the closing price data of green stock market and green bond market from April 19, 2018 to April 18, 2023, this paper combines the traditional time series model and deep learning model to fit and forecast the yield volatility of green financial market, and quantifies the risk value of green financial market. The results show that: (1) Both the green stock market and the green bond market have the characteristics of leptokurtosis and fat-tail, volatility aggregation and long-term memory to varying degrees. (2) The volatility risk in the green financial market has obvious differentiation characteristics, and the volatility in the green stock market is significantly higher than that in the green bond market. (3) The prediction accuracy of LSTM long-term and short-term memory model is significantly higher than that of GARCH family model. The GARCH-LSTM mixed model can further improve the prediction accuracy. According to the research results, the combination of modern artificial intelligence algorithm and classical econometric theory can become an important means to identify the volatility risk of green financial market, and put forward meaningful suggestions for the risk management of green financial market.

Keywords: Green Financial Market; Financial Volatility Forecast; GARCH Family Model; LSTM Model; VaR.

1. Introduction

The 20^{th} National Congress of the Communist Party of China (CPC) inaugurated at 10: 00 a.m. on October 16, 2022 at the Great Hall of the People. As the party's Report to the 20^{th} CPC National Congress proposed: "We should speed up the green transformation of development mode, implement the comprehensive conservation strategy, develop green and low-carbon industries, advocate green consumption, and promote the formation of green and low-carbon production mode and lifestyle." Among them, developing green finance and giving full play to the role of finance in supporting green development are beneficial measures to realize green transformation of development mode and high-quality development of economy and finance.

In recent years, the development of China's green finance has initially achieved some milestone results, with a relatively complete green financial policy system and market system. However, at the same time, China's green financial market system is still imperfect, and no special green financial institution has been established. The green financial business is mainly carried out by the traditional commercial banks. In the process of green financial products trading, the price of non-green financial products is higher than that of green financial products, the research on price fluctuation and market risk is more mature, and the predictability is more certain, which makes it easier to obtain a larger profit. Therefore, green financial products may be greatly impacted, affecting the healthy development of the green financial industry.

Based on this, the research on the transaction price volatility of green financial products, effective prevention of large fluctuations in the market and giving full play to the early warning function of price volatility on the risks of green financial market have become the top priorities of the current research in the field of green finance. Regarding the definition of green finance, White and Labatt (2003) believed that green finance is a financial instrument that guides the flow of capital to the resource-saving and environmental protection industries and corresponding technological development to meet the financing needs of the environmental protection industries. Hu Angang and Zhou Shaojie (2014) believed that green finance is an indirect policy tool that calls on enterprises to focus on sustainable development and ecological environment protection, and in the meanwhile prevent and control major environmental risks, and promote the green development of industry and the implementation of green consumption concept. In conclusion, this paper holds that green finance refers to a series of financial policies and measures that utilize green financial instruments such as green stocks, green bonds and other green financial derivatives to provide financing needs and financial services for the ecological and environmental protection industries and promote the coordinated and sustainable development of the financial industry and environmental protection. The party's Report to the 20th CPC National Congress pointed out that many major problems still need to be solved to prevent financial risks. We should strengthen the financial stability guarantee system and guard against systemic risks. It can be seen that China has taken the financial risk into a broader consideration, but the research on green financial risk related to sustainable and high-quality development is very rare. Therefore, the importance of studying the volatility of green financial assets and quantifying market risk is evident.

In the theory of Investment Science, risk is defined as the uncertainty of asset returns, which is usually quantified by the variance of asset returns and the covariance between portfolio returns. One of the most important risks is market risk. Market risk refers to the uncertainty of interest rate, exchange rate, stock price and commodity price, which has a certain adverse impact on the enterprise to achieve its profit target. At present, the most important method that can be used to assess financial market risk should be Value at Risk, which is called VaR method. No matter foreign scholars or domestic experts, the research on VaR mainly focuses on the level of quantification and estimation. The estimation methods can be roughly divided into three categories, which are historical simulation method, variance and covariance analysis method and Monte Carlo simulation method. For example, Song Min, Xin Qiang and He Yinan (2020) selected the daily carbon trading data of five carbon emissions trading exchanges in China's carbon financial market as the research objects, and calculated the VaR values of the five exchanges under different confidence levels respectively by using GARCH (1, 1) model to conduct an empirical analysis of the risks in China's carbon financial trading market. Liu Hui, Yao Haixiang and Ma Qinghua (2017) believed that the VaR calculated by historical simulation method can effectively measure the market risk without changing the market volatility. Xie Heliang and Huang Qing (2017) introduced the GARCH model to characterize the volatility aggregation of financial data, and then introduced the Markov chain Monte Carlo method to overcome the estimation error caused by the parameter estimation constraints of the GARCH model, in order to solve the shortcomings of the Monte Carlo method in the calculation of VaR. Tian Chengshi (2008) took a more flexible measure of stock market risk into consideration: conditional VaR model.

When using VaR method, the prediction and estimation of the volatility of financial markets is the core part of the model. Financial market volatility refers to the volatility of financial asset prices, which is an important indicator to measure the risk level of financial assets. Time series analysis is the optimal solution for quantitative modeling of financial market risk in the period when the field of big data and artificial intelligence has not yet arisen. For the analysis of financial asset volatility, the most commonly used volatility model is the ARCH family model proposed by Engle (1982), which mainly includes ARCH-M model, TARCH and NARCH model. Bollerslev (1986) proposed the GARCH model, which considers that the variance of random disturbance term is not only related to the variance of lag disturbance term, but also related to lag disturbance term itself. With the deepening of research, GARCH family model has been continuously expanded: CGARCH, FIGARCH and multivariate GARCH model have been proposed. The domestic scholars' research on volatility model also presents a gradually deepening development trend. For example, Wang Jiani and Li Wenhao (2005) discussed in detail several commonly used models and measurement indicators for predicting volatility, and considered that the fitting effect of GARCH model was poor. Ma Yulin and Wang Xiquan (2007) compared the results of two kinds of VaR forecasts based on GARCH model and actual volatility model, and found that the forecast effect of VaR based on actual volatility is significantly better than that based on GARCH model. Liu Qing, Dai Jingyue and Yang Chao (2015) took the earnings data of the stock market as samples, and made an empirical comparative study on six common GARCH family structures. The results show that, compared with other GARCH family structures, the EGARCH model can better predict the fluctuation process of the return on assets. From this, we can see that the content of the GRACH family models have been continuously improved after a long period of development and evolution, forming a relatively mature theoretical system, which plays a significant role in the fitting and prediction of volatility. However, with the advent of the big data era, faced with massive data, stock market volatility often presents some fluctuations that are difficult to fit, explain and predict using GARCH family models, so the fitting effect is not good and the prediction error is too large, which affects the accuracy of volatility analysis and thus affects the effective estimation of financial market risk. Nonetheless, the fields of machine learning and artificial intelligence have developed from their rise to their present maturity, which provides us with a new way to realize the accurate prediction of volatility. Among them, the most representative is a series of in-depth learning algorithms headed by neural network. For example, Chen Weihua (2018) used in the field of high-frequency volatility prediction for the first time to make out-of-sample prediction of volatility using deep learning to improve the volatility prediction accuracy. The research found that LSTM model had the best prediction effect in the test model, and the prediction effect of the model tended to be stable with the increase of number of historical days.

Financial derivatives pricing and financial risk management have always been one of the core research directions of financial mathematics. The essence of these two research directions is the measurement and prediction of real volatility. In view of the above domestic and foreign research and the actual situation of the development of green finance, this paper aims to apply market risk research and volatility analysis to the green financial market system to fill the research gap related to green financial market. First of all, this paper chooses the green stock sub-market in the green financial market and the representative securities index in the green bond sub-market for statistical analysis to analyze the differences within the green financial market from a quantitative perspective. Secondly, this paper intends to fit and forecast the volatility of the domestic green financial securities market return rate, and applies the forecasted volatility to the VaR risk measurement model, which provides a certain reference for the further improvement of domestic green financial market risk management. Finally, on the quantitative method of volatility, this paper intends to combine the ARMA-GARCH family model with LSTM model, which is a less used deep learning method in previous volatility research, and take the green finance-related securities data as the research object, to fit and predict the volatility of financial time series, which provides a more direct, accurate and convincing prediction method for the analysis of volatility risk in green financial markets.

2. Theories and Hypotheses

The traditional financial time series will show some specific features, among which the most important features are shown in the following aspects: (1) Leptokurtosis and fat-tail: for the true distribution of financial asset returns, the interval near the average has higher probability distribution density than the standard normal distribution when compared with the tail interval with larger deviation from the average. (2) Volatility aggregation: the volatility of financial assets' return rates will show similar aggregation effect in a certain range, and a larger volatility will usually be accompanied by a larger volatility. (3) Long-term memory: as a stationary time series, the self-correlation function of long-term memory converges to zero at a slower rate and exhibits negative power exponential decay. Starting from the connection between the green financial market and the traditional financial market, Cai Yuping and Zhang Yuanpeng (2014) believed that compared with the traditional industry, the emerging green industry is more uncertain. As the green industry or the low-carbon industry requires a large amount of long-term capital investment, financial institutions are not motivated to carry out green finance and the green finance is at greater risk. Therefore, the first hypothesis in this paper is obtained:

Hypothesis 1: The volatility of green financial market yields will present leptokurtosis and fat-tail, volatility aggregation and long-term memory.

In the green financial market, the green bond market and the green stock market are the two most important components of the green financial system. According to the definition of the World Bank, green bonds are financial instruments that are specifically designed to provide financial support to enterprises or units that target ecological benefits or climate change benefits. The green stock index is a stock index that is calculated based on the green attributes of an enterprise, pays attention to its stock price and designs specific selection criteria. It is considered as a good indicator to measure the fluctuation of asset prices in the green stock market. As a result, the volatility of financial asset yields in the two markets may be different. This leads to the following hypotheses:

Hypothesis 2: The risk of price fluctuation of financial assets within the green financial market has different characteristics.

Hypothesis 3: The green stock market has more significant yield volatility characteristics than the green bond market.

After the above theoretical analysis, it is not difficult to see that the fluctuation of the market price of green financial assets will affect the enthusiasm of market participants, making green bonds and shares unable to effectively play the role of guiding and assisting sustainable development. In addition, the traditional econometric model represented by GARCH family model and the modern artificial intelligence algorithm represented by LSTM model have different emphases on the volatility forecast for the choice of forecast model. The GARCH model has formed a mature theoretical system in analyzing and predicting the volatility of the yield series, and can become a good tool for predicting volatility. Meanwhile, the combination of ARMA model and GARCH model to analyze the horizontal changes and fluctuations of the yield series can also form a complete description of the yield model. However, such traditional econometric models also have some defects. Due to the complex characteristics of non-linearity and volatility of financial time series data, theoretically using traditional econometric models alone may result in the defects of large non-linearity prediction error and inaccurate lag order. Therefore, combining GARCH model with other methods has become a hot topic in the research. In the meantime, deep learning is mostly a data-driven model, which can improve the accuracy of prediction to a great extent. However, in-depth learning model also weakens the significance of explanatory variables and is difficult to depict and describe the unique characteristics of financial data, resulting in the lack of rigorous theoretical explanation. In order to better combine deep learning with financial theory, this paper proposes to establish a GARCH-LSTM mixed model. Therefore, this paper puts forward the following hypotheses:

Hypothesis 4: LSTM long-term and short-term memory models have higher prediction accuracy than the traditional GARCH family model.

Hypothesis 5: The GARCH-LSTM mixed model can effectively combine the respective advantages of the two models to further improve the prediction accuracy.

3. Data Processing and Model Building

3.1 Data Sources and Sample Selection

The relevant data of green financial assets in this paper are derived from China Securities Index. The green stock index in this paper selects Shanghai and Shenzhen 300 Green Leading Stock Index (931037), which selects 100 stocks with the highest scores as index samples according to a series of green evaluation criteria. The green bond index used in this paper is the China Securities Exchange Green Bond AAA Index (931146), which consists of green bonds with credit rating of AAA and remaining maturity of more than one month. The data spans from April 19, 2018 to April 18, 2023, for a total of 1,215 data. At the same time, this paper uses the logarithmic return rate to conduct the relevant research, so as to make the closing price P_t at time t, and the corresponding daily return rate is r_t, which is defined as the following:

$$r_t = 100 \times \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

In the research of forecast of volatility σ_t, realized volatility is often used to compare with the forecast value of volatility to test the forecast precision of the model. Realized volatility is mainly estimated by using high-frequency data during the trading day. However, as the low-frequency data used in this paper cannot directly observe the volatility of financial assets' earnings, this paper estimates realized volatility based on a calculation method of realized volatility based on time window sliding technology introduced by Gao Tong and Zhu Hailong (2020). The specific definition is as the following:

$$RV_t = \sqrt{\frac{1}{n}\sum_{i=t-\frac{n}{2}}^{t+\frac{n}{2}}\left(r_t - \bar{r}\right)^2} \tag{2}$$

Where RV_t is the realized volatility, $\bar{r}$ is the average of daily yield r_t. In this paper, n=4 is taken, i.e. the realized volatility is equal to the standard deviation of daily yields on the t[th] trading day and a total of 5 trading days before and after that.

3.2 Construction of ARMA-GARCH Family Model

Autoregressive moving average model is a mixed model of autoregressive model and moving average model used to process stationary time series. The model is defined as follows:

$$r_t = \eta_0 + \sum_{i=1}^{m}\eta_i r_{t-i} + \varepsilon_t + \sum_{j=1}^{n}\gamma_j \varepsilon_{t-j} \tag{3}$$

Where, r_t represents the time series of returns on the price of green financial assets; and ε_t represents the residual perturbation term of the time series of returns on t[th] day. η_i is the coefficient of AR term and the order of hysteresis is m. γ_j is the MA term coefficient and the order of the hysteresis is n.

Generalized autoregressive conditional heteroscedasticity model, i.e. GARCH model, can better solve the heteroscedasticity phenomenon of green financial asset price series. The model is defined as follows:

$$\begin{cases} r_t = \mu_t + \varepsilon_t \\ \varepsilon_t \sim \mathrm{N}\left(0, \sigma_t^2\right) \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j \sigma_{t-j}^2 \end{cases} \tag{4}$$

Where, μ_t represents the time series mean value of the return rate of the price of green financial assets. ${\sigma_t}^2$ is the conditional variance of the yield time series. α_i is the ARCH term coefficient and the lag order is q. β_i is the GARCH term coefficient and the lag order is p. The ARMA model can be used as a mean value equation to form the ARMA-GARCH model.

The threshold GARCH model, i.e. the TGARCH model, takes into account the existence of different information in the green financial market that will have a positive and negative impact on the price of financial assets. The model is defined as follows:

$$\begin{cases} r_t = \mu_t + \varepsilon_t \\ \varepsilon_t \sim \mathrm{N}\left(0, \sigma_t^2\right) \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{r} \alpha_k^{'} \varepsilon_{t-k}^2 I_{t-k} \\ I_{t-k} = \begin{cases} 1, \varepsilon_{t-k} < 0 \\ 0, \varepsilon_{t-k} > 0 \end{cases} \end{cases} \tag{5}$$

Where I_t is a virtual variable, $\sum_{k=1}^{r} \alpha_k^{'} \varepsilon_{t-k}^2 I_{t-k}$ is an asymmetric effect term, which represents the disturbing effect of different information on the price of financial assets.

The exponential GARCH model, i.e. the EGACH model, takes $\ln\left(\sigma_t^2\right)$ into account in the volatility equation that the conditional variance varies to varying degrees when the random variance is positive or negative, which more accurately describes the volatility of the green financial asset price series. The model is defined as follows:

$$\begin{cases} r_t = \mu_t + \varepsilon_t \\ \varepsilon_t \sim \mathrm{N}\left(0, \sigma_t^2\right) \\ \ln\left(\sigma_t^2\right) = \alpha_0 + \sum_{k=1}^{r} \theta_k \frac{\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^{q} \alpha_i \frac{\left|\varepsilon_{t-i}\right|}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \ln\left(\sigma_{t-j}^2\right) \end{cases} \tag{6}$$

Where, the parameter θ_t measures the magnitude and direction of the impact of positive and negative market shocks on volatility.

3.3 Construction of Long-term and Short-term Memory Neural Network (LSTM) Model

LSTM long-term and short-term memory neural network is a gated RNN form. The method can effectively solve the problems of gradient explosion and disappearance existing in a common RNN network, and is widely applied to predicting time series data with long characteristic intervals and long delays. The analysis of the price series of green financial assets in this paper is a useful supplement. The LSTM model consists of an input layer, a hidden layer and an output layer, in which the circulation body of the hidden layer consists of memory cells, and a memory cell consists of a forgetting gate, an input gate and an output gate.

(1) Forgetting gate: LSTM needs to judge whether the information is retained when the data volume is relatively large. Therefore, the corresponding control function is set as follows:

$$f_t = \sigma\left(W_f\left[h_{t-1}, x_t\right] + b_f\right) \tag{7}$$

Where $f_t \in [0,1]$, h_{t-1} is the output variable, X_t is the input variable, W_f is the weight of the variable, b_f is the intercept term, and σ is a sigmoid function.

(2) Input gate: determining whether the information needs to be updated and the amount of updates by combining the information of the forgetting gate:

$$\begin{cases} i_t = \sigma\left(W_i\left[h_{t-1}, x_t\right] + b_i\right) \\ \widetilde{c_t} = \tanh\left(W_c\left[h_{t-1}, x_t\right] + b_c\right) \\ c_t = f_t * c_{t-1} + i_t * \tilde{c}_t \end{cases} \tag{8}$$

Where the i_t value is 0 or 1, W_i and W_c represent the corresponding weight, b_i and b_c represent the corresponding intercept term, $\tanh$ is the tangent excitation function, C_{t-1} is the cell state value, $\widetilde{c_t}$ is the saved input variable, and C_t is the new cell state value.

(3) Output gate: enter the output gate after the information selection is completed to determine the information to be output:

$$\begin{cases} o_t = \sigma\left(W_o\left[h_{t-1}, x_t\right] + b_o\right) \\ h_t = o_t * \tanh\left(c_t\right) \end{cases} \tag{9}$$

Where W_o and b_o represent the weight and intercept term of the output gate and h_t is the output value of the new cell.

3.4 Construction of GARCH-LSTM Combination Model

According to the above hypotheses, theoretically, the GARCH family model may have a large prediction error. Meanwhile, the LSTM model also lacks a specific description of financial attributes, so the GARCH-LSTM combination model is constructed as follows:

(1) Use the GARCH family model after determining the order to make out-of-sample forecast of volatility, the forecast value of volatility is obtained as $\widetilde{\sigma_t}$.

(2) Perform GARCH family model fitting on the intra-sample return rate data, and compare them with the original volatility estimation value to obtain a residual sequence.

(3) The residual sequence is taken as the input data of the LSTM model. An out-of-sample prediction value of the residual sequence ε_t is obtained.

(4) Superimpose the prediction results of the LSTM model and the GARCH model to obtain the prediction values of the GARCH-LSTM combined model: $\widehat{\sigma_t} = \widetilde{\sigma_t} + \varepsilon_t$.

3.5 Model Evaluation Index Construction

In this paper, four loss functions, namely, mean square error (MSE), mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE), are selected as the evaluation criteria for the model. Specific definitions are as follows:

$$\begin{gathered} MSE = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{\sigma}_t - RV_t\right)^2 \\ MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{\sigma}_t - RV_t\right| \\ RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{\sigma}_t - RV_t\right)^2} \\ MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\hat{\sigma}_t - RV_t}{RV_t}\right| \end{gathered} \tag{10}$$

Where, RV_t is the realized volatility, and $\hat{\sigma}_t$ is the model-fitted volatility. When the predicted value is completely consistent with the actual value, the values of the four loss functions are equal to 0. The greater the error, the greater the values of the four loss functions.

3.6 VaR Model based on GARCH-LSTM Mixed Model

This paper intends to use the parameter estimation method to calculate the VaR value of green financial asset price. The general normal distribution parameter estimation method assumes that the return rate is in normal distribution, and the value of VaR is equal to the standard deviation of the return rate multiplied by the quantile under the corresponding confidence interval:

$$VaR = Z_\alpha \sigma \sqrt{\Delta t} \tag{11}$$

Where Z_α is the quantile of the standard normal distribution under the confidence level α, σ is the standard deviation of the yield and Δt is the holding period.

From the hypotheses, it can be seen that the price fluctuations of green financial assets may not follow the normal distribution, which will lead to a large error in the results. Based on this, this paper proposes to use the volatility series predicted by GARCH-LSTM mixed model to calculate the VaR value. At the same time, this paper uses the failure frequency back test method to compare the calculated VaR value with the actual loss value. If the actual loss exceeds the VaR value, it is recorded as a failure. The first hypothesis is:

$$H_0 : p = p^* \quad vs \quad H_1 : p \neq p^*$$

Where p^*, at the confidence level c, leads to the expected probability of failure $p^* = 1 - c$, and p is the probability that the actual loss exceeds VaR. The VaR value of T days is calculated, and the actual failure happens for N days, then the failure probability is $p = \frac{N}{T}$. Now we only need to test whether the failure probability p is equal to the expected probability p^*, and we can have a more accurate judgment on the accuracy of the model. This paper quotes the likelihood ratio test statistic proposed by Kupiec:

$$LR = -2\ln\left(\frac{(1-p^*)^{T-N} p^{*N}}{\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}}\right) \tag{12}$$

LR obeys the χ^2 distribution with degree of freedom of 1, i.e. $LR \sim \chi^2(1)$. The actual number of failed days to obtain specific LR statistics for retest is brought in.

4. Empirical Analysis

4.1 Descriptive Statistics

Before the empirical analysis, this paper makes a descriptive statistical analysis of the data. The descriptive statistics of logarithmic yield and yield volatility of green stock index and green bond index are calculated, including mean, median, maximum, minimum, skewness, kurtosis and J-B statistics. The specific results are shown in Table 1.

Observing the sample range, from the yield point of view, the average logarithmic yield of green stock index and green bond index are both positive, which indicates that the indexes of both markets have an upward trend and the average yield of green bond index is higher than that of green stock index. The fluctuation upper and lower bound of the green stock index yield is significantly higher than that of the green bond index yield, and the standard deviation of the green stock index yield is larger than that of the green bond index yield, indicating that the fluctuation range of the green stock index yield is larger than that of the green bond index. From the perspective of volatility, the volatility of green stock index is higher than that of green bond index, regardless of the mean, maximum, minimum and standard deviation. From the above analysis, it can be concluded that the bond market has relatively fixed returns and relatively small risks, while the stock market has the characteristics of high returns and high risks. Therefore, it can be determined that there are differences in the green financial market. J-B statistics show that both the yield and volatility of the two markets do not meet the normal distribution. The green stock index returns have left-sided characteristics, and the data are more densely distributed at the left tail, while the green bond index returns have right-sided characteristics, and the data are more densely distributed at the right tail. The volatility of both green stock index and green bond index have right-sided phenomenon. It can be concluded that both market return series have leptokurtosis and fat-tail characteristics.

Table 1. Descriptive Statistics

	Shanghai and Shenzhen 300 Green Leading Stock Index		China Securities Exchange Green Bond AAA Index	
	Yield rate	Volatility	Yield rate	Volatility
Average/mean value	0.007715	0.501778	0.019712	0.019548
Maximum	6.452134	1.837334	0.546378	0.106071
Minimum value	-8.10385	0.093162	-0.33142	0.001955
Skewness	-0.28602	1.717964	0.215416	2.274582
Kurtosis	6.166114	7.621995	15.84357	9.662038
Standard deviation	1.244489	0.241244	0.055887	0.015568
J-B statistics	523.6134	1677.7704	8353.4840	3291.8437

4.2 Construction of GARCH Family Model

The ADF unit root test is used to test whether the yield series is stable. The test results are shown in Table 2:

Table 2. Unit Root Test Results

ADF statistical test	T statistics	P value
Green stock index	-35.8209	0.0001
Green bond index	-20.5773	0.0001
1% threshold	-2.57008	

As can be seen from the above table, the T statistic value of the ADF test of the green stock index and green bond index yield series is -35.8209 and -20.5773 respectively, which is lower than the threshold value of the 1% confidence interval. Therefore, the original hypothesis that the yield series has a unit root can be rejected, which proves that the green stock index and green bond index yield series are stationary. The green stock index and green bond index yield series can be regressed through time series. The lagging order of the yield series is determined by the autocorrelation (AFC) function and partial autocorrelation (PAFC) function distribution of the green stock index and green bond index yield series with respect to the lagging order of 20. The specific results are shown in Figure 1:

From the figure, we can judge that the daily logarithmic return series of the green stock index has weak autocorrelation, while the autocorrelation function and partial autocorrelation function of the green bond index return series both have obvious “tailing” phenomenon and have obvious autocorrelation. The results show that the two markets have different degrees of long-term memory. Therefore, the ARMA model is established for the average value equation of green stock index and green bond index yield series. According to the AIC minimum criterion, the average value equation of green stock index yield series is determined as ARMA (2,2) model and the average value equation of green bond index yield series is determined as ARMA (1,1) model. Looking at the distribution diagram of the autocorrelation (AFC) function and partial autocorrelation (PAFC) function of the 20-order lag of the square of the green stock index and green bond index yield sequence, we can see that the square of the logarithmic yield sequence is a correlation sequence, and the original sequence has a dependency, not a white noise sequence. The specific results are shown in Figure 2.

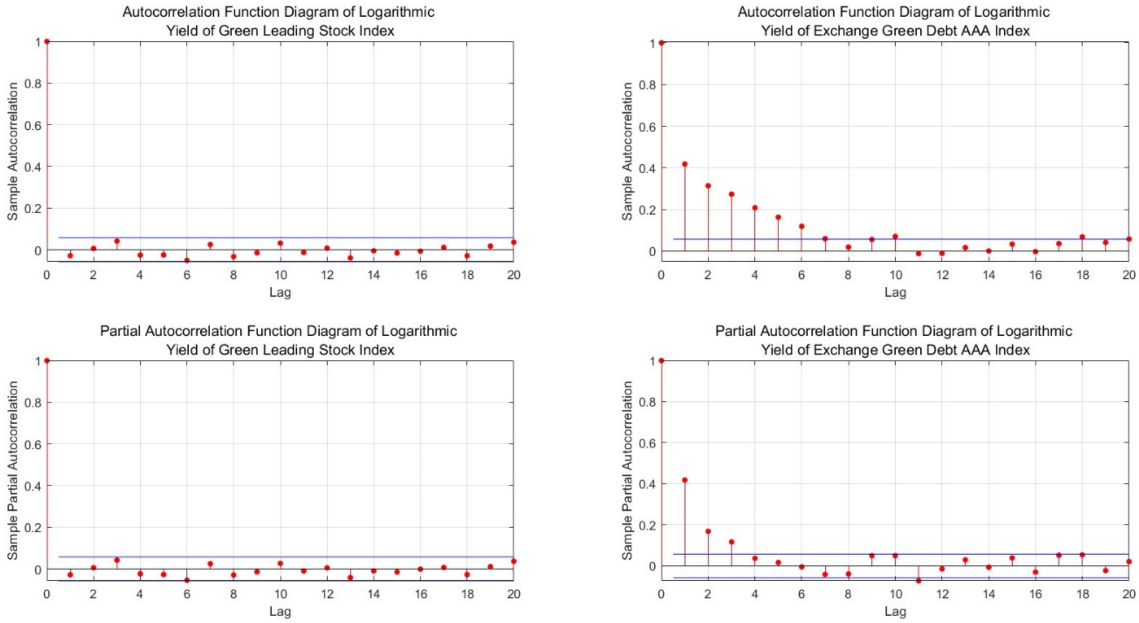

Figure 1. Distribution of Autocorrelation (AFC) Function and Partial Autocorrelation (PAFC) Function

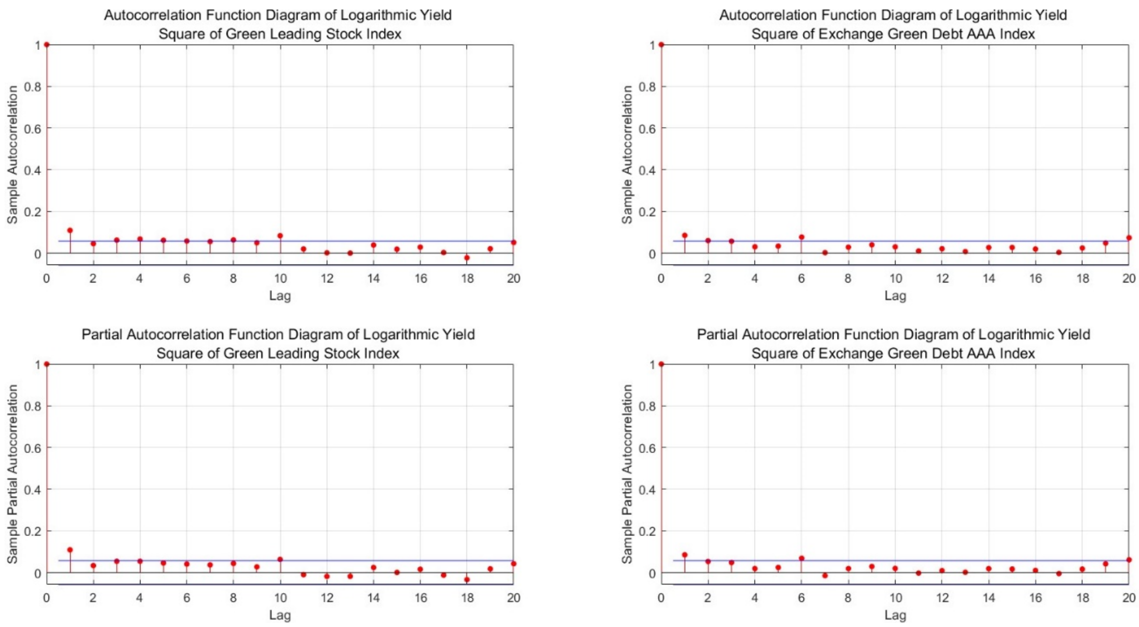

Figure 2. Distribution of Autocorrelation (AFC) Function and Partial Autocorrelation (PAFC) Function of Square Sequence

Next, we do LM test on the green stock index and green bond index yield series. The original assumption is that there is no ARCH effect until the degree of order P in the sequence, and whether the sequence has ARCH effect is judged by constructing F statistics to test whether the regression coefficients of the ARCH model are zero at the same time. The specific results are shown in Table 3.

Table 3. LM Test Results

LM test statistics	F statistics	P value
Green stock index	14.4354	0.0001
Green bond index	16.2691	0.00005
5% threshold	3.8415	

Through LM test, the F statistic value of the green stock index and green bond index yield series is 14.4354 and 16.2691 respectively, which is much higher than the threshold of 5% confidence interval. Therefore, it can be concluded that the green stock index and green bond index yield series have obvious ARCH effect. According to the above analysis, the green stock index and green bond index yield series are suitable for establishing GARCH family model. After trial fitting, according to the AIC minimum principle, both ARCH term and GARCH term are optimal with lag of order 1. Therefore, the following are models obtained with lag of order 1. The specific results are shown in Table 4 (the P values in brackets are the corresponding parameter estimates):

Table 4. Parameters of GARCH Family Model and Its Significance Test Results

Parameter	Green stock index			Green bond index		
	GARCH	TGARCH	EGARCH	GARCH	TGARCH	EGARCH
η_0	———	———	———	0.02243 (0.0000)	0.021207 (0.0000)	0.018465 (0.0000)
η_1	0.869404 (0.0000)	-0.21589 (0.0023)	1.048017 (0.0000)	0.783682 (0.0000)	0.827117 (0.0000)	0.811667 (0.0000)
η_2	0.090839 (0.006)	-0.90453 (0.0000)	-0.06879 (0.0749)	———	———	———
γ_1	-0.88265 (0.0000)	0.196776 (0.0068)	-1.06514 (0.0000)	-0.421214 (0.0000)	-0.47872	-0.469153 (0.0000)
γ_2	-0.09183 (0.0000)	0.896832 (0.0000)	0.078809 (0.0000)	———	———	———
α_0	0.09594 (0.0001)	0.086315 (0.0001)	-0.14416 (0.0000)	0.0000781 (0.0000)	0.000137 (0.0000)	-0.450516 (0.0000)
α_1	0.122256 (0.0000)	0.078902 (0.0000)	0.218779 (0.0000)	0.051522 (0.0000)	0.035104 (0.0001)	-0.025641 (0.0159)
β_1	0.822442 (0.0000)	0.839398 (0.0000)	0.934137 (0.0000)	0.917181 (0.0000)	0.869576 (0.0000)	0.943352 (0.0000)
$\alpha_1^{'}$	———	0.062021 (0.0001)	———	———	0.085991 (0.0000)	———
θ_1	———	———	-0.04805 (0.0000)	———	———	0.158683 (0.0000)

Table 5. ARCH Effect Test of Residual Sequence

ARCH effect test	Model	F statistics	P value	T*R2 statistics	P value
Green stock index	GARCH (1,1)	0.522	0.470	0.523	0.470
	TGARCH (1,1)	0.234	0.628	0.235	0.628
	EGARCH (1,1)	0.940	0.332	0.941	0.332
Green bond index	GARCH (1,1)	0.429	0.513	0.429	0.512
	TGARCH (1,1)	0.659	0.417	0.660	0.417
	EGARCH (1,1)	0.950	0.330	0.951	0.330

From the parameter estimation results, it can be seen that the model parameters passed the significance test, and then the ARCH effect test was performed on the residual sequence of the model. The results are shown in Table 5.

The P values are all greater than 0.05, which indicates that the model can remove the ARCH effect of the residual sequence well. The effect of volatility fitting is shown in Figure 3:

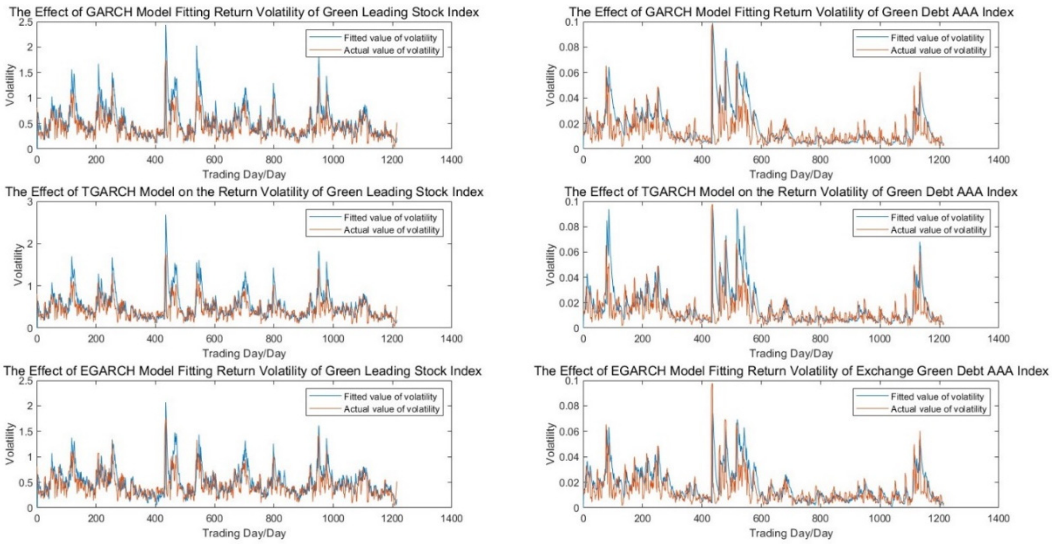

Figure 3. Effect of GARCH Family Model Fitting Return Volatility

In order to quantify the fitting effect of the model, the following four loss functions, namely, mean square error, average absolute error, root mean square error and average absolute percentage error, are calculated and evaluated comprehensively. The specific results are shown in Table 6:

Table 6. Fitting Effect Index of the Model

		MSE	MAE	RMSE	MAPE
	GARCH(1,1)	0.064810	0.172599	0.254577	0.461715
Green leading stock index	TGARCH(1,1)	0.067570	0.174318	0.259943	0.461518
	EGARCH(1,1)	0.048891	0.159621	0.259943	0.440206
	GARCH(1,1)	0.000202	0.008441	0.014206	0.815414
Exchange green debt AAA index	TGARCH(1,1)	0.000195	0.008244	0.013974	0.733838
	EGARCH(1,1)	0.000139	0.007656	0.011809	0.737782

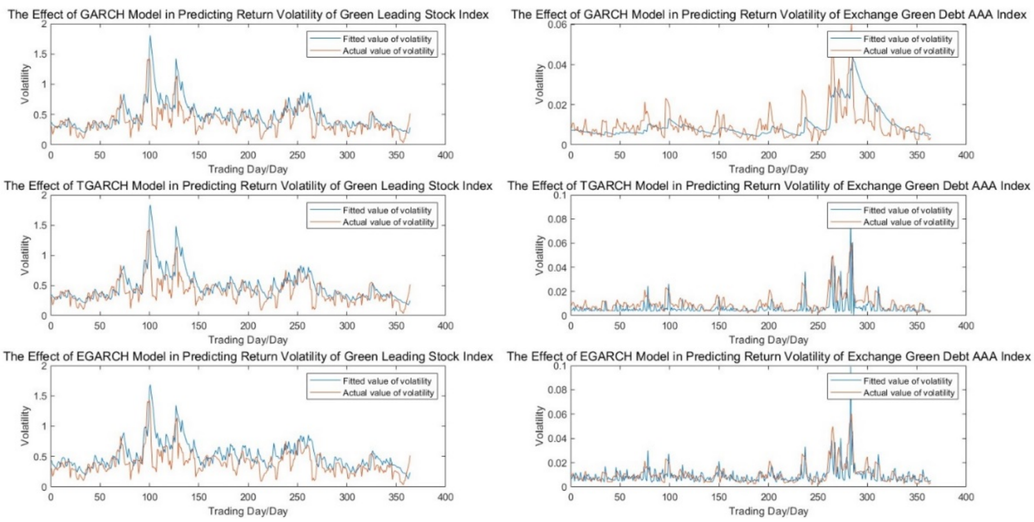

Figure 4. Effect of Out-of-sample Forecast of Return Volatility of GARCH Family Model

From the value of loss functions, it can be seen that EGACH (1, 1) is the best model for fitting the return volatility of green leading stock index and the return volatility of green debt AAA index. After that, the sample interval is divided into the inner interval and the outer interval according to the ratio of 7: 3. First, the GARCH family model is used to model the data within the sample, and then the data outside the sample are predicted. Then, the forecast effect is evaluated by comparing with the realized volatility. The forecast effect is shown in Figure 4:

In order to quantify the prediction effect of the model, four loss functions, namely, mean square error, average absolute error, root mean square error and average absolute percentage error, are calculated and evaluated comprehensively. The specific results are shown in Table 7:

Table 7. Forecast Effect Indicators of the Model

		MSE	MAE	RMSE	MAPE
Green leading stock index	GARCH(1,1)	0.040799	0.138530	0.201987	0.480162
	TGARCH(1,1)	0.046121	0.142246	0.214758	0.488276
	EGARCH(1,1)	0.045182	0.157120	0.214758	0.558300
Exchange green debt AAA index	GARCH(1,1)	0.000051	0.004595	0.007138	0.479555
	TGARCH(1,1)	0.000070	0.005017	0.008375	0.402895
	EGARCH(1,1)	0.000052	0.004462	0.007236	0.443567

According to the value of loss functions, GARCH (1, 1) is the best model for the out-of-sample forecast of the return volatility of green leading stock index. For the out-of-sample forecast of the return volatility of green debt AAA index, GARCH (1, 1) and EGARCH (1, 1) have both gains and losses, but the forecast effects are better than TGARCH (1, 1).

4.3 Construction of LSTM Model

We set the time step at 24. The LSTM neural network is composed of three hidden layers: input layer, three hidden layers and output layer. This paper divides the sample intervals according to the ratio of 7: 3. In terms of parameter selection, the LSTM neural network unit specified in this paper has 100 implicit units and 500 rounds of training. The gradient threshold value to 1 is set to prevent gradient explosion, the initial learning rate is set at 0.005. After 400 rounds of training, the learning rate was reduced by multiplying by a factor of 0.1. The results of prediction and fitting are shown in Figure 5 and Figure 6:

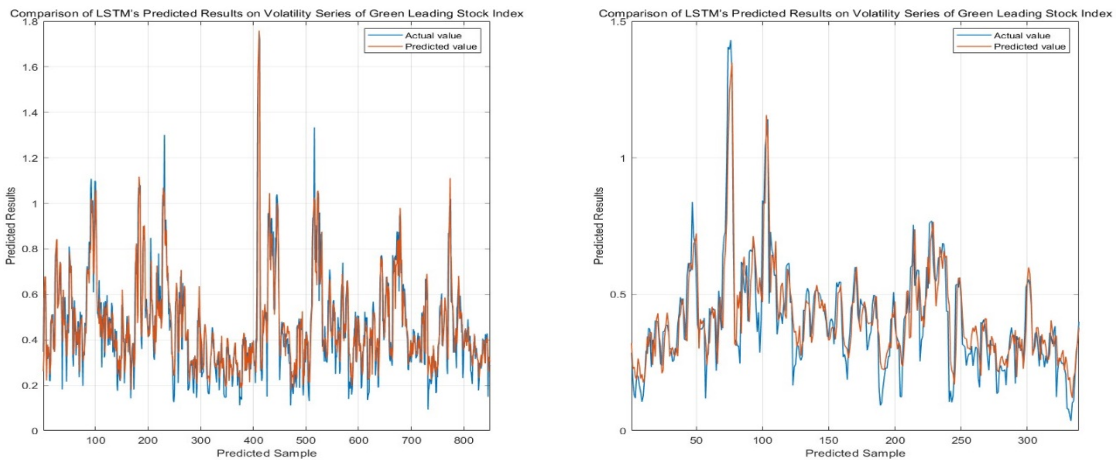

Figure 5. Effect of LSTM Model in Predicting Volatility of Green Stock Index

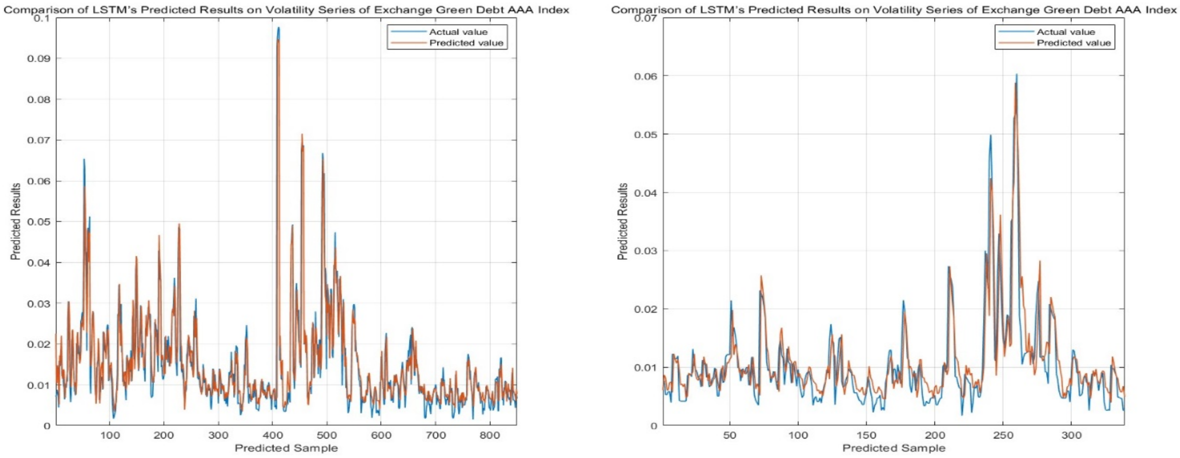

Figure 6. Effect of LSTM Model in Predicting Volatility of Green Bond Index

From the figure, it can be found that the volatility forecast curve formed by LSTM neural network modeling is close to the actual volatility unbiased estimation curve, and the change is also relatively consistent, which better reflects the true value of volatility. In order to quantify the prediction effect, four loss functions, namely, mean square error, average absolute error, root mean square error and average absolute percentage error, are calculated and the model is comprehensively evaluated. The specific results are shown in Table 8:

Table 8. Forecast Effect Indicators of LSTM Model

		MSE	MAE	RMSE	MAPE
Green leading stock index	Training set	0.009422	0.067920	0.097067	0.185452
	Test set	0.010358	0.084472	0.101775	0.265169
Exchange green debt AAA index	Training set	0.000028	0.002943	0.005294	0.276671
	Test set	0.000018	0.002671	0.004213	0.327327

4.4 Construction of GARCH-LSTM Mixed Model

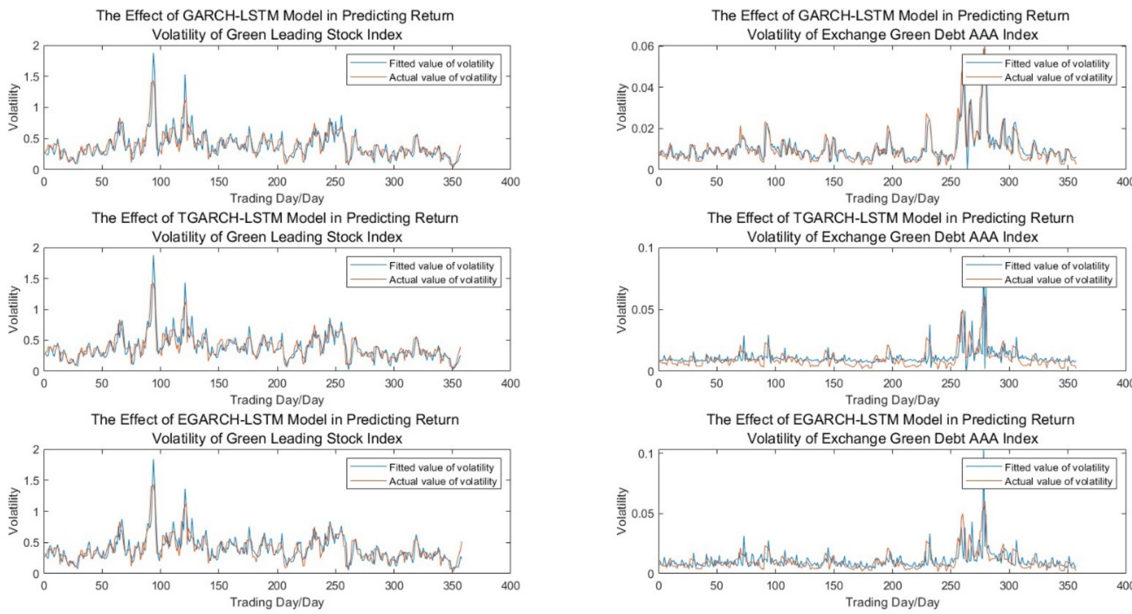

Figure 7. Effect of Out-of-sample Forecast of Return Volatility of GARCH-LSTM Mixed Model

Firstly, the volatility series is divided into intervals according to the ratio of 7: 3, and then the GARCH family model is used for prediction to obtain the predicted value $\widetilde{\sigma_t}$. Secondly, a GARCH family model is respectively established for fitting the training set volatility series to obtain a residual sequence between the training set volatility series and the realized volatility, an LSTM model is established, and the residual is taken as an input of the LSTM model to obtain an out-of-sample residual prediction sequence ε_t . Finally, the final out-of-sample volatility forecast sequence is $\widehat{\sigma_t} = \widetilde{\sigma_t} + \varepsilon_t$. The specific results are shown in Figure 7.

In order to quantify the prediction effect, four loss functions, namely, mean square error, average absolute error, root mean square error and average absolute percentage error, are calculated and the model is comprehensively evaluated. The specific results are shown in Table 9:

Table 9. Comparison of Forecast Effect Indicators of Each Model

		MSE	MAE	RMSE	MAPE
Green leading stock index	GARCH-LSTM	0.013857	0.084340	0.117718	0.236350
	TGARCH-LSTM	0.013860	0.084690	0.117730	0.237003
	EGARCH-LSTM	0.013500	0.085928	0.117730	0.253142
Exchange green debt AAA index	GARCH-LSTM	0.000017	0.002625	0.004134	0.310368
	TGARCH-LSTM	0.000056	0.004745	0.007492	0.564132
	EGARCH-LSTM	0.000051	0.004667	0.007140	0.517951

Finally, according to different models and the magnitude of the absolute value of the values, each indicator of the four loss functions is sorted, each indicator is given the same weight, and the comprehensive ranking of each indicator is obtained after weighting. The results are shown in Table 10:

Table 10. Comprehensive Ranking of Forecast Effect Indicators of Each Model

	Model	MSE ranking	MAE ranking	RMSE ranking	MAPE ranking	Comprehensive score	Ranking
Green leading stock index	GARCH(1,1)	5	5	5	5	20	5
	TGARCH(1,1)	7	6	6	6	25	6
	EGARCH(1,1)	6	7	6	7	26	7
	LSTM	1	2	1	4	8	2
	GARCH-LSTM	3	1	2	1	7	1
	TGARCH-LSTM	4	3	3	2	12	3
	EGARCH-LSTM	2	4	3	3	12	3
Exchange green debt AAA index	GARCH(1,1)	3	4	3	5	15	3
	TGARCH(1,1)	7	7	7	3	24	6
	EGARCH(1,1)	5	3	5	4	17	4
	LSTM	2	2	2	2	8	2
	GARCH-LSTM	1	1	1	1	4	1
	TGARCH-LSTM	6	6	6	7	25	7
	EGARCH-LSTM	3	5	4	6	18	5

By comparing the data in the table, it can be found that the LSTM model is better than the GARCH family model under the four loss functions for both the green leading stock index and the exchange green debt AAA index. Secondly, the GARCH-LSTM mixed model is generally better than the single model. From the results, no matter in the green leading stock index forecast or the exchange green debt AAA index forecast, the forecast effect of GARCH-LSTM model is better than the single use of LSTM model and GARCH model. GARCH model combined with LSTM model can produce better prediction effect on nonlinear random error in green stock and green bond data volatility. On the other hand, after LSTM combines with GARCH model, it learns the aggregation and long-memory characteristics in the volatility of green stock and green bond data. After LSTM combines with TGARCH and EGARCH model, it has the leverage effect set up in the model to simulate financial data, and the prediction effect is greatly improved. However, for the exchange's green debt AAA index, the prediction effects of LSTM combined with TGARCH and EGARCH model have been somewhat reduced. The reason for these situations may be that the volatility of each index changes differently. The volatility of green leading stock index changes frequently and the non-linear characteristics are more obvious. Therefore, the use of deep learning model has achieved better results. However, the volatility of the green debt AAA index on the exchange does not fluctuate as frequently and violently as the green leading stock index, so the GARCH family model can well present the volatility information, and after adding the LSTM model, it may easily fall into the over-fitting problem, thus affecting the prediction ability of the model.

4.5 Simulation Results of GARCH-LSTM-VaR Model

In this part, the volatility data predicted by GARCH-LSTM model with the best prediction effect is selected as the standard deviation of VaR model. The VaR values for 357 trading days are calculated at confidence levels of 95%, 97.5% and 99%, respectively. The specific calculation results are shown in Figure 8:

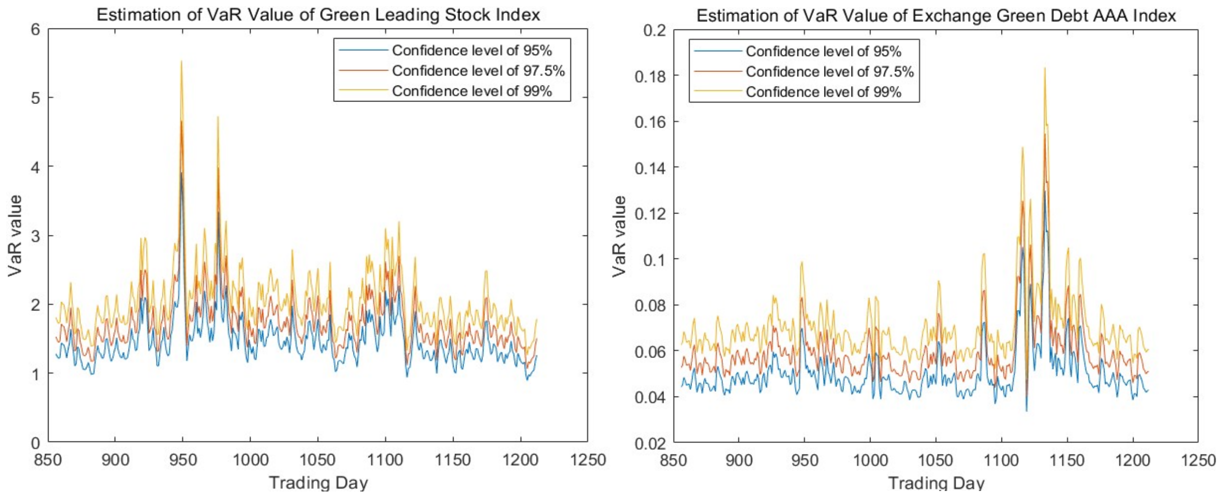

Figure 8. VaR Values at Different Confidence Levels

The following is a descriptive statistical analysis of the calculated VaR values, the results are shown in Table 11:

Table 11. Descriptive Statistical Analysis of VaR Values

	Different confidence levels	Average/mean value	Maximum	Minimum value	Standard deviation
Green leading stock index	95%	1.4588	3.9108	0.8937	0.3500
	97.50%	1.7383	4.6600	1.0649	0.4171
	99%	2.0632	5.5311	1.2639	0.4950
Exchange green debt AAA index	95%	0.0503	0.1297	0.0335	0.0117
	97.50%	0.0599	0.1545	0.0399	0.0139
	99%	0.0711	0.1834	0.0474	0.0165

Through the descriptive statistical analysis of the VaR value distribution and VaR value of the two markets, it can be concluded that the VaR value of the green leading stock index is generally higher than the exchange green debt AAA index, and the standard deviation of the VaR value of the green leading stock index is also significantly higher than the exchange green debt AAA index. It means that the green leading stock index is more exposed to market price volatility risk and the exchange green debt AAA index is less exposed to market price volatility risk. Next, it is compared with the actual log yield data, and the comparison result is shown in Figure 9:

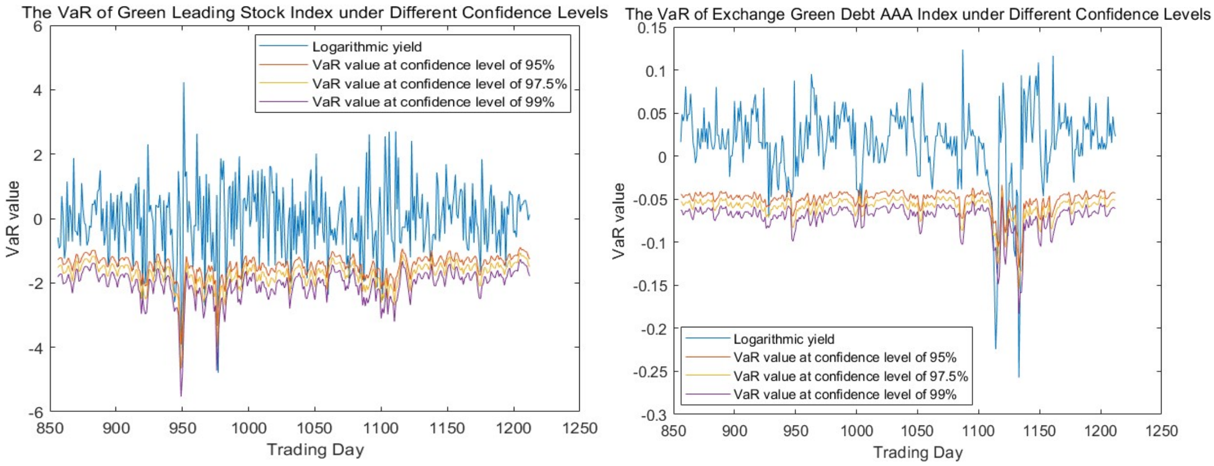

Figure 9. Comparison of VaR Values with Logarithmic Yield Data under Different Confidence Levels

Most of the VaR series curves are located below the logarithmic return series curves of the two green financial indexes, which indicates that it can well describe the daily fluctuation of the logarithmic return series of the green financial indexes. At the same time, the trend of the VaR series curve is similar to that of the logarithmic return series curve of the green financial index, indicating that there is a high correlation between its fluctuation changes, that is, the VaR series can well reflect the change trend of the risk of the return series. In order to ensure the effectiveness of the above VaR values, they are back tested through the Kupiec test. The results are shown in Table 12:

Table 12. Kupiec Test Results

	Green leading stock index			Exchange green debt AAA index		
Different confidence levels	95%	97.50%	99%	95%	97.50%	99%
Number of days for back test	357	357	357	357	357	357
Expected number of days of failure	17.85	8.925	3.57	17.85	8.925	3.57
Actual number of days of failure	25	15	4	13	11	7
Expected probability of failure	5%	2.50%	1%	5%	2.50%	1%
Actual probability of failure	9.80%	4.20%	1.10%	3.60%	3.00%	1.90%
LR statistics	2.695424	3.532466	0.050353	1.525621	0.461251	2.60022
P value	0.100637	0.060178	0.822449	0.216771	0.49704	0.106849

From the test results, it can be seen that the VaR value based on the GARCH-LSTM mixed model, the actual number of days of failure is higher than the expected number of days of failure under each confidence level, which indicates that the VaR value based on the GARCH-LSTM mixed model underestimates the actual risk to a certain extent, but the Kupiec test P values of the test results are all greater than 0.05, and the original hypothesis cannot be rejected, that is, the volatility estimated by the GARCH-LSTM mixed model is underestimated within a reasonable range, which proves that the estimation effect is better when the volatility value estimated by the GARCH-LSTM mixed model is added to the calculation of the VaR value.

5. Research Conclusion and Policy Suggestions

Firstly, based on the Shanghai and Shenzhen 300 Green Leading Stock Index and the China Securities Exchange Green Bond AAA Index data from April 19, 2018 to April 18, 2023, this paper establishes three types of GARCH models to analyze the volatility characteristics and forecast the volatility of domestic green financial markets. Secondly, the LSTM long-term and short-term memory neural network model is introduced and the LSTM-GARCH mixed model is established to further forecast the volatility series in the out-of-sample interval. After that, the forecast accuracy of volatility of different models is compared based on the four loss functions, and it is concluded that the application of depth learning method overcomes many constraints of traditional time series measurement model, which is beneficial to improve the forecast accuracy. Finally, the forecast results of LSTM-GARCH mixed model are applied to the calculation of value at risk. The research findings are:

Firstly, the statistical analysis of the green stock index and green bond index yield series shows that both the green stock market and the green bond market have the characteristics of leptokurtosis and fat-tail, volatility aggregation and varying degrees of autocorrelation, i.e. long-term memory, which proves the Hypothesis 1 proposed in this paper.

The significance of Hypothesis 1 lies in: the obvious leptokurtosis and fat-tail and volatility aggregation prove that the herd effect among participants in green financial market is very significant, i.e. the same trading strategy is adopted within a certain concentration time period, and simultaneously buying or selling results in a large fluctuation in the price and return rate of financial products. From the perspective of economics, this phenomenon reflects that market participants are worried about the uncertainty of the green financial market. The reason may be that market participants lack the protection of relevant laws and regulations and industry standards in the process of participating in market transactions, which makes them have a high risk awareness. In addition, the lag and asymmetry of information make it easy to blindly follow the trend of trading strategies.

Secondly, after analyzing the fluctuation of the green financial market price and return rate, fitting the GARCH family model, and calculating the VaR value, we can find that there are significant differences in various indicators and parameters between the green stock market and the green bond market, which indirectly indicates that there are obvious differences in the volatility risk within the green financial market, and proves the Hypothesis 2 proposed in this paper. In a word, the volatility of the return rate and VaR value of green bond market is smaller and has obvious range than that of green stock market, which proves the Hypothesis 3 proposed in this paper.

The practical significance of Hypothesis 2 and Hypothesis 3 lies in that different volatility characteristics are only superficial data responses, but in essence, the green stock market and the green bond market have different risk intensity and composition, which makes the green stock market and the green bond market show different characteristics in the degree of risk responses. From the perspective of the operating mechanism of the stock market and the bond market, the green stock market and the green bond market have different market trading mechanisms, which results in obvious differences in the price formation process of green financial products within their respective markets and further results in different market risk levels. On the other hand, for the domestic green financial market, the market construction of the green bond market is ahead of the market construction of the green stock market. The specific performance of the two markets is that they are different in their sensitivity to the introduction of government policies, and they also have different levels of response to relevant market information. Therefore, the performance of the two markets in dealing with risks also presents huge differences.

Thirdly, by applying GARCH family model to forecast, LSTM model to forecast and GARCH-LSTM mixed model to forecast, the forecast result of LSTM model is obviously better than that of GARCH family model, which proves the Hypothesis 4 proposed in this paper. On the other hand, although TGARCH-LSTM and EGARCH-LSTM models are not as effective as a single model in forecasting, GARCH-LSTM is still the best model in forecasting the volatility of green stock market and green bond market, which proves Hypothesis 5 proposed in this paper.

The significance of Hypothesis 4 and Hypothesis 5 is to provide a new modeling approach for the future research on volatility prediction of green financial markets. The GARCH model has a sound mathematical background and relatively strict financial logic. Applying it to prediction can obtain prediction results with strong interpretation ability, but in the process of parameter estimation, there are more or less certain subjective factors and error factors that cannot be explained by the model, so that the final results may be unsatisfactory. In contrast, the LSTM model can extract non-linear and unobservable effective information from green financial time series data to improve the prediction accuracy. However, the LSTM model also ignores the fact that the financial field is an integral part of social science, and its natural science prediction logic simulating human neural network cannot be completely nested in financial time series. Secondly, the LSTM model is likely to get the result of large deviation in the output data under the premise of the same data input, and its stability has also been criticized. Therefore, this paper combines the advantages of GARCH model in extracting linear and deterministic information with the advantages of LSTM model in extracting non-deterministic information in the residual series to improve the prediction accuracy of the model.

Finally, this paper also extracts the characteristics of asymmetric effects of green financial markets that were not considered in the previous Hypotheses. Judging from the parameter estimation results of TGARCH model, the asymmetric term coefficients of both markets are not 0, which means that there are significant asymmetric effects in green stock market and green bond market. Also, the asymmetric term coefficients of both markets are greater than 0, indicating that bad information has a greater impact on volatility. When there is a significant asymmetric effect, it means that the market is vulnerable to perceived factors and sensitive to policies and information, further amplifying the risk of price fluctuations in green financial markets.

Based on the above analysis and result demonstration, this paper intends to put forward some policy suggestions for the risk management of green financial market, the specific contents are as follows:

(1) Improve the legal, regulatory and policy framework for green financial markets and build a standardized green financial trading market system. Actively learn from the legal construction of green finance in developed countries and combine it with the actual situation in China. Taking the "People's Republic of China (PRC) Environmental Protection Law" as the cornerstone, comprehensively promote the construction of the environmental and ecological protection evaluation system of green financial market, the trading rules of green financial market and punishment methods that violate the principle of green development. At the same time, establish the access conditions of green financial market to curb the occurrence and spread of risks. A legal and regulatory system and a policy framework for a green financial market in line with China's national conditions have been formed.

(2) Improve the information disclosure system of the green financial market and promote the openness and sharing of information. Strengthening the information disclosure of green financial markets requires the establishment of public information disclosure scope and standards. Market participants who obtain funding sources through green stocks and bonds should disclose in detail the use of corporate funds, the implementation progress of corporate projects and the profitability of the company, etc., so as to reduce the complexity for participants in green financial markets to obtain information and improve participants' recognition of green financial markets, which is helpful to effectively prevent and resolve major financial risks.

(3) Actively developing innovative green financial instruments. China's green financial market is in the ascendant, the trading instruments are still mainly green bonds, the market structure is single, the green stock market has no mature operating conditions, and the ability to prevent risks is insufficient. Therefore, China's green financial market should flexibly use and regroup financial instruments such as the price, interest rate, term, method of repayment, means of transaction and risk factor of stocks and bonds, create new green finance tools, and vigorously promote the development of green financial derivatives. Also, it should enrich the asset structure of the green financial market, which is beneficial to preventing the risk of price fluctuation in the green financial market.

This paper is still limited to a certain extent and is expected to be improved in future research. First of all, the green stock index and green bond index selected in this paper can represent the green financial market to a certain extent, but it does not guarantee that other indexes and segments can reach the conclusion of this paper. China's green financial market also contains many rich green financial derivatives which can be used for volatility analysis and pricing research. The applicability of the conclusion is not demonstrated in this paper. Secondly, the data in this paper are selected from low-frequency time series data. The estimation of realized volatility using low-frequency data may have the possibility of incomplete information and large error. In the following research, this method can be applied to high-frequency data, and its prediction effect will be more convincing. Finally, this paper applies only one deep learning algorithm to the mixed model, and the subsequent research can completely apply other machine learning and deep learning models to the same prediction research method in this paper. It is hoped that this paper can provide a useful perspective for the study of volatility and value at risk of green financial products.

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